Communications Satellites

In 1972, Canada became the first country to use satellites for communications within its own borders. The Anik A1 satellite carried radio and television programs to all parts of the country. The satellite was in a geostationary orbit, which means that it moved in time with the Earth's rotation and stayed a constant distance above a fixed point on the Earth's surface. Therefore, the satellite's orbit was circular.

Anik F1, launched in 2000, carries the bulk of Canada's television signals. This satellite is in a geostationary orbit 35,880 km above a point on the Earth's equator, at a longitude of 107.3°W. Canada has five teleports for uplinking signals from TV and radio stations to geostationary satellites. The teleports are in Montréal, Toronto, Calgary, Edmonton, and Vancouver.

1. The radius of the Earth is about 6370 km. To the nearest kilometre, how far does Anik F1 travel in one day?

2. At what velocity is Anik F1 revolving about the Earth's axis, to the nearest metre per second?

3. At what velocity is a point on the Earth's equator revolving about the Earth's axis, to the nearest metre per second?

4. How many times faster, to the nearest tenth, is Anik F1 revolving than a point on the Earth's equator?

5. The velocity, \( v \) metres per second, of a satellite about the Earth's axis is related to the distance from the centre of the Earth, \( r \) metres, and the acceleration of gravity, \( g \) metres per second per second, by the following equation.

\[
v = \sqrt{gr}
\]

a) Solve the equation for \( g \).

b) Calculate \( g \), at the location of the satellite, to the nearest hundredth of a metre per second per second.

c) The value of \( g \) at the Earth's surface is about 9.8 m/s\(^2\). How many times as great is this value as the value of \( g \) at the location of the satellite, to the nearest whole number?

6. Which Canadian teleport is closest to a longitude of 107.3°W?
Review of Prerequisite Skills

If you need help with any of the skills named in purple below, refer to Appendix A.

1. **Simplifying expressions** Expand and simplify.
   a) \(5x(x - 3) - 2x^2(x - 1)\)
   b) \((2x - 3)^2 - (4x + 5)^2\)
   c) \(2(3x + 2)^2 - (x - 1)^2\)
   d) \(3(x - 1)^2 + 5(1 - 4x)^2\)

2. **Length of a line segment** Find the length of the line segment joining each pair of points. Round answers to the nearest tenth.
   a) \((2, 5)\) and \((7, 2)\)
   b) \((-2, 3)\) and \((5, 1)\)
   c) \((5, -2)\) and \((0, -3)\)
   d) \((-1, -3)\) and \((-6, -9)\)

3. **Midpoint formula** Find the coordinates of the midpoint of the line segment with the given endpoints.
   a) \((-2, 5)\) and \((4, -3)\)
   b) \((3, -2)\) and \((7, 10)\)
   c) \((-3, -5)\) and \((2, -4)\)

4. **Graphing equations** Graph each equation.
   a) \(y = x - 4\)
   b) \(y = 2x - 1\)
   c) \(y = -x + 2\)
   d) \(2x - y = -1\)

5. **Solve for y.**
   a) \(3x + y = 4\)
   b) \(x - 4y = 2\)
   c) \(y^2 = 25\)
   d) \(x^2 + y^2 = 25\)

6. **Solving quadratic equations by factoring** Solve by factoring.
   a) \(x^2 - x - 6 = 0\)
   b) \(2x^2 + 3x - 2 = 0\)
   c) \(4x^2 - 13x + 3 = 0\)
   d) \(6x^2 - 5x + 1 = 0\)
   e) \(6x^2 - 13x - 5 = 0\)
   f) \(5x^2 - 36x + 7 = 0\)

7. **The quadratic formula** Solve using the quadratic formula. Round answers to the nearest tenth, if necessary.
   a) \(x^2 + 3x - 10 = 0\)
   b) \(2x^2 + 5x = 3\)
   c) \(3x^2 + 2 = -7x\)
   d) \(2x^2 + x - 4 = 0\)
   e) \(4x^2 + x - 2 = 0\)
   f) \(12x^2 = 5 - 16x\)

8. **Determine the constant to be added to each polynomial to make it a perfect square trinomial.**
   a) \(x^2 + 12x\)
   b) \(x^2 - 8x\)
   c) \(x^2 + 3x\)
   d) \(x^2 - 6x\)
   e) \(x^2 - 5x\)
   f) \(x^2 + x\)

9. **Rewriting in the Form \(y = a(x - h)^2 + k\), \(a \neq 1\)** Rewrite each of the following the form \(y = a(x - h)^2 + k\). State the maximum or minimum value of \(y\), and the the value of \(x\) when it occurs.
   a) \(y = x^2 + 4x - 5\)
   b) \(y = x^2 - 6x - 10\)
   c) \(y = -x^2 - x + 30\)
   d) \(y = x^2 - 11x + 2\)
   e) \(y = -x^2 - 8x\)
   f) \(y = x^2 + 5x\)

10. **Rewriting in the Form \(y = a(x - h)^2 + k\), \(a = 1\)** Rewrite each of the following the form \(y = a(x - h)^2 + k\). State the maximum or minimum value of \(y\), and the the value of \(x\) when it occurs.
   a) \(y = 2x^2 + 8x - 16\)
   b) \(y = -3x^2 + 6x + 6\)
   c) \(y = 3x^2 + 6x - 8\)
   d) \(y = 4x^2 - 12x\)
   e) \(y = 0.1x^2 + 2x + 1\)
   f) \(y = -0.2x^2 - 6x\)

11. **Solving linear systems** Solve and check.
   a) \(3x + 2y = 7\)
   b) \(4x - y = 7\)
   c) \(3x + 2y = 11\)
   d) \(3x + 4y = -3\)
   e) \(x - 3y = 11\)
   f) \(5x - 2y = -4\)

12. **Write an equation in standard form for the line through the given points.**
   a) \((5, 6)\) and \((4, 8)\)
   b) \((-3, 8)\) and \((0, -1)\)
   c) \((-2, -5)\) and \((-4, -6)\)
   d) \((-3, -4)\) and \((2, -2)\)