7.8 Mortgages

A knowledge of mortgages enables you to make decisions about financial arrangements when buying property. The variety in amortization periods, terms, amounts of mortgages, and even interest rates, allows you to make or combine choices. These choices can have a significant impact on your payments.

**Investigate & Inquire**

This amortization table shows monthly mortgage payments for different interest rates. Each payment includes the interest on the principal still owed and some of the principal. A mortgage is arranged for a specific length of time, called the term of the mortgage.

<table>
<thead>
<tr>
<th>Interest Rate (%)</th>
<th>10 years</th>
<th>15 years</th>
<th>20 years</th>
<th>25 years</th>
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<tr>
<td>5.0</td>
<td>10.581 493</td>
<td>7.881 238</td>
<td>6.571 250</td>
<td>5.816 050</td>
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<td>6.843 913</td>
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<tr>
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<td>9.481 529</td>
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<td>14.180 269</td>
<td>11.816 096</td>
<td>10.809 741</td>
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</table>
1. Use the amortization table.
   a) What happens to the monthly payment as the interest rate increases? Why does this make sense?
   b) What happens to the monthly payment as the number of years for the amortization period decreases? Why does this make sense?
   c) How much is the monthly payment for an interest rate of 7%, amortized over 25 years for each amount?
      i) $1000    ii) $2000    iii) $3000    iv) $4000    v) $5000
      vi) $100 000 vii) $200 000 viii) $300 000 ix) $400 000 x) $500 000
   d) How much is the monthly payment for an interest rate of 11.5%, amortized over 10 years for each amount?
      i) $1000    ii) $10 000 iii) $100 000 iv) $1 000 000

2. Suppose you have a mortgage of $100 000 and the current interest rate is 9% for a 3-year term.
   a) What would your monthly mortgage payment be if your mortgage is amortized over 25 years?
   b) What would you have paid by the end of 1 year?
   c) What would you have paid by the end of the 3-year term?

3. Suppose you have a mortgage of $100 000 and the current interest rate is 9% for a 3-year term.
   a) What would your mortgage payment be per month if your mortgage is amortized over 20 years?
   b) What would you have paid by the end of 1 year?
   c) What would you have paid by the end of the 3-year term?

4. Suppose you have a mortgage of $100 000 and the current interest rate is 9% for a 3-year term.
   a) What would your mortgage payment be per month if your mortgage is amortized over 15 years?
   b) What would you have paid by the end of 1 year?
   c) What would you have paid by the end of the 3-year term?

5. Suppose you have a mortgage of $100 000 and the current interest rate is 9% for a 3-year term.
   a) What would your mortgage payment be per month if your mortgage is amortized over 10 years?
   b) What would you have paid by the end of 1 year?
   c) What would you have paid by the end of the 3-year term?
6. a) Make a generalization about the effect of the amortization period on the amount of a mortgage.
b) Explain why someone might want a shorter amortization period.
c) Explain why someone might want a longer amortization period.

7. What factors might affect a decision whether to increase a down payment to decrease the mortgage payments?

8. a) Why do you think the amounts in the table are not rounded to the nearest cent?
b) Use an example to show what would happen if the amounts were rounded to the nearest cent.

**Example 1  Finding Mortgage Payments**

Victor is buying a house for $196,500. He makes a down payment of 25% of the price and negotiates a mortgage at 7.5%, amortized over 25 years, for the balance of the price.

a) How much is Victor's mortgage?
b) How much are Victor's monthly payments?

**Solution 1  Paper-and-Pencil and Table Method**

a) The down payment is 25% of the price, or 25% of $196,500.
25% of 196,500 = 49,125
The mortgage is the price of $196,500 less the down payment of $49,125.
196,500 − 49,125 = 147,375
Victor’s mortgage is $147,375.

b) Use the amortization table in the Investigate & Inquire. Since the table gives monthly mortgage payments for each $1000, divide the mortgage of $147,375 by 1000 to find how many thousands are in $147,375.
147,375 ÷ 1000 = 147.375
At 7.5%, amortized over 25 years, the monthly payment for each $1000 is $7.315 549, so multiply 147.375 times $7.315 549.
147.375 × 7.315 549 = 1,078.13
Victor’s monthly mortgage payments are $1,078.13.
**Solution 2  Graphing-Calculator Method**

a) Change the **mode settings** to 2 decimal places. From the **Finance menu**, choose **TVM Solver**.

Enter the known values.
There are 12 payments a year for 25 years, so \( N = 25 \times 12 \).
The interest rate is 7.5% per annum, so \( I = 7.5 \).
The mortgage is the house price of $196 500 less the down payment of 25%, so \( PV = 196 500 - 0.25 \times 196 500 \).
A payment is made each month, so \( P/Y = 12 \).
The interest is compounded semi-annually, so \( C/Y = 2 \).
The payments are made at the end of each payment interval, so select **END**.

Move the cursor to **PMT** to find the mortgage payment, and press **ALPHA SOLVE**. Since the mortgage payment is paid out, **PMT** is negative.

Victor’s mortgage is $147 375.

b) Victor’s monthly payments are $1078.13, as shown in **Solution 1**.

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**Example 2  Finding the Cost of a Mortgage**

The Pin family is negotiating a mortgage on a condominium for $85 000, amortized over 20 years, at an interest rate of 8.9%.

a) What is the Pins’ monthly mortgage payment?

b) Mortgages are renegotiated after the end of the term, which is often 5 years. However, if the Pin family did keep the mortgage for 20 years and the interest rate remained unchanged, how much would it cost?

**Solution 1  Graphing-Calculator Method**

a) Change the **mode settings** to 2 decimal places. From the **Finance menu**, choose **TVM Solver**.

Enter the known values.
There are 12 payments a year for 20 years, so \( N = 20 \times 12 \).
The interest rate is 8.9% per annum, so \( I = 8.9 \).
The mortgage is $85 000, so \( PV = 85 000 \).
A payment is made each month, so \( P/Y = 12 \).
The interest is compounded semi-annually, so \( C/Y = 2 \).
The payments are made at the end of each payment interval, so select END.

Move the cursor to PMT to find the mortgage payment, and press ALPHA SOLVE. Since the mortgage payment is paid out, PMT is negative.

The Pins' monthly payment is $750.58.

b) There are 12 monthly payments for 20 years, each $750.58.
   \[ 12 \times 20 \times 750.58 = 180\,139.20 \]
The mortgage would cost $180,139.20.

**SOLUTION 2  Spreadsheet Method**

a) The Pins' monthly payment of $750.58 is found with a graphing calculator, as in Solution 1.

b) Use a graphing calculator to find the interest rate per month. Change the mode settings to 9 decimal places. From the Finance menu, choose TVM Solver.

Enter values for $1 for 1 month of this mortgage.

Enter the known values.
The interest rate per month is being calculated, so \( N = 1 \).
The interest rate is 8.9\% per annum, so \( I = 8.9 \).
Use the negative amount $1 to find the value per dollar paid out, so \( PV = -1 \).
The payments are monthly, so \( P/Y = 12 \).
The interest is compounded semi-annually, so \( C/Y = 2 \).
The payments are made at the end of each payment interval, so select END.

Move the cursor to FV to find the future value of $1, and press ALPHA SOLVE.

The future value of $1 is $1.007,282,775.

Since the future value is the amount compounded for the present value, or the investment for $1, \( A = P + i \), where \( i \) represents the monthly interest rate.
\[ A = P + i \]

Substitute known values: \[ 1.007282775 = 1 + i \]

Isolate \( i \):

\[ i = 0.007282775 \]

So, the interest rate per month is 0.007 282 775, or 0.728 277 5%.

Create a spreadsheet with formulas like those shown below. Format the cells to express amounts of money to 2 decimal places.

The Pin family’s mortgage is $85 000, so enter 85 000 in cell A1.
The interest rate per annum is 8.9%, so enter 0.089 in cell A2.
The calculated interest rate per month as a decimal is 0.007 282 775, so enter 0.007 282 775 in cell A3.
The monthly payment is $750.58, so enter 750.58 in cell A4.

Copy the formulas down until the rows show payments for 20 years.

The sum in cell D248 is 180 139.20.

If the Pin family kept the mortgage for 20 years, it would cost $180 139.20.

The solution with a graphing calculator shows that the mortgage would cost $180 139.20, instead of $180 138.12. Such differences occur with rounding.
**Example 3** Comparing the Effects of the Frequency of Payments

Noela is getting a $100,000 mortgage, with an interest rate of 9.2%, amortized over 15 years, on the cottage she is buying. She is able to choose whether to make the payments monthly or biweekly. If Noela pays biweekly, she plans to pay half the amount of the monthly payment. Use a graphing calculator to decide how this choice of the frequency of payments would affect the length of time taken to pay off the mortgage.

**Solution**

To find the length of time for biweekly payments, first calculate the monthly payment.

Change the **mode settings** to 2 decimal places. From the **Finance menu**, choose **TVM Solver**.

Enter the known values for monthly payments.

- There are 12 payments a year for 15 years, so \( N = 15 \times 12 \).
- The interest rate is 9.2% per annum, so \( I = 9.2 \).
- The mortgage is $100,000, so \( PV = 100,000 \).
- A payment is made each month, so \( P/Y = 12 \).
- The interest is compounded semi-annually, so \( C/Y = 2 \).
- The payments are made at the end of each payment interval, so select **END**.

Move the cursor to **PMT** to find the mortgage payment, and press **ALPHA SOLVE**.

Since the mortgage payment is paid out, **PMT** is negative.

Each monthly payment would be $1015.96.

Enter the values for biweekly payments.

- The interest rate is 9.2% per annum, so \( I = 9.2 \).
- The mortgage is $100,000, so \( PV = 100,000 \).
- Since the payment is paid out, **PMT** is negative. Noela plans that each biweekly payment would be half the amount of the monthly payment, so **PMT** = \(-1015.96 \div 2\).
- There are 52 weeks in a year and payments are made every 2 weeks, so \( P/Y = 52 \div 2 \).
- The interest is compounded semi-annually, so \( C/Y = 2 \).
- The payments are made at the end of each payment interval, so select **END**.

When you store a value to **P/Y**, **C/Y automatically changes to match, so go back to **C/Y** and enter 2.
Move the cursor to \( N \) to find the number of payments, and press \( \text{ALPHA SOLVE} \).

For biweekly payments, the number of payments would be 331.37.

Since there are 26 biweekly payments in a year, divide the number of payments by 26 to find the number of years.

\[
\frac{331.37}{26} = 12.745
\]

With these biweekly payments, it would take 12.745 years to pay off the mortgage.

To find the number of months in 0.745 years, multiply 0.745 times 12.

\[
0.745 \times 12 = 8.94
\]

For these biweekly payments it would take 12 years and 8.94 months, or 12 years and 9 months with the last payment less than $507.98. With monthly payments, it would take 15 years to pay off the mortgage.

**Key Concepts**

- The total amount paid for a mortgage depends on the amount of the mortgage, the interest rate, and the frequency of payments. Changing the term of a mortgage affects the total amount of interest and the total amount of the payments.
- Monthly mortgage payments can be calculated using an amortization table, a graphing calculator, or a spreadsheet program.
- Mortgage payments are usually made monthly, but other lengths of time such as biweekly are possible.

**Communicate Your Understanding**

1. List five terms related to mortgages, and explain each.
2. Describe how you would use the amortization table in the Investigate & Inquire to find the monthly mortgage payment for a mortgage of $60 000, at an interest rate of 6.5%, amortized over 25 years.
3. Describe how you would use a graphing calculator to find the monthly mortgage payment for a mortgage of $91 000, at an interest rate of 9.9%, amortized over 18 years.
4. Explain why the compounding period is not given in the examples.
Practise

A

1. What is the amount of each down payment?
   a) 75% down on a price of $214 000
   b) 29% down on a price of $92 800
   c) 80% down on a price of $579 900
   d) 50% down on a price of $74 440

2. What is the mortgage for each property?
   a) a house costing $161 500 with 79% down
   b) a condominium costing $73 000 with $40 000 down
   c) a farm costing $850 000 with 90% down
   d) a cabin costing $19 000 with $12 000 down

3. For each of the following mortgages, use the amortization table in the Investigate & Inquire to find the monthly payment for each $1000.
   a) 10% interest on a mortgage, amortized over 25 years
   b) 5.5% interest on a mortgage, amortized over 10 years
   c) 10.50% interest on a mortgage, amortized over 15 years

4. Why is the amount not needed for the mortgages in question 3?

5. For each mortgage, use the amortization table in the Investigate & Inquire to find the monthly payment.
   a) 9% interest on a $52 000 mortgage, amortized over 20 years
   b) 6.5% interest on a $154 800 mortgage, amortized over 25 years
   c) 11.5% interest on a $87 200 mortgage, amortized over 10 years
   d) 8% interest on a $600 000 mortgage, amortized over 15 years

6. Use a graphing calculator to find the monthly payment for each mortgage.
   a) 7.13% interest on a $52 000 mortgage, amortized over 25 years
   b) 14.42% interest on a $154 800 mortgage, amortized over 15 years
   c) 8.7% interest on a $87 200 mortgage, amortized over 23 years
   d) 6.49% interest on a $250 000 mortgage, amortized over 16 years

7. For each mortgage in question 6, what would be the total cost, assuming that the mortgage was kept for the full amortization period?

Apply, Solve, Communicate

8. Country lot Jesse is arranging a mortgage for $78 000 to buy a lot in the country. The rate is 8.5%, amortized over 25 years.
   a) What are Jesse's monthly mortgage payments?
   b) If Jesse kept the mortgage for 25 years, how much would it cost?
9. **Offices** A group of investors bought two units in an industrial plaza to rent to companies. The two units sold for $780 000. The investors made a down payment and arranged a mortgage for the remaining 70% of the cost. The mortgage is at 9.5%, paid monthly for a 9-year term and amortized over 20 years.

a) What is the monthly payment?

b) How much will the investors owe after 9 years?

10. **Condominium**

a) Josie would like to buy a condo priced at $150 000. She has $80 000 for a down payment. Find her monthly mortgage payments for a 12% mortgage, amortized over

   i) 10 years
   ii) 15 years
   iii) 20 years
   iv) 25 years

b) What happens to the mortgage payment as the amortization period increases? Why do you think this happens?

11. **Assuming a mortgage**

a) Three years ago, the Edward family arranged a mortgage of $81 000 on their cottage. The mortgage is amortized over 25 years, at a rate of 6% for a 5-year term. What is the monthly payment?

b) The Samuel family is buying the cottage, and assuming the Edwards’ mortgage. How much is still owed on this mortgage?

12. **Application** Naomi and Kevin are buying a house for $345 500 with a down payment of $260 000. The owners agreed to take back the mortgage, which means Naomi and Kevin will arrange the mortgage with them, and make the payments to them. The owners are offering an interest rate of 5%, amortized over 15 years with either monthly payments or biweekly payments. If the biweekly offer is accepted, each payment will be half what the monthly payment would be. Which plan would allow the mortgage to be paid off faster? How much faster would it be?

13. **Assumptions** A $100 000 mortgage loan is taken at 6.5% per annum, amortized over 20 years.

a) What is the monthly payment of the loan?

b) Assuming the interest rate remains the same over the period of the loan, how much is the total amount paid for the loan? How likely is this assumption? Explain reasons for your answer.
14. **Inquiry/Problem Solving**  
a) The Camerons are buying a farm for $700,000. After a down payment of $425,000, they are considering a mortgage for the remainder at an interest rate of 6.5%, amortized over 25 years. What would their mortgage payments be?  
b) If the mortgage rate changes to 6% before they make their arrangements, what would their mortgage payments be?  
c) If the mortgage rate changes to 7% before they make their arrangements, what would their mortgage payments be?  
d) Make a generalization about how mortgage rates affect mortgage payments. Why does this make sense?  

15. **Communication**  
a) What are the advantages of using the amortization table to find the monthly payment for a mortgage?  
b) What are the advantages of using a graphing calculator to find the monthly payment for a mortgage?  

16. **Making financial decisions**  
In 1982, Barbara bought a house worth $158,000 with a down payment of $94,000 and a mortgage on the balance at 19.25% per annum, amortized over 15 years. Since the mortgage rate was unusually high, Barbara arranged a 1-year term.  
a) What were her monthly mortgage payments?  
b) How much would she have paid for the house altogether if the mortgage had continued for 15 years?  
c) A year later, when the mortgage rate was 13%, Barbara negotiated a new mortgage, amortized over 14 years, for the remainder she owed. How much were her mortgage payments then?  
d) What factors might influence people as they negotiate the term of a mortgage?  

17. **Paying off a mortgage**  
Mark and Sandy bought a house for $192,000 with a down payment of $120,000. Their mortgage rate is 9%, amortized over 25 years, for a 5-year term.  
a) What is their monthly mortgage payment?  
b) After the 5-year term, they renegotiate for a 3-year term at 6.5%. What is their new monthly payment?  
c) If they continue to make the same payments as they did for the 9% rate, how long would it take to pay off the mortgage?
18. **Estimating**
   a) Use the amortization table in the Investigate & Inquire to estimate the monthly payment for each $1000 of a mortgage, amortized over 25 years, with an interest rate of
   i) 6.3%  
   ii) 19.5%
   b) Explain why your estimation strategies in part a) are reasonable.
   c) Use your estimates from part a) to calculate the monthly payments on a mortgage of $200 000, amortized over 25 years with an interest rate of
   i) 6.3%  
   ii) 19.5%
   d) Use technology to find the monthly mortgage payments at these rates.
   i) 6.3%  
   ii) 19.5%

19. **Increase in interest rates**
   a) Predict, or state from your understanding of mortgages, how an increase in interest rates affects the length of time required to pay off the mortgage.
   b) Explain how you arrived at your prediction.
   c) Devise a way to verify your prediction. Apply this method.
   d) Explain how your answer from part c) compares with your prediction from part a).

20. **Research**
   a) Research financial information about mortgages with regard to the following questions.
      • How are mortgage rates influenced by the economy?
      • How do mortgage rates influence house sales?
      • What techniques are suggested for determining the amount you can afford to carry in a mortgage?
      • What additional costs are involved in buying real estate and setting up a mortgage?
      • What different types of mortgages are available?
   b) Use your research to describe a situation in which someone would need to make a financial decision about a mortgage. Explain what you would recommend for the situation, and justify your explanation with facts and with logical reasoning.

**Achievement Check**

In July 1979, mortgage rates in Canada averaged around 11%. Two years later, they reached the unprecedented level of 22%. For a $100 000 mortgage to be repaid over 25 years, find the required monthly payment at the two different interest rates. What total amount would be paid in each case? Determine the total amount that would be paid using the rates and terms offered for mortgages currently.