Banks and financial institutions offer a variety of accounts and investments. When you invest or borrow money, the interest rate can greatly affect the amount of interest you pay or receive. Researching and comparing interest rates is an important step in arranging a plan that suits you.

For compound interest, the money you invest or borrow is called the principal. The interest rate is the percent of the principal that is earned or paid as interest. The sum of the interest and the principal is the amount. The amount is the principal for the next compounding period.

**Compound Interest**

1. Complete a table like the following for $1000 invested with compound interest at a rate of 5% per annum, compounded annually for 10 years.

<table>
<thead>
<tr>
<th>Number of years</th>
<th>Principal ($)</th>
<th>Interest rate</th>
<th>Interest ($)</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000.00</td>
<td>0.05</td>
<td>50.00</td>
<td>1050.00</td>
</tr>
<tr>
<td>2</td>
<td>1050.00</td>
<td></td>
<td></td>
<td>1102.50</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Plot the points in the table from question 1 with the time in years along the horizontal axis and the amount in dollars along the vertical axis. Join the points.

3. **a)** What kind of sequence is represented by the values in the Amount column? How do you know?

**b)** What is the value of $a$?

**c)** What is the value of $r$?

**d)** Use the values of $a$ and $r$ to write a formula for the $n$th term for the sequence in the Amount column. Use the table to check your formula for $t_n$. 

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4. a) Substitute \( n = 1, 2, 3, \ldots, 10 \) into your formula for \( t_n \) from part d) of question 3 to find \( t_1, t_2, t_3, \ldots, t_{10} \).

b) Graph the values from part a) with \( n \) along the horizontal axis and \( t_n \) along the vertical axis. Join the points.

5. a) Use the formula for compound interest to find the amount after each year up to 10 years.

b) Graph the values from part a) with \( n \) along the horizontal axis and \( A \) along the vertical axis. Join the points.

c) Use your formula from part a) to write a formula with the variables \( x \) and \( y \). Since \( n \) is represented along the horizontal axis, replace \( n \) with \( x \). Since \( A \) is represented along the vertical axis, replace \( A \) with \( y \).

6. Consider your graphs from questions 2, 4, and 5.

a) What shape are your graphs?

b) What is the \( y \)-intercept of each graph?

c) What does each \( y \)-intercept represent for the investment of $1000?

d) What do the points along the curve of each graph represent for the investment of $1000?

7. A function in which a variable is an exponent is called an **exponential function**. The graph of an exponential function models exponential growth.

a) Explain whether the formulas representing the investment are exponential functions.

b) Explain whether the graphs representing the investment are linear or non-linear.

c) Explain whether the graphs representing the investment model exponential growth.

8. a) Why is it reasonable to join the points of each graph?

b) Use one of your graphs to estimate the amount for a time from 1 to 10 years. Use your formula for \( t_n \) to check your estimate.

c) Use your formula for \( t_n \) to find the amount after 20 years.

d) Use your formula for \( A \) to find the amount after 20 years.

e) Compare the amounts from parts c) and d). Explain the result.

9. Suppose you invested $1000 earning compound interest at a rate of 6.75% per annum, compounded annually.

a) Predict the appearance of a graph representing this investment with the time in years along the horizontal axis and the amount in dollars along the vertical axis.
b) What amount would you have at the end of
i) 1 year?  ii) 2 years?  iii) 3 years?  iv) 4 years?  v) 5 years?
c) Name the sequence for the amounts in part b).
d) State the first term and the common ratio for the sequence.
e) Write the formula for $t_n$ of the sequence. Check your formula for $n = 1$, 3, and 5.
f) Graph the formula from part e) with $n$ along the horizontal axis and $t_n$ along the vertical axis. Join the points.
g) Compare your graph with your prediction from part a).

10. Use your results to describe the relationship for compound interest, geometric sequences, and exponential growth. Refer to your table, graphs, and formulas in your explanation.

### Comparing Compound Interest and Simple Interest

Two kinds of Canada Savings Bonds are regular Canada Savings Bonds and compound Canada Savings Bonds. Regular Canada Savings Bonds earn simple interest. Each year the interest is deposited into the owner’s bank account or mailed to the owner. Compound Canada Savings Bonds earn compound interest, so the interest is reinvested and the whole amount of the bond is paid when it is cashed.

11. Copy and complete this table for a regular Canada Savings Bond at simple interest of 6% per annum, over 8 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Principal ($)</th>
<th>Interest ($)</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500.00</td>
<td>30.00</td>
<td>530.00</td>
</tr>
<tr>
<td>2</td>
<td>530.00</td>
<td>31.80</td>
<td>561.80</td>
</tr>
<tr>
<td>3</td>
<td>561.80</td>
<td>33.71</td>
<td>595.51</td>
</tr>
<tr>
<td>4</td>
<td>595.51</td>
<td>35.73</td>
<td>631.24</td>
</tr>
<tr>
<td>5</td>
<td>631.24</td>
<td>37.88</td>
<td>669.12</td>
</tr>
<tr>
<td>6</td>
<td>669.12</td>
<td>39.09</td>
<td>708.21</td>
</tr>
<tr>
<td>7</td>
<td>708.21</td>
<td>40.33</td>
<td>748.54</td>
</tr>
<tr>
<td>8</td>
<td>748.54</td>
<td>41.59</td>
<td>790.13</td>
</tr>
</tbody>
</table>

12. Identify the type of sequence in the table. Write the formula for the $n$th term, $t_n$.

13. Use the formula to find $t_1$, $t_5$, and $t_8$.

14. Use the formula $A = P + Prt$ to find the amount after 5 years and 6 years.

15. Write the equation for the amount in the form $y = mx + b$. 

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16. Complete a table like the following for a compound Canada Savings Bond at an interest rate of 6% per annum, compounded annually, over 8 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Principal ($)</th>
<th>Interest ($)</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500.00</td>
<td>30.00</td>
<td>531.00</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>561.80</td>
</tr>
</tbody>
</table>

17. Identify the sequence in the table. Write the formula for the $n$th term, $t_n$.

18. Use the formula to find $t_1$, $t_5$, and $t_8$.

19. Use the formula for compound interest to find the amount after 5 years and 6 years.

20. In the formula for $t_n$, replace $n$ with $x$ and $t_n$ with $y$ to write an equation with the variables $x$ and $y$.

21. This graph compares amounts for the bonds in the above tables. Decide whether each graph represents linear or exponential growth. Explain your decision. Use the words simple interest, compound interest, arithmetic sequence, geometric sequence, linear growth, and exponential growth in your explanation.

22. a) Describe the reasons someone might choose to buy regular Canada Savings Bonds.
b) Describe the reasons someone might choose to buy compound Canada Savings Bonds.
c) If you were investing $100,000, would you choose regular Canada Savings Bonds, compound Canada Savings Bonds, some of each, or neither? Explain how you would support this choice.

Web Connection

To investigate the types of Canada Savings Bonds currently available and their interest rates, visit the above web site. Go to Math Resources, then to MATHEMATICS 11, to find out where to go next. Write a report about the current series of bonds available.