Investigate & Apply

Relating Sequences and Systems of Equations

1. Solve the following system of equations.
   \[ 2x + 5y = 8 \]
   \[ 7x + 3y = -1 \]
   Note that in each equation, the coefficients, including the constant term, form an arithmetic sequence.

2. For the system of equations
   \[ Ax + By = C \]
   \[ Px + Qy = R \]
   the coefficients \( A, B, C \) form an arithmetic sequence and the coefficients \( P, Q, R \) form another arithmetic sequence. Prove that \( (x, y) = (-1, 2) \) is always an exact solution to this system.

3. What if the coefficients form a geometric sequence? Is there an exact solution? Is there a general solution? If there is a solution, develop it. If not, show that it does not exist.

4. Extend the above ideas to at least one other type of sequence. Develop a feasible solution or show that none exists.