Store Profits

A company is hiring staff for its new mega-bookstore. If there are too few staff members, they will not be able to run the store effectively. If there are too many staff members, their salary costs will be too high.

A consultant has advised the company that the average weekly profit per staff member, \( P \) dollars, will be related to the number of staff members, \( s \), by the function

\[
P = -s^2 + 50s - 400.
\]

1. What is the maximum possible weekly profit per staff member?

2. What number of staff members gives the maximum weekly profit per staff member?

3. What is the least number of staff members that will result in a profit?

4. What is the greatest number of staff members that will result in a profit?

5. What numbers of staff members will result in a weekly profit of at least $200 per staff member?

6. The total weekly profit of the store is the product of the number of staff members and the weekly profit per staff member.
   a) Use guess and check to determine whether your answers to questions 1 and 2 result in the greatest total weekly profit for the store.
   b) What should the company do to make the greatest total weekly profit?
   c) What is the greatest total profit the store could make in a year?
Review of Prerequisite Skills

If you need help with any of the skills named in purple below, refer to Appendix A.

1. **Evaluating radicals** Evaluate.
   a) \(\sqrt{121}\)  
   b) \(\sqrt{225}\)  
   c) \(\sqrt{0.09}\)  
   d) \(\sqrt{1.69}\)  
   e) \(\sqrt{0.0016}\)  
   f) \(\sqrt{2 - 3^2}\)  
   g) \(\sqrt{9^2 + 12^2}\)  
   h) \(\sqrt{2 \times 200}\)  
   i) \(\sqrt{g^2 - 4(2)(4)}\)  
   j) \(\sqrt{6^2 - 4(8)(-2)}\)  

2. **Graphing quadratic functions** Sketch the graph of each function and find the coordinates of the vertex, the equation of the axis of symmetry, the maximum or minimum value, and any intercepts.
   a) \(y = 2x^2 - 8\)  
   b) \(y = -3x^2 + 6\)  
   c) \(y = (x - 2)^2 + 3\)  
   d) \(y = -2(x + 1)^2 + 8\)

3. **Solving quadratic equations by graphing** Solve by graphing. Check your solutions.
   a) \(x^2 - x - 6 = 0\)  
   b) \(x^2 + 5x + 4 = 0\)  
   c) \(x^2 - 4 = 0\)  
   d) \(x^2 - 6x = 0\)

4. **Solving quadratic equations by factoring** Solve by factoring. Check your solutions.
   a) \(x^2 + x - 12 = 0\)  
   b) \(x^2 - 10x + 25 = 0\)  
   c) \(y^2 + 8y + 15 = 0\)  
   d) \(t^2 - 4t - 32 = 0\)  
   e) \(2z^2 = 5z + 3\)  
   f) \(9s^2 + 6s + 1 = 0\)  
   g) \(6w^2 - w = 12\)  
   h) \(2x^2 - 12 = 5x\)  
   i) \(0 = 5m^2 + 8m + 3\)  
   j) \(4x^2 = 15x + 4\)  
   k) \(2x^2 - 3x = 0\)  
   l) \(9x^2 - 25 = 0\)

5. **The quadratic formula** Solve using the quadratic formula. Check your solutions.
   a) \(x^2 + 6x + 8 = 0\)  
   b) \(y^2 - 2y - 15 = 0\)  
   c) \(4x^2 = 3 + x\)  
   d) \(2r^2 - r = 3\)  
   e) \(6x^2 + 5x = 6\)  
   f) \(6x^2 + 7x - 20 = 0\)

6. **The quadratic formula** Solve using the quadratic formula. Express answers as exact roots and as approximate roots, to the nearest hundredth.
   a) \(x^2 - 3x + 1 = 0\)  
   b) \(y^2 + 3y - 3 = 0\)  
   c) \(2m^2 - m = 5\)  
   d) \(3t^2 = t + 1\)  
   e) \(4s^2 + 7s = -1\)  
   f) \(x^2 - x - 1 = 0\)  
   g) \(2w^2 + 5w + 1 = 0\)  
   h) \(3x + 3 = 5x^2\)

7. Find the value of \(c\) that will make each expression a perfect square trinomial.
   a) \(x^2 + 10x + c\)  
   b) \(x^2 - 12x + c\)  
   c) \(x^2 - 2x + c\)  
   d) \(x^2 + 8x + c\)  
   e) \(x^2 - 14x + c\)  
   f) \(x^2 + 4x + c\)  
   g) \(x^2 - 30x + c\)  
   h) \(x^2 + 18x + c\)

8. **Rewriting in the form** \(y = a(x - h)^2 + k, a = 1\) Rewrite each of the following in the form \(y = a(x - h)^2 + k\), and state the maximum or minimum value of \(y\) and the value of \(x\) when it occurs.
   a) \(y = x^2 + 2x - 5\)  
   b) \(y = x^2 - 4x + 6\)  
   c) \(y = x^2 + 6x + 2\)  
   d) \(y = -x^2 + 8x - 6\)  
   e) \(y = -x^2 - 6x + 3\)  
   f) \(y = -x^2 + 2x - 5\)  
   g) \(y = x^2 + 10x\)  
   h) \(y = -x^2 + 4x + 1\)

9. **Rewriting in the form** \(y = a(x - h)^2 + k, a \neq 1\) Rewrite each of the following in the form \(y = a(x - h)^2 + k\), and state the maximum or minimum value of \(y\) and the value of \(x\) when it occurs.
   a) \(y = 2x^2 - 8x + 3\)  
   b) \(y = 3x^2 + 6x - 7\)  
   c) \(y = -2x^2 - 12x - 9\)  
   d) \(y = -4x^2 + 8x - 2\)  
   e) \(y = 2x^2 - 20x + 11\)  
   f) \(y = -3x^2 + 18x + 5\)  
   g) \(y = 6x^2 - 12x\)  
   h) \(y = -5x^2 - 20x + 2\)