1.4 Review: Adding, Subtracting, and Multiplying Polynomials

Key Concepts
- To add polynomials, collect like terms.
- To subtract polynomials, add the opposite.
- To multiply a polynomial by a monomial, use the distributive property to multiply each term in the polynomial by the monomial.
- To multiply two binomials, use the distributive property or use FOIL.
- To multiply any two polynomials, multiply each term of one polynomial by each term of the other polynomial.

Communicate Your Understanding
1. Describe how you would simplify each of the following.
   a) \((2x^2 + 5x + 1) + (x^2 - 3x - 4)\)
   b) \((3x^2 - 2x - 4) - (x^2 + x - 5)\)
2. Describe how you would expand and simplify \(2x(x + 1) - 3x(x - 4)\).
3. Describe how you would expand and simplify each of the following.
   a) \((3x + 1)(2x - 3)\)
   b) \(-2(x - 2)(2x + 1)\)
   c) \((x + 1)(x - 2) - (x - 3)(x + 2)\)

Adding and Subtracting Polynomials
Recall that terms such as \(8x\) and \(5x\), which have the same variable factors, are known as like terms. To simplify an expression containing like terms, add their coefficients.

Example 1 Adding Polynomials
Simplify \((4x^2 - 7x - 5) + (2x^2 - x + 3)\).

Solution
To add polynomials, collect like terms.
\[
(4x^2 - 7x - 5) + (2x^2 - x + 3) = 4x^2 - 7x - 5 + 2x^2 - x + 3 \\
= 4x^2 + 2x^2 - 7x - x - 5 + 3 \\
= 6x^2 - 8x - 2
\]
**Example 2** Subtracting Polynomials

Simplify \((4s^2 + 5sr - 7r^2) - (6s^2 + 3sr - 2r^2)\).

**Solution**

To subtract, add the opposite.
Multiply each term to be subtracted by \(-1\).

\[
(4s^2 + 5sr - 7r^2) - (6s^2 + 3sr - 2r^2) = (4s^2 + 5sr - 7r^2) - 1(6s^2 + 3sr - 2r^2)
= 4s^2 + 5sr - 7r^2 - 6s^2 - 3sr + 2r^2
= -2s^2 + 2sr - 5r^2
\]

**Practise**

1. Add.
   a) \((3x^2 - x + 2) + (4x^2 + 3x - 1)\)
   b) \((2r^2 + 5r - 7) + (3r^2 - 4r + 6)\)
   c) \((7m^2 - mn - 8n^2) + (6m^2 + 9mn + 11n^2)\)
   d) \((-4y^2 + 2xy - 6x^2) + (5y^2 - 6xy + 7y^2)\)
   e) \((3xy - 2x + 7) + (6xy + 5x - 3)\)
   f) \((5x + 3y - 8xy) + (6xy + 2x - 5y)\)

2. Subtract.
   a) \((3x^2 - 7x + 3) - (x^2 + 5x - 2)\)
   b) \((5s^2 + 8s - 12) - (6s^2 - s + 4)\)
   c) \((9x^2 - 4xy - y^2) - (6y^2 + 3xy + 10x^2)\)
   d) \((-r^2 + 4rs + s^2) - (6r^2 - rs + 11s^2)\)
   e) \((3x + 4y - 5z) - (x - y - z)\)
   f) \((5m - 3n) - (2m - 7n + 4)\)

3. Add the sum of \(3x^2 - 6x + 5\) and \(-3x^2 + 6\) to \(-x^2 - x - 1\).

4. Add \(4x + 2y - 7\) to the sum of \(-2x + 3y - 2\) and \(3x + y - 4\).

5. Subtract \(3r^2 + 4r - 7\) from the sum of \(2r^2 - 5r + 3\) and \(4r^2 + 2r + 3\).

6. Subtract the sum of \(m^2 + 2m - 3\) and \(4m^2 - m + 2\) from \(3m^2 + 4m - 1\).

7. **Measurement** The perimeter of a triangle is \(5x - 2y + 3z\). If two sides have lengths \(3y + z\) and \(4x - y + z\), what is the length of the third side?
Multiplying Polynomials by Monomials

To multiply a polynomial by a monomial, use the distributive property to multiply each term in the polynomial by the monomial.

**Example 3 Expanding**

Expand \(3a(2a^2 - 4a - 5)\).

**Solution**

Use the distributive property.

\[
3a(2a^2 - 4a - 5) = 3a(2a^2 - 4a - 5) = 6a^3 - 12a^2 - 15a
\]

**Example 4 Expanding and Simplifying**

Expand and simplify \(2x(3x - 5) - 4x(x - 7) + 3x(x - 1)\).

**Solution**

Use the distributive property to remove the brackets.

Then, collect like terms.

\[
2x(3x - 5) - 4x(x - 7) + 3x(x - 1) = 2x(3x - 5) - 4x(x - 7) + 3x(x - 1) = 6x^2 - 10x - 4x^2 + 28x + 3x^2 - 3x = 6x^2 - 4x^2 + 3x^2 - 10x + 28x - 3x = 5x^2 + 15x
\]

When more than one set of brackets is used, simplify to remove the innermost brackets first.

**Example 5 More Than One Set of Brackets**

Expand and simplify \(2[3(2x + 3) - 2(x - 1)]\).

**Solution**

\[
2[3(2x + 3) - 2(x - 1)] = 2(6x + 9 - 2x + 2) = 2(4x + 11) = 8x + 22
\]
Practise
8. Expand.
   a) $2(3x + 4)$  
   b) $-5(2 - 3x)$ 
   c) $4y(2y - 3)$  
   d) $-3(3m + 2n)$  
   e) $2x(4s - 5x)$ 
   f) $4(2b^2 + b - 1)$  
   g) $-2(q^2 - 5b - 4)$  
   h) $3p(2p^2 - p + 4)$ 
   i) $-4g(1 + 3g - 3g^2)$ 
9. Expand and simplify.
   a) $(2x - 4) - 3(x - 5)$  
   b) $(y^2 - 9x + 5) - 5(y - 4)$  
   c) $5(3x - 4y) - (2x - 5y) + 7$  
   d) $4(a - 2b - c) - 6(4a + 2b - 6c)$  
   e) $3(2x - 9) - 3 - (4x + 1) + 2$  
   f) $7(3x - 1) - 4(5x + 2) - 6$ 

Multiplying Polynomials
The distributive property can be used to multiply two binomials.

Example 6 Using the Distributive Property
Expand and simplify $(2x + 3)(4x - 5)$.

Solution
$(2x + 3)(4x - 5) = 2x(4x - 5) + 3(4x - 5)$
$= 8x^2 - 10x + 12x - 15$
$= 8x^2 + 2x - 15$

In Example 6, the same result is obtained if each term in the first binomial is multiplied by each term in the second binomial. You can remember this method with the acronym FOIL, which stands for adding the products of First terms, Outside terms, Inside terms, and Last terms.

$(2x + 3)(4x - 5) = (2x + 3)(4x - 5)$
$= 8x^2 - 10x + 12x - 15$
$= 8x^2 + 2x - 15$

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**Example 7  Expanding and Simplifying**

Expand and simplify \(3(x - 4)(x + 2) - 2(x + 5)(x - 3)\).

**Solution**

\[
3(x - 4)(x + 2) - 2(x + 5)(x - 3) = 3(x^2 - 2x - 8) - 2(x^2 + 2x - 15)
\]
\[
= 3x^2 - 6x - 24 - 2x^2 - 4x + 30
\]
\[
= x^2 - 10x + 6
\]

Recall that some graphing calculators, such as the TI-92 and TI-92 Plus, have the capability to perform operations with polynomials.

To find the product of any two polynomials, multiply each term of one of the polynomials by each term of the other polynomial. Then, collect like terms.

**Example 8  Multiplying Trinomials**

Expand and simplify \((x^2 - 3x - 1)(2x^2 + x - 2)\).

**Solution**

\[
(x^2 - 3x - 1)(2x^2 + x - 2) = 2x^4 + x^3 - 6x^2 - 3x^2 + 6x - 2x^2 - x + 2
\]
\[
= 2x^4 + x^3 - 6x^2 - 2x^2 - 3x^2 - 2x^2 + 6x - x + 2
\]
\[
= 2x^4 - 5x^3 - 7x^2 + 5x + 2
\]
**Example 9 Squaring a Trinomial**

Expand and simplify \((y^2 - 2y + 5)^2\).

**Solution**

\[(y^2 - 2y + 5)^2 = (y^2 - 2y + 5)(y^2 - 2y + 5)\]
\[= y^4 - 2y^3 + 5y^2 - 2y^3 + 4y^2 - 10y + 5y^2 - 10y + 25\]
\[= y^4 - 4y^3 + 14y^2 - 20y + 25\]

**Practise**

11. Expand and simplify.
   a) \((x - 7)(x + 6)\)
   b) \((t - 5)(t + 8)\)
   c) \((y - 3)(y - 9)\)
   d) \((3y - 1)(4y + 7)\)
   e) \((4x + 3)(2x + 7)\)
   f) \((5 + 2m)(3 - 4m)\)
   g) \(2(8 - x)(5x + 2)\)
   h) \(3(2x - 5)^2\)
   i) \(-(5x - 6)(5x + 6)\)

12. Expand and simplify.
   a) \((7x + 2y)(8x - 7y)\)
   b) \((3s + r)(2s - 3r)\)
   c) \((4x - 5y)(3x - 10y)\)
   d) \((6w - 11x)(w + 3x)\)
   e) \((5x^2 - 4x)(3x^2 + 2x)\)
   f) \((2m - 3m^2)(m^2 + 2m)\)
   g) \((3x - 4y)^2\)
   h) \(-2(5x + 6y)(5x - 6y)\)
   i) \(5(1 - xy)(1 + xy)\)

13. Expand and simplify.
   a) \((x - 7)(x + 1) + (x + 6)(x + 2)\)
   b) \((2t - 1)(t + 4) - (t + 6)(3t + 2)\)
   c) \(2(x - 4)(x + 3) + 5(2x - 1)(x + 6)\)
   d) \(2(2y - 5)(y - 4) - (5y - 3)(y + 4)\)
   e) \(2(m - 3)(m - 4) - 3(m + 5)^2\)
   f) \(3(2x + 3)^2 - (x - 5)^2 - (3x - 4)(x - 5)\)
   g) \(5(2y - 5)(2y + 5) - 4(y - 2)(y + 3) - (2y + 1)^2\)
   h) \(5r^2 - (r - 3)^2 - 2(r^2 - 5r) + 2(2r + 3)^2\)
   i) \(4(x^2 - 3xy) - (x + y)^3 - 2(x - y)(x + y) + 5\)
   j) \((2r + 3s)(r - t) - 4(r - 2t)^2 + 5(r^2 - t^2)\)

**Apply, Solve, Communicate**

14. **Communication**
   a) Explain how the diagram illustrates the product \((2x + 1)(x + 2y + 3)\).
   b) State the product in simplified form.
15. Expand and simplify.
   a) $(x + 3)(x^2 + 2x + 4)$  
   b) $(y - 2)(y^2 - y - 5)$
   c) $(3m + 2)(2m^2 + 3m - 4)$  
   d) $(x^2 - 5x - 7)(2x + 1)$
   e) $(x^2 + 2x - 1)(x^2 - x - 4)$  
   f) $(y - 2)(y^3 - 2y^2 + 3y - 1)$
   g) $(3a^2 - 4a + 2)(a^2 - a - 5)$  
   h) $(x^3 - 7)(3x^3 + 7)$
   i) $(x^2 - 4x + 1)^2$  
   j) $(2n^2 - n - 1)^2$
   k) $(2a - b + 3c)^2$  
   l) $(2x - 1)(x^2 - 2x^2 + 5x - 3)$
   m) $2(x - 1)(x^2 - 3x + 2) - (2x^2 - 3x + 4)(2x + 3)$
   n) $4(x - y + z)(x - 2y - 3z) - (x + y + z)^2 - (x - y - 2z)$
   o) $(3x - 5)(3 + (2x + 4)(x - 1))$

16. a) Multiply $(x + 1)(x + 2)$. Then, multiply the result by $x - 3$ and simplify.
   b) Multiply $(x + 1)(x - 3)$. Then, multiply the result by $x + 2$ and simplify.
   c) Multiply $(x - 3)(x + 2)$. Then, multiply the result by $x + 1$ and simplify.
   d) Does the order in which you multiply three binomials affect the result?

17. Expand and simplify.
   a) $(2x + 1)(x - 3)(4x - 5)$  
   b) $(x + 2y)(x - 3y)(2x - y)$
   c) $(a + b + c + d)^2$

18. **Measurement** The dimensions of a rectangular prism are represented by binomials, as shown.
   a) Write a simplified expression that represents the surface area of this prism.
   b) Write a simplified expression that represents the volume of this prism.
   c) If $x$ represents 7 cm, what are the surface area and the volume of the prism?

19. **Application** Write and simplify an expression to represent the area of the shaded region.

20. **Inquiry/Problem Solving** Is the product of two binomials always a trinomial? Explain.

21. Expand and simplify.
   a) $\left(\frac{x + \frac{1}{x}}{x - \frac{1}{x}}\right)$  
   b) $\left(\frac{y - \frac{2}{y}}{y + \frac{3}{y}}\right)$