Summary of Formulas and Equations

Note: The order of the formulas follows the order of the chapters.

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Equation</th>
<th>Translation Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Translation</td>
<td>$y = f(x) + k$</td>
<td>If $k &gt; 0$, translate upward $k$ units.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If $k &lt; 0$, translate downward $k$ units.</td>
</tr>
<tr>
<td>Horizontal Translation</td>
<td>$y = f(x - h)$</td>
<td>If $h &gt; 0$, translate right $h$ units.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If $h &lt; 0$, translate left $h$ units.</td>
</tr>
<tr>
<td>Vertical Stretch</td>
<td>$y = af(x)$</td>
<td>If $a &gt; 1$, expand vertically by a factor of $a$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If $0 &lt; a &lt; 1$, compress vertically by a factor of $a$.</td>
</tr>
<tr>
<td>Horizontal Stretch</td>
<td>$y = f(kx)$</td>
<td>If $k &gt; 1$, compress horizontally by a factor of $\frac{1}{k}$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If $0 &lt; k &lt; 1$, expand horizontally by a factor of $\frac{1}{k}$.</td>
</tr>
</tbody>
</table>

Reflections

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected in the $x$-axis.

The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected in the $y$-axis.

The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

Trigonometry of Right Triangles

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
Trigonometric Ratios of Angles

\[ \sin \theta = \frac{y}{r} \]
\[ \cos \theta = \frac{x}{r} \]
\[ \tan \theta = \frac{y}{x} \]

The Sine Law and the Cosine Law

**Sine Law**
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

**Cosine Law**
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
b^2 = a^2 + c^2 - 2ac \cos B
\]
\[
c^2 = a^2 + b^2 - 2ab \cos C
\]

Special Triangles

- **45°-45°-90°**
  - 1:1:1
  - \( \sqrt{2} \)\(^2\) = 2

- **30°-60°-90°**
  - 2:1:2
  - \( \sqrt{3} \)\(^2\) = 3

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Special Values of Trigonometric Functions

<table>
<thead>
<tr>
<th>θ (degrees)</th>
<th>θ (radians)</th>
<th>sinθ</th>
<th>cosθ</th>
<th>tanθ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>π/6</td>
<td>1/2</td>
<td>√3/2</td>
<td>1/√3</td>
</tr>
<tr>
<td>45°</td>
<td>π/4</td>
<td>1/√2</td>
<td>1/√2</td>
<td>1</td>
</tr>
<tr>
<td>60°</td>
<td>π/3</td>
<td>√3/2</td>
<td>1/2</td>
<td>√3</td>
</tr>
<tr>
<td>90°</td>
<td>π/2</td>
<td>1</td>
<td>0</td>
<td>not defined</td>
</tr>
<tr>
<td>180°</td>
<td>π</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>270°</td>
<td>3π/2</td>
<td>-1</td>
<td>0</td>
<td>not defined</td>
</tr>
<tr>
<td>360°</td>
<td>2π</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Trigonometric Identities

Quotient Identity

\[ \tan x = \frac{\sin x}{\cos x} \]

Pythagorean Identity

\[ \sin^2 x + \cos^2 x = 1 \]
**Trigonometric Functions**

\[ y = a \sin k(x - d) + c \]
\[ y = a \cos k(x - d) + c \]

If \( a > 1 \), the graph is expanded vertically by a factor of \( a \).
If \( 0 < a < 1 \), the graph is compressed vertically by a factor of \( a \).
The amplitude is \( |a| \).

If \( k > 1 \), the graph is compressed horizontally by a factor of \( \frac{1}{k} \).
If \( 0 < k < 1 \), the graph is expanded horizontally by a factor of \( \frac{1}{k} \).

If \( d > 0 \), the graph is translated to the right \( d \) units.
If \( d < 0 \), the graph is translated to the left \( d \) units.

If \( c > 0 \), the graph is translated upward \( c \) units.
If \( c < 0 \), the graph is translated downward \( c \) units.

**Sequences and Series**

**Arithmetic**

\[ a, a + d, a + 2d, a + 3d, \ldots \]
\[ t_n = a + (n - 1)d \]
\[ S_n = \frac{n}{2} [2a + (n - 1)d] \]
\[ S_n = \frac{n}{2} (a + t_n) \]

**Geometric**

\[ a, ar, ar^2, ar^3, \ldots \]
\[ t_n = ar^{n-1} \]
\[ S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1 \]
Finance

Compound Interest

\[ A = P (1 + i)^n \]

A is the amount at the end of the time for the investment or loan, P is the principal invested, i is the interest rate per compounding period, and n is the number of compounding periods.

Present Value

\[ PV = \frac{A}{(1 + i)^n} \quad \text{or} \quad PV = A(1 + i)^{-n} \]

PV is the present value, A is the amount at the end of the investment, i is the interest rate per compounding period, and n is the number of compounding periods.

Amount of an Annuity

\[ A = R \left[ \frac{(1 + i)^n - 1}{i} \right] \]

A is the amount at the time of the last investment, R is the payment made at the end of each compounding period, n is the number of compounding periods, and i is the interest rate per compounding period.

Present Value of an Annuity

\[ PV = R \left[ 1 - (1 + i)^{-n} \right] \]

PV is the present value, R is the payment made at the end of each compounding period, n is the number of compounding periods, and i is the interest rate per compounding period.

Length and Midpoint Formulas

Length of a line segment joining \((x_1, y_1)\) and \((x_2, y_2)\):

\[ l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Midpoint of a line segment joining \((x_1, y_1)\) and \((x_2, y_2)\):

\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
**Equations of Lines**

Slope of a line: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

Point-slope form: \( y - y_1 = m(x - x_1) \)

Slope and y-intercept form: \( y = mx + b \)

Standard form: \( Ax + By + C = 0 \)

**Conic Sections**

**Circle**

\( x^2 + y^2 = r^2 \)

centre \((0, 0)\), radius \(r\)

\( (x - h)^2 + (y - k)^2 = r^2 \)

centre \((h, k)\), radius \(r\)

**Ellipse**

\( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), \( a > b > 0 \)

\( a^2 = b^2 + c^2 \)
\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad a > b > 0
\]
\[
a^2 = b^2 + c^2
\]

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1, \quad a > b > 0
\]
\[
a^2 = b^2 + c^2
\]

\[
\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1, \quad a > b > 0
\]
\[
a^2 = b^2 + c^2
\]
Hyperbola

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

\[a^2 + b^2 = c^2\]
\[
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \\
a^2 + b^2 = c^2
\]
Parabola

\[ y = \frac{1}{4p} x^2 \]

\[ x = \frac{1}{4p} y^2 \]

\[ y - k = \frac{1}{4p} (x - h)^2 \]

\[ x - h = \frac{1}{4p} (y - k)^2 \]