The blimp that provided overhead television coverage of the first World Series played in Canada was based in Miami. The blimp flew 1610 km from Miami to Washington, D.C., and then 634 km to Toronto.

The time taken to fly from Miami to Washington was \( \frac{1610}{s} \) hours, where \( s \) was the average speed in kilometres per hour.

The time taken to fly from Washington to Toronto was \( \frac{634}{s} \) hours.

The total flying time from Miami to Toronto was \( \frac{1610}{s} + \frac{634}{s} \) hours.

The expression \( \frac{1610}{s} + \frac{634}{s} \) is the sum of two rational expressions with the same denominator.

**INVESTIGATE & INQUIRE**

Rectangle A and Rectangle B have different areas but the same width.

Rectangle C is formed by placing Rectangles A and B end to end.

<table>
<thead>
<tr>
<th>Rectangle A</th>
<th>Rectangle B</th>
<th>Rectangle C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + 2 )</td>
<td>( x + 2 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\text{Area}}{x^2 + 3x + 1} )</td>
<td>( \frac{\text{Area}}{x^2 + 4x + 2} )</td>
<td></td>
</tr>
</tbody>
</table>

1. a) Using the areas of rectangles A and B, write and simplify an expression that represents the area of rectangle C.
   b) What is the width of rectangle C?

2. Write, but do not divide, a rational expression that represents the length of
   a) rectangle A
   b) rectangle B
3. Using the area and the width of rectangle C, write, but do not divide, a rational expression that represents the length of rectangle C.

4. How does the expression you wrote in question 3 compare with the two expressions you wrote in question 2? Explain.

5. Write a rule for adding two rational expressions with the same denominator.

6. Add.
   a) \( \frac{x}{3} + \frac{4x}{3} \)
   b) \( \frac{5}{3t} + \frac{2}{3t} \)
   c) \( \frac{n + 1}{n + 3} + \frac{n - 1}{n + 3} \)
   d) \( \frac{x^2 + 1}{x^2} + \frac{2x^2 + 1}{x^2} \)

7. The flying time of the blimp from Miami to Toronto was \( \frac{1610}{s} + \frac{634}{s} \) hours.
   a) Add the rational expressions.
   b) If the average speed, \( s \), of the blimp was 85 km/h, what was the total flying time, in hours?

---

Rational expressions with a common denominator can be added or subtracted in the same way as fractions with a common denominator.

\[
\frac{5}{7} + \frac{1}{7} - \frac{2}{7} = \frac{5 + 1 - 2}{7} = \frac{4}{7}
\]

**Example 1 Adding and Subtracting With Common Denominators**

Simplify each of the following. State the restriction on the variable.

a) \( \frac{3}{x^2} + \frac{5}{x^2} - \frac{2}{x^2} \)

b) \( \frac{4x - 1}{x + 2} - \frac{x + 3}{x + 2} \)
**Solution**

a) \[
\frac{3}{x^2} + \frac{5}{x^2} - \frac{2}{x^2} = \frac{3 + 5 - 2}{x^2} = \frac{6}{x^2}
\]
Write with the common denominator: 
Add or subtract the numerators: 
Exclude values for which \(x^2 = 0\). 
\[x = 0\]
Therefore, \(\frac{3}{x^2} + \frac{5}{x^2} - \frac{2}{x^2} = \frac{6}{x^2}, x \neq 0\).

b) \[
\frac{4x - 1}{x + 2} - \frac{x + 3}{x + 2} = \frac{(4x - 1) - (x + 3)}{x + 2}
\]
Write with the common denominator: 
Subtract the numerators: 
Simplify: 
Exclude values for which \(x + 2 = 0\). 
\[x = -2\]
Therefore, \(\frac{4x - 1}{x + 2} - \frac{x + 3}{x + 2} = \frac{3x - 4}{x + 2}, x \neq -2\).

Rational expressions with different denominators can be added or subtracted in the same way as fractions with different denominators.

\[
\frac{1}{6} + \frac{3}{4}
\]
Rewrite with a common denominator: 
Add the numerators: 
\[
\frac{2}{12} + \frac{9}{12} = \frac{11}{12}
\]
\[
\frac{3}{5} - \frac{1}{2}
\]
Rewrite with a common denominator: 
Subtract the numerators: 
\[
\frac{6}{10} - \frac{5}{10} = \frac{1}{10}
\]
Note that the least common denominator (LCD) is normally used but is not necessary. If a greater common denominator is used, the result will reduce to give the same answer.

\[
\frac{1}{6} + \frac{3}{4} = \frac{4}{24} + \frac{18}{24} = \frac{22}{24} = \frac{11}{12}
\]

**Example 2**  Adding and Subtracting With Whole-Number Denominators

Simplify \(\frac{3x + 2}{4} + \frac{x - 4}{8} - \frac{2x - 1}{6}\).

**Solution**

To find the LCD, find the least common multiple (LCM) of the denominators 4, 8, and 6. The LCM can be found by factoring. It must contain all the separate factors of 4, 8, and 6.

- \(4 = 2 \times 2\)
- \(8 = 2 \times 2 \times 2\)
- \(6 = 2 \times 3\)

So, the LCD is 24.

The LCM is \(2 \times 2 \times 2 \times 3 = 24\).

Rewrite with a common denominator:

\[
\frac{3x + 2}{4} + \frac{x - 4}{8} - \frac{2x - 1}{6} = \frac{6(3x + 2)}{24} + \frac{3(x - 4)}{24} - \frac{4(2x - 1)}{24}
\]

Add or subtract the numerators:

\[
\frac{6(3x + 2) + 3(x - 4) - 4(2x - 1)}{24} = \frac{18x + 12 + 3x - 12 - 8x + 4}{24} = \frac{13x + 4}{24}
\]

Simplify:
Therefore, 
\[ \frac{3x + 2}{4} + \frac{x - 4}{8} - \frac{2x - 1}{6} = \frac{13x + 4}{24}. \]

Sometimes it is necessary to factor \(-1\) from one of the denominators to recognize the common denominator.

**Example 3 Factoring \(-1\) From a Denominator**

Simplify \(\frac{5}{x - 3} + \frac{2}{3 - x}\). State the restriction on the variable.

**Solution**

Factor \(-1\) from the denominator \(3 - x\).

\[ 3 - x = -1(-3 + x) = -(x - 3) \]

Rewrite \(\frac{2}{3 - x}\) so that there is a common denominator.

\[
\frac{5}{x - 3} + \frac{2}{3 - x} = \frac{5}{x - 3} - \frac{-2}{x - 3}
\]

\[ = \frac{5}{x - 3} + \frac{2}{x - 3}
\]

\[ = \frac{5 + 2}{x - 3}
\]

\[ = \frac{3}{x - 3} \]

Exclude values for which \(x - 3 = 0\) or \(3 - x = 0\).

\[ x = 3 \text{ or } x = 3 \]

Therefore, \(\frac{5}{x - 3} + \frac{2}{3 - x} = \frac{3}{x - 3}, x \neq 3\).

**Key Concepts**

- To add or subtract rational expressions with a common denominator, write the numerators over the common denominator, and add or subtract the numerators.
- To add or subtract rational expressions with different denominators, rewrite the expressions with a common denominator. Then, write the numerators over the common denominator, and add or subtract the numerators.
Communicate Your Understanding

1. a) Describe how you would simplify \( \frac{5x}{x+4} - \frac{2x}{x+4} \).
   
   b) What is the restriction on the variable?

2. Describe how you would simplify \( \frac{x+4}{2} + \frac{x-3}{6} - \frac{x+5}{4} \).

Practise

In each of the following, state any restrictions on the variables.

A

1. Simplify.
   
   a) \( \frac{2}{y} + \frac{4}{y} - \frac{5}{y} \)
   
   b) \( \frac{5x^2 - 3}{x^2 + 6} \)
   
   c) \( \frac{4}{x+3} + \frac{5}{x+3} \)
   
   d) \( \frac{x}{x-2} - \frac{y}{x-2} \)

2. Simplify.
   
   a) \( \frac{x+7}{2} + \frac{x+4}{2} \)
   
   b) \( \frac{2y-1}{3} + \frac{3y-6}{3} \)
   
   c) \( \frac{3a-1}{a} - \frac{4a+2}{a} \)
   
   d) \( \frac{5x-y}{3x} - \frac{4x+y}{3x} \)
   
   e) \( \frac{a^2}{x+1} + \frac{2a^2}{x+1} \)
   
   f) \( \frac{6t-8}{7} + \frac{3-5t}{7} \)
   
   g) \( \frac{5z}{2z-1} - \frac{z-3}{2z-1} \)
   
   h) \( \frac{2x+3}{x^2-1} + \frac{3x-4}{x^2-1} \)
   
   i) \( \frac{4x+1}{x^2+5x+6} + \frac{3x+2}{x^2+5x+6} \)
   
   j) \( \frac{1-2y}{2x^2+3x+1} - \frac{5y+3}{2x^2+3x+1} \)

3. Find the LCM.
   
   a) 4, 5, 6
   
   b) 4, 9, 12
   
   c) 8, 10, 12
   
   d) 20, 15, 10

4. Simplify.
   
   a) \( \frac{2x}{2} + \frac{x}{3} \)
   
   b) \( \frac{3a}{4} + \frac{a}{2} - \frac{2a}{6} \)
   
   c) \( \frac{x}{5} - \frac{y}{2} + \frac{7}{10} \)
   
   d) \( \frac{3m}{8} - \frac{m}{6} - \frac{2m}{3} \)

5. Simplify.
   
   a) \( \frac{2m+3}{2} + \frac{3m+4}{7} \)
   
   b) \( \frac{4x-3}{4} + \frac{x+2}{3} \)
   
   c) \( \frac{y-5}{6} - \frac{2y-3}{4} \)
   
   d) \( \frac{2x+3y}{5} - \frac{4x-y}{2} \)
   
   e) \( \frac{4t-1}{6} + \frac{3t+2}{2} - \frac{2t+1}{3} \)
   
   f) \( \frac{3a-b}{9} - \frac{a-2b}{3} - \frac{4a-3b}{6} \)
   
   g) \( \frac{5x-1}{5} + 1 - \frac{4x-3}{6} \)
   a) \( \frac{3}{2-x} + \frac{2}{x-2} \)
   b) \( \frac{1}{x-1} - \frac{1}{1-x} \)
   c) \( \frac{a-2}{2a-3} + \frac{a+3}{3-2a} \)

7. **Flying times**
   a) Write an expression that represents the time, in hours, it takes a plane to fly 1191 km from Winnipeg to Calgary at an average speed of \( s \) kilometres per hour.
   b) Write an expression that represents the time, in hours, it takes a plane to fly 685 km from Calgary to Vancouver at the same speed as in part a).
   c) Write and simplify an expression that represents the total flying time for a trip from Winnipeg to Vancouver via Calgary.
   d) If the average speed of the plane is 700 km/h, what is the total flying time, in hours, for the trip in part c)?

8. **Application**
   A backgammon game board consists of two rectangles of the same size, known as tables, separated by a divider, called the bar.
   a) The area of each table on a backgammon board can be modelled by the expression \( x^2 + 8x \), and the width of each table by \( x \). Write and simplify an expression that represents the width, \( w \), of the whole board in terms of \( x \).
   b) If the width of the bar is \( \frac{x}{5} \), write and simplify an expression that represents the length of the whole board in terms of \( x \).
   c) If \( x \) represents 15 cm, what are the dimensions of each table? of the whole board?
9. **Application**  Two triangles have the same base length, represented by \(x\). The height of one triangle is \(x + 1\). The height of the other triangle is \(x + 3\). Write and simplify an expression that represents the total area of the two triangles.

10. **Communication**  Rectangle A and rectangle B each have a length of \(2x + 1\). Rectangle A has an area of \(6x^2 + 5x + 1\), and rectangle B has an area of \(4x^2 - 4x - 3\).
   a) Write but do not simplify an expression for the width of rectangle A.
   b) Write but do not simplify an expression for the width of rectangle B.
   c) Subtract the width of rectangle A from the width of rectangle B. Simplify the resulting expression.
   d) Subtract the width of rectangle B from the width of rectangle A. Simplify the resulting expression.
   e) How do the results of parts c) and d) compare? Explain.

11. **Modelling problems algebraically**  The diameter of the smaller circle is \(d\). The diameter of the larger circle is \(d + 1\).
   a) Write an expression that represents the area of the smaller circle in terms of \(d\).
   b) Write an expression that represents the area of the larger circle in terms of \(d\).
   c) Write and simplify an expression that represents the area of the shaded part of the diagram in terms of \(d\).
   d) If \(d\) represents 10 cm, find the area of the shaded part of the diagram, to the nearest tenth of a square centimetre.

12. **Measurement**  The diagram shows trapezoid ABCD divided into rhombus ABCE and isosceles triangle ADE.
   a) Write an expression that represents the area of the triangle in terms of \(x\).
   b) Write an expression that represents the area of the rhombus in terms of \(x\).
   c) Add and simplify the expressions you wrote in parts a) and b).
   d) Write and simplify an expression that represents the longer base of the trapezoid in terms of \(x\).
   e) Use the formula for the area of a trapezoid to write and simplify an expression that represents the area of the trapezoid in terms of \(x\).
   f) Compare your expressions from parts c) and e).
13. **Pattern** Triangular numbers of objects can be arranged to form triangles. The first four triangular numbers are as shown.

a) An expression for finding the $n$th triangular number can be written in the form $\frac{n(n + \blacksquare)}{\square}$, where $\blacksquare$ and $\square$ represent whole numbers. Copy and complete the expression by finding the numbers represented by $\blacksquare$ and $\square$.

b) Write the 5th, 6th, 7th, 8th, and 9th triangular numbers.

c) Add any two consecutive triangular numbers. What kind of number results?

d) Write an expression that represents the $(n + 1)$th triangular number.

e) Add your expressions from parts a) and d). Simplify the result and express it in factored form.

f) How does your result from part e) explain your result from part c)?

**Achievement Check**

Your company makes fridge magnets. The materials for each magnet cost $0.14.

Your company has additional expenses of $27,000 a year. The per-magnet cost is $\underline{\text{total costs per year}}$. If your company can make and sell twice as many number produced per year magnets next year as this year, the per-magnet cost will be reduced by $0.90. How many magnets is your company making and selling this year?