1.1 Reviewing the Exponent Laws

Investigate & Inquire

An order of magnitude is an approximate size of a quantity, expressed as a power of 10.

The table shows some speeds in metres per second, expressed to the nearest order of magnitude.

<table>
<thead>
<tr>
<th>Entity</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light (in space)</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Sound (in air)</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Horse (galloping)</td>
<td>$10^1$</td>
</tr>
<tr>
<td>Human (walking)</td>
<td>$10^0$</td>
</tr>
<tr>
<td>Garden snail</td>
<td>$10^{-3}$</td>
</tr>
</tbody>
</table>

1. Express $10^0$ metres per second in standard form.

2. Use division to determine, to the nearest order of magnitude, how many times as fast
   a) light is as sound
   b) a horse is as a snail

3. Write the rule you used to divide two powers of 10.

4. To the nearest order of magnitude, the moon orbits the Earth $10^6$ times as fast as a snail can travel. Use multiplication to express the speed of the moon in metres per second, to the nearest order of magnitude.

5. Write the rule you used to multiply two powers of 10.

The following summary shows the exponent laws for integral exponents.

Exponent Law for Multiplication

$3^2 \times 3^4 = (3 \times 3)(3 \times 3 \times 3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$a^m \times a^n = (a \times a \times \ldots \times a)(a \times a \times \ldots \times a)$

$m$ factors $n$ factors

$a \times a \times a \times \ldots \times a$

$m + n$ factors

$a^m \times a^n = a^{m+n}$
Exponent Law for Division

\[
\frac{6^5}{6^2} = \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6} = 6 \times 6 \times 6 = 6^3
\]

\[
\frac{a^m}{a^n} = \frac{a \times a \times a \ldots \times a}{a \times a \ldots \times a}, \quad a \neq 0
\]

\[
= a \times a \times a \times \ldots \times a
\]

\[
= a^{m-n}
\]

Power Law

\[
(5^3)^3 = (5 \times 5)(5 \times 5)(5 \times 5)
\]

\[
= 5 \times 5 \times 5 \times 5 \times 5 \times 5
\]

\[
= 5^6
\]

\[
(a^n)^m = (a \times a \times \ldots \times a)^m
\]

\[
= (a \times a \times \ldots \times a) \times (a \times a \times \ldots \times a) \times \ldots \times (a \times a \times \ldots \times a)
\]

\[
= a \times a \times a \times \ldots \times a
\]

\[
= a^{m \times n}
\]

Power of a Product

\[
(5 \times 2)^3 = (5 \times 2)(5 \times 2)(5 \times 2)
\]

\[
= 5 \times 5 \times 5 \times 2 \times 2 \times 2
\]

\[
= 5^3 \times 2^3
\]

\[
(ab)^n = (ab) \times (ab) \times \ldots \times (ab)
\]

\[
= (a \times a \times \ldots \times a) \times (b \times b \times \ldots \times b)
\]

\[
= a^m \times b^n
\]

Power of a Quotient

\[
\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}
\]

\[
= \frac{2 \times 2 \times 2}{5 \times 5 \times 5}
\]

\[
= \frac{2^3}{5^3}
\]

\[
\left(\frac{a}{b}\right)^m = \frac{a}{b} \times \frac{a}{b} \times \ldots \times \frac{a}{b}
\]

\[
= \frac{a \times a \times \ldots \times a}{b \times b \times \ldots \times b}
\]

\[
= \frac{a^m}{b^m} \quad b \neq 0
\]
A **power** is an expression in the form \(a^m\). The exponent laws can be used to simplify expressions with powers.

### Example 1  Simplifying Expressions With Powers

Simplify.

a) \((3a^2b)(-2a^3b^2)\)  
b) \((m^3)^4\)  
c) \((-4p^3q^2)^3\)

#### Solution

a) \((3a^2b)(-2a^3b^2) = 3 \times (-2) \times a^2 \times a^3 \times b \times b^2 = -6a^5b^3\)

b) \((m^3)^4 = m^{3 \times 4} = m^{12}\)

c) \((-4p^3q^2)^3 = (-4)^3 \times (p^3)^3 \times (q^2)^3 = -64p^9q^6\)

### Example 2  Simplifying a Power of a Quotient

Simplify \(\left(\frac{6x^3y^3}{8y^4}\right)^2\).

#### Solution 1

Use the power of a quotient law first.

\[
\left(\frac{6x^3y^3}{8y^4}\right)^2 = \frac{(6)^2(x^3)^2(y^3)^2}{(8)^2(y^4)^2}
\]

\[
= \frac{36x^6y^6}{64y^8}
\]

\[
= \frac{9x^6}{16y^2}
\]

#### Solution 2

Simplify the quotient first.

\[
\left(\frac{6x^3y^3}{8y^4}\right)^2 = \left(\frac{3x^3}{4y}\right)^2
\]

\[
= \frac{(3)^2(x^3)^2}{(4)^2(y)^2}
\]

\[
= \frac{9x^6}{16y^2}
\]

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The following summarizes the rules for zero and negative exponents.

**Zero Exponent**

\[
\frac{2^3}{2^3} = 2^{3-3} = 2^0 = 1
\]

but \( \frac{2^3}{2^3} = 1 \)

so \( 2^0 = 1 \)

Note that \( 0^0 \) is not defined.

**Negative Exponents**

\[
2^3 \times 2^{-3} = 2^{3+(-3)} = 2^0 = 1
\]

so \( 2^3 \times 2^{-3} = 1 \)

\[
\frac{2^2 \times 2^{-3}}{2^3} = \frac{1}{2^3}
\]

Divide both sides by \( 2^3 \).

\[
2^{-3} = \frac{1}{2^3}
\]

Similarly, if \( a \neq 0 \), \( a^{-m} = \frac{1}{a^m} \)

**Example 3  Simplifying Expressions With Negative Exponents**

Simplify \( \frac{-6x^2y(-9x^{-5}y^{-2})}{3x^2y^{-4}} \). Express the answer with positive exponents.

**Solution**

\[
\frac{(-6x^2y)(-9x^{-5}y^{-2})}{3x^2y^{-4}} = \frac{54x^{-7}y^{-1}}{3x^2y^{-4}} = 18x^{-9}y^3 = \frac{18y^3}{x^9}
\]

**Example 4  Evaluating Expressions With Zero and Negative Exponents**

Evaluate.

a) \( \left( \frac{3}{4} \right)^{-2} \)  

b) \( \frac{(-6)^0}{2^{-3}} \)  

c) \( 2^{-4} + 2^{-6} \)

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**Solution 1** Paper-and-Pencil Method

a) \( \left( \frac{3}{4} \right)^{-2} = \left( \frac{3}{4} \right)^{-2} = \frac{1}{9} \cdot \frac{16}{16} = \frac{16}{9} \)

b) \( \frac{(-6)^0}{2^{-3}} = \frac{1}{2^3} \)

c) \( \frac{2^{-4} + 2^{-6}}{2^{-3}} = \frac{\frac{1}{2^4} + \frac{1}{2^6}}{\frac{1}{2^3}} = \frac{2^2 + 1}{2^6} \cdot \frac{2^3}{1} = \frac{5}{8} \)

**Solution 2** Graphing-Calculator Method

The first answer given by a graphing calculator may be a decimal. If necessary, convert the decimal to a fraction using the Frac function.

a) \( (\frac{3}{4})^{-2} = \frac{1}{9} \cdot \frac{16}{16} = \frac{16}{9} \)

b) \( (-6)^0 / 2^{-3} = \frac{1}{2^3} \)

c) \( \frac{2^{-4} + 2^{-6}}{2^{-3}} = \frac{\frac{1}{2^4} + \frac{1}{2^6}}{\frac{1}{2^3}} = \frac{2^2 + 1}{2^6} \cdot \frac{2^3}{1} = \frac{5}{8} \)

**Key Concepts**

- Exponent law for multiplication: \( a^m \times a^n = a^{m+n} \)
- Exponent law for division: \( a^m \div a^n = a^{m-n} \)
- Power law: \( (a^m)^n = a^{mn} \)
- Power of a product law: \( (ab)^m = a^m b^m \)
- Power of a quotient law: \( \left( \frac{a}{b} \right)^m = \frac{a^m}{b^m} \)
- Zero exponent property: \( a^0 = 1 \)
- Negative exponent property: \( a^{-m} = \frac{1}{a^m} \) and \( \frac{1}{a^{-m}} = a^m \)
Communicate Your Understanding
1. Describe how you would simplify \((-4x^2y^3)(3xy^4)\).
2. Describe how you would evaluate \(\frac{3^{-2}}{3^{-1} + 3^{-2}}\) using paper and pencil.
3. What is the value of \(0^4\)? Explain.
4. Explain why \(a \neq 0\) for the negative exponent property.

Practice

A
1. Express as a power of 2.
   a) \(2^4 \times 2^3\)   b) \(2^6 \div 2^2\)   c) \((2^4)^3\)
   d) \(2 \times 2^2\)   e) \(2^3 \times 2^m\)   f) \(2^7 \div 2^1\)
   g) \(2^4 \div 2^4\)   h) \((2^3)^y\)   i) \(2^{-3} \times 2^4\)
   j) \(2^{-2} \div 2^{-5}\)
2. Evaluate.
   a) \(3^{-2}\)   b) \(5^0\)   c) \((-2)^{-4}\)
   d) \((2^{-1})^2\)   e) \(-(3)^0\)   f) \(\frac{1}{5^2}\)
   g) \(\frac{1}{(-4)^{-1}}\)   h) \(-(2^3)^{-2}\)
3. Simplify. Express each answer with positive exponents.
   a) \(a^4 \times a^3\)   b) \((m^6)(m^2)\)
   c) \(b^5 \times b^6 \times b\)   d) \(a \times b^2 \times a^4\)
   e) \((x^2)(y)(x^4)(x^5)\)   f) \((x^3)(x^{-5})\)
   g) \(m^{-4} \times m^{-3}\)   h) \(y^{-1} \times y^{-3} \times y^2\)
   i) \(a^5 \times a^0\)   j) \((a^{-3})(b^{-2})(a^2)\)
4. Simplify. Express each answer with positive exponents.
   a) \(x^6 \div x^2\)   b) \(m^7 \div m\)   c) \(t^4 \div t^{-2}\)
   d) \(y^{-5} \div y^{-3}\)   e) \(m^4 \div m^0\)   f) \(t^0 \div t^{-5}\)
5. Simplify. Express each answer with positive exponents.
   a) \((x^2)^2\)   b) \((a^3b^3)^4\)   c) \((x^3)^{-1}\)
   d) \((t^4)^0\)   e) \((a^{-1}b^2)^{-2}\)   f) \((x^2y^3)^{-3}\)
6. Simplify. Express each answer with positive exponents.
   a) \(\left(\frac{x^2}{2}\right)^3\)
   b) \(\left(\frac{a^4}{b}\right)^4\)
   c) \(\left(\frac{x^2}{y^3}\right)^5\)
   d) \(\left(\frac{x}{3}\right)^{-1}\)
   e) \(\left(\frac{a^2}{b^3}\right)^{-2}\)
7. Simplify. Express each answer with positive exponents.
   a) \(5m^4 \times 3m^2\)   b) \((4ab^4)(-5a^3b^7)\)
   c) \(5a(-2ab^2)(-3b^3)\)   d) \((-6m^3n^2)(-4mn^5)\)
   e) \((7x^2)(6x^{-2})\)   f) \((3x^{-2}y^2)(-2x^2y^{-3})\)
   g) \((-6a^{-1}b^2)(-a^{-3}b^{-4})\)   h) \((-10x^3) \div (-2x)\)
   i) \(\frac{45a^2b^4}{9ab^2}\)   j) \(\frac{\frac{4a^4b^3}{15a^2b^6}}{\frac{-a^3}{ab^6} \times \frac{-a^3}{b^2}}\)
   k) \(\frac{3ab^3 \times 10a^4b^2}{15a^5b^6}\)   l) \(-54a^5b^{-7}\)
   m) \((35x^5) \div (5x^{-3})\)   n) \(-6a^2b^{-3}\)
   o) \((-6m^{-4}n^2) \div (2m^{-1}n^{-6})\)
   p) \(-2x^3y)(-12x^{-4}y^2)\)
   q) \(6xy^{-3}\)
8. Simplify. Express each answer with positive exponents.
   a) \(2m^3\)\(^2\)   b) \((-4x^3)^3\)
   c) \((-3m^2n^2)^2\)   d) \((5c^{-3}d^3)^{-2}\)
   e) \((2a^{-3}b^2)^{-3}\)   f) \((-3x^2y^{-2})^{-4}\)
Apply, Solve, Communicate

10. History The Burgess Shale in British Columbia's Yoho National Park contains one of the world's best fossil collections. The fossils are about $5.4 \times 10^8$ years old. This is about $4.5 \times 10^4$ times as old as the first known human settlement in British Columbia. About how many years ago did humans first settle in British Columbia?

11. Application A piece of wood burns completely in one second at 600°C. The time the wood takes to burn is doubled for every 10°C drop in temperature and halved for every 10°C increase in temperature. In how many seconds would the wood burn at
   a) 500°C?  
   b) 650°C?

12. Inquiry/Problem Solving Use brackets to make each statement true. Justify your solution.
   a) $2^{-2} \times 2^2 + 2^2 - 2^0 = 2^0$  
   b) $3^{-4} - 3^{-2} \div 3^0 - 3^2 = 3^{-4}$

13. Without evaluating the expressions, determine which is greater, $20^{100}$ or $400^{40}$.

14. Evaluate.
   a) $\frac{6^1 + 6^{-1}}{6^1 - 6^{-1}}$  
   b) $\frac{5^{-4} - 5^{-6}}{5^{-3} + 5^{-5}}$  
   c) $2^n(2^n - 2^{1+n})$  
   d) $3\left(3^{2x} - \frac{1}{3^{2x}}\right)$

15. Communication a) For which non-zero real values of $x$ is $-x^{-4} = (-x)^{-4}$? Explain.
   b) For which non-zero real values of $x$ is $-x^{-3} = (-x)^{-3}$? Explain.

16. Equations Determine the value of $x$.
   a) $x^2 \times x^3 = 32$  
   b) $x^5 \div x^2 = 64$  
   c) $x^{-1} \times x^3 = \frac{1}{81}$  
   d) $x^2 \div x^5 = \frac{1}{125}$

17. For which values of $x$ is $x^{-4} \div x^{-4} = 1$ true? Explain.