7.1 Investigation: Simple Interest, Arithmetic Sequences, and Linear Growth

When you invest money in a bank or other financial institution, you are paid interest for the use of your money. The financial institution uses your money to earn money. When you borrow money, you pay interest for the loan.

Interest can be calculated as simple interest, which means that only the money originally invested earns interest. If you invest $1000 at a simple interest rate of 5% per annum for several years, only the original investment of $1000 earns interest each year.

**Simple Interest**

The money you invest or borrow is called the principal. The interest rate is the percent of the principal that is earned, or paid, as interest. The sum of the principal and the interest is called the amount.

1. Suppose you invest $1000 that earns simple interest at a rate of 5% per annum.
   a) How much interest would you earn in the first year?
   b) What amount would you have after 1 year?
   c) Since simple interest is being calculated, your interest from the first year is not reinvested. How much interest would you earn in the second year?
   d) How much interest would you have altogether after 2 years?
   e) What amount would you have after 2 years?
2. Complete a table like the following for 10 years for $1000 invested with simple interest at a rate of 5% per annum. Use your answers from question 1 in the second row.

<table>
<thead>
<tr>
<th>Number of years</th>
<th>Principal ($)</th>
<th>Interest rate</th>
<th>Interest ($)</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>0.05</td>
<td>50</td>
<td>1050</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Plot the points in the table from question 2, with the time, in years, along the horizontal axis and the amount, in dollars, along the vertical axis. Join the points.

4. Consider the values in the Amount column of the table.
   a) What kind of sequence do you notice? Explain.
   b) What is the first term in this sequence?
   c) What is the difference between consecutive terms in this sequence? What is the difference called?
   d) Use your answers from parts b) and c) to write a formula for the $n$th term for the sequence in the Amount column. Use the table to check your formula for $t_n$.

5. a) Substitute $n = 1, 2, 3, \ldots, 10$ into your formula for $t_n$ from part d) of question 4 to find $t_1, t_2, t_3, \ldots, t_{10}$.
   b) Graph the values from part a) with $n$ along the horizontal axis and $t_n$ along the vertical axis. Join the points.

6. Use the table from question 2.
   a) Let $P$ be the principal in dollars, $r$ be the interest rate expressed as a decimal, and $t$ be the time in years. Write an expression for finding the interest.
   b) Let $I$ be the interest in dollars. Use your expression from part a) to write a formula showing the relationship between $I, P, r,$ and $t$. Check your formula by applying it to values in the table.
   c) Let $A$ be the amount in dollars. Write a formula showing the relationship between $A, P,$ and $I$. Check your formula with the table.
   d) Substitute for $I$ in your formula from part c) to write a formula expressing $A$ in terms of $P, r,$ and $t$. Check this formula with the table.

7. a) Substitute $t = 1, 2, 3, \ldots, 10$ into your formula for $A$ from part d) of question 6 to find the amount after each year up to 10 years.
   b) Graph the values from part a) with $t$ along the horizontal axis and $A$ along the vertical axis. Join the points.
8. Consider your graphs from questions 3, 5, and 7.
   a) What shape are your graphs?
   b) What is the y-intercept of each graph?
   c) What does each y-intercept represent for the investment of $1000?
   d) What is the slope of each graph?
   e) What does each slope represent for the investment of $1000?
   f) Use the y-intercept and slope to write an equation in the form \( y = mx + b \).
   g) Graph the equation from part f).

9. If the points representing a relationship lie on a straight line, the relationship is linear.
   a) Are the graphs linear or non-linear? Explain.
   b) Why is it reasonable to join the points of each graph?
   c) Use one graph to estimate the amount for a time from 1 to 10 years.
   d) Use a formula to check your estimate.
   e) Use your formula for \( t_n \) to find the amount after 20 years.
   f) Use your formula for \( A \) to find the amount after 20 years. Compare this amount with the value for \( t_{20} \) in part e). Explain the result.

10. Suppose you invested $1000 earning simple interest at a rate of 6.75% per annum.
    a) Predict the appearance of a graph representing this investment with the time in years along the horizontal axis and the amount in dollars along the vertical axis.
    b) What amount would you have at the end of
       i) 1 year?
       ii) 2 years?
       iii) 3 years?
       iv) 4 years?
       v) 5 years?
    c) Name the type of sequence for the amounts in part b).
    d) State the first term and the common difference for the sequence.
    e) Write the formula for \( t_n \) of the sequence. Check your formula for \( n = 1, 3, \text{ and } 5 \).
    f) Graph the formula from part e) with \( n \) along the horizontal axis and \( t_n \) along the vertical axis. Join the points.
    g) Write an equation in the form \( y = mx + b \) to represent your graph.
    h) Compare your graph with your prediction from part a).

11. Use your results to describe the relationship between simple interest, arithmetic sequences, and linear growth. Refer to your table, graphs, and formulas in your explanation.