2.3 Solving Quadratic Equations

The process of completing the square, which was used in Section 2.2 to find the maximum or minimum of a quadratic function, is also one of the ways to solve quadratic equations.

In Roman times, a city forum was a large open space surrounded by buildings. It provided a meeting place and a centre for public life.

The Roman city of Pompeii was destroyed by the eruption of Mount Vesuvius in A.D. 79, but a coating of ash protected the city’s buildings and streets. The continuing excavation of Pompeii allows visitors to see how citizens of the Roman Empire lived.

**Investigate & Inquire**

The forum in Pompeii was a rectangle whose length was 120 m greater than the width.

1. a) Let the width of the forum be $x$. Write an expression in expanded form to represent the area of the forum.
   b) The area of the forum was 6400 m$^2$. Write an equation with the expression from part a) on the left side and the numerical value of the area on the right side.

2. a) Add a number to the left side of the equation to make the left side a perfect square.
   b) Why must this number also be added to the right side of the equation?
   c) Write the left side as the square of a binomial, and simplify the right side.
   d) Take the square root of both sides.
   e) Solve for $x$.


4. For the forum in Pompeii, what was
   a) the width?  
   b) the length?
An equation may have a solution in one set of numbers but not in another. For example, the equation $x + 1 = 0$ has no solution in the set of whole numbers. However, it has a solution, $x = -1$, in the set of integers. Similarly, the equation $x^2 - 2 = 0$ has no solution in the set of rational numbers. However, it has two solutions, $x = \pm \sqrt{2}$, in the set of real numbers.

The equation $x^2 + 1 = 0$ has no solution in the set of real numbers. An attempt to solve the equation gives

$$x^2 = -1$$

and $x = \pm \sqrt{-1}$.

Since $i = \sqrt{-1}$, the solutions of the equation $x^2 + 1 = 0$ can be written as $i$ and $-i$. These solutions are in the set of pure imaginary numbers. Many quadratic equations have roots that are pure imaginary numbers or imaginary numbers.

All quadratic functions have two zeros, that is, two values of the independent variable that make the value of the function equal to zero. If the graph of a quadratic function intersects the x-axis, the zeros of the function are real zeros, that is, they are real numbers. If there are two real zeros, they can be distinct or equal. If the graph of a quadratic function does not intersect the x-axis, the function has two imaginary zeros. The three possibilities are shown in the graphs.

![Graphs showing two distinct real zeros, two equal real zeros, and two imaginary zeros.](image)

The quadratic equations that correspond to the functions shown in the graphs have two distinct real roots, two equal real roots, and two imaginary roots.

5. a) Describe two other algebraic methods for solving the same equation.
b) Which of the three methods do you prefer? Explain.

6. Solve each of the following equations using a method of your choice.
a) $x^2 + 4x = 12$
b) $x^2 - 2x = 3$
c) $x^2 + 6x = -8$
The algebraic method of completing the square gives exact solutions to a quadratic equation. One of the steps in the process involves using the square root principle. For example, if \((x + 2)^2 = 9\) then \(x + 2 = \pm 3\)

\[
x + 2 = 3 \quad \text{or} \quad x + 2 = -3
\]

\[
x = 1 \quad \text{or} \quad x = -5
\]

**Example 1 Solving by Completing the Square**

Solve \(x^2 - 6x - 27 = 0\) by completing the square.

**Solution**

Add 27 to both sides:

\[
x^2 - 6x = 27
\]

Add the square of half the coefficient of \(x\) to both sides:

\[
x^2 - 6x + 9 = 27 + 9
\]

Write the left side as the square of a binomial:

\[
(x - 3)^2 = 36
\]

Take the square root of both sides:

\[
x - 3 = \pm 6
\]

Solve for \(x\):

\[
x = 9 \quad \text{or} \quad x = -3
\]

The roots are 9 and -3.

The solution to Example 1 can be modelled graphically. The equation has two distinct real roots. The graph of the corresponding function has two distinct real zeros.

Note that the roots of the equation \(x^2 - 6x - 27 = 0\) are the \(x\)-intercepts of the graph of the corresponding function, \(y = x^2 - 6x - 27\).
To solve by completing the square when the coefficient of \(x^2\) is not 1, divide each term of the equation by the coefficient of \(x^2\) before completing the square.

**Example 2  Solving by Completing the Square, \(a \neq 1\)**

Solve \(2x^2 - 5x - 1 = 0\) by completing the square. Express answers as exact roots and as approximate roots, to the nearest hundredth.

**Solution**

\[
\begin{align*}
2x^2 - 5x - 1 &= 0 \\
x^2 - \frac{5}{2}x - \frac{1}{2} &= 0 \\
\end{align*}
\]

Divide both sides by 2:

\[
\begin{align*}
x^2 - \frac{5}{2}x &= \frac{1}{2} \\
\end{align*}
\]

Add \(\frac{1}{2}\) to both sides:

\[
\begin{align*}
x^2 - \frac{5}{2}x + \frac{25}{16} &= \frac{1}{2} + \frac{25}{16} \\
\end{align*}
\]

Complete the square:

\[
\begin{align*}
\left(x - \frac{5}{4}\right)^2 &= \frac{33}{16} \\
x - \frac{5}{4} &= \pm\frac{\sqrt{33}}{4} \\
x &= \frac{5 \pm \sqrt{33}}{4}
\end{align*}
\]

Write the left side as the square of a binomial:

\[
\begin{align*}
\end{align*}
\]

Take the square root of both sides:

Solve for \(x\):

The exact roots are \(\frac{5 + \sqrt{33}}{4}\) and \(\frac{5 - \sqrt{33}}{4}\).

**Estimate**

\[
\begin{align*}
\sqrt{33} & \approx 6 \\
\frac{5 + 6}{4} & = 3 \\
\frac{5 - 6}{4} & = -\frac{1}{4}
\end{align*}
\]

The approximate roots are 2.69 and \(-0.19\), to the nearest hundredth.
Example 3 Finding Complex Roots by Completing the Square

Solve $3x^2 + 2x + 6 = 0$ by completing the square.

Solution

Divide both sides by 3:

$3x^2 + 2x + 6 = 0$

$x^2 + \frac{2}{3}x + 2 = 0$

$x^2 + \frac{2}{3}x = -2$

$x^2 + \frac{2}{3}x + \frac{1}{9} = -2 + \frac{1}{9}$

$(x + \frac{1}{3})^2 = -\frac{17}{9}$

$x + \frac{1}{3} = \pm \sqrt{-17} \times \frac{3}{3}$

$x + \frac{1}{3} = \pm \sqrt{17} \times \frac{3}{3}$

$x = -1 \pm \frac{\sqrt{17}}{3}$

The roots are $\frac{-1 + \sqrt{17}}{3}$ and $\frac{-1 - \sqrt{17}}{3}$.

The solution to Example 3 can be modelled graphically. The equation has two imaginary roots. The graph of the corresponding function does not intersect the $x$-axis and has two imaginary zeros.
Example 4 Solving by Factoring

Solve \(4x^2 - 11x = x - 9\) by factoring. Check the solution.

Solution

Write in the form \(ax^2 + bx + c = 0\):

\[
4x^2 - 12x + 9 = 0
\]

Factor the left side:

\[
(2x - 3)(2x - 3) = 0
\]

Use the zero product property:

\[
2x - 3 = 0 \quad \text{or} \quad 2x - 3 = 0
\]

\[
x = \frac{3}{2} \quad \text{or} \quad x = \frac{3}{2}
\]

Check.

For \(x = \frac{3}{2}\),

\[
\text{L.S.} = 4x^2 - 11x
\]

\[
= 4\left(\frac{3}{2}\right)^2 - 11\left(\frac{3}{2}\right)
\]

\[
= 4\left(\frac{9}{4}\right) - \frac{33}{2}
\]

\[
= 9 - \frac{33}{2}
\]

\[
= \frac{18 - 33}{2}
\]

\[
= -\frac{15}{2}
\]

\[
\text{L.S.} = \text{R.S.}
\]

There are two equal roots, \(\frac{3}{2}\) and \(\frac{3}{2}\).

The solution to Example 4 can be modelled graphically. The equation has two equal real roots. The graph of the corresponding function has two equal real zeros.

\[
y = 4x^2 - 12x + 9
\]

\((1, 0)\)
**Example 5 Solving Using the Quadratic Formula**

Solve $x^2 - 2x + 3 = 0$ using the quadratic formula.

**Solution**

For $x^2 - 2x + 3 = 0$, $a = 1$, $b = -2$, and $c = 3$.

Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 12}}{2}$$

$$= \frac{2 \pm \sqrt{-8}}{2}$$

$$= \frac{2 \pm 2i\sqrt{2}}{2}$$

$$= 1 \pm i\sqrt{2}$$

The roots are $1 + i\sqrt{2}$ and $1 - i\sqrt{2}$.

**Example 6 Checking Imaginary Roots**

Solve and check $x^2 + 4 = 0$.

**Solution**

$$x^2 + 4 = 0$$

Subtract 4 from both sides: $x^2 = -4$

Take the square root of both sides: $x = \pm \sqrt{-4}$

Simplify:

$$x = \pm \sqrt{-1 \times 4}$$

$$= \pm 2i$$

$$x = 2i$$ or $$x = -2i$$
2.3 Solving Quadratic Equations

Check.
For \( x = 2i \),
\[
\begin{align*}
\text{L.S.} &= x^2 + 4 \\
&= (2i)^2 + 4 \\
&= 4 \times (-1) + 4 \\
&= -4 + 4 \\
&= 0
\end{align*}
\]
\[
\begin{align*}
\text{R.S.} &= 0
\end{align*}
\]
L.S. = R.S.

For \( x = -2i \),
\[
\begin{align*}
\text{L.S.} &= x^2 + 4 \\
&= (-2i)^2 + 4 \\
&= 4 \times (-1) + 4 \\
&= -4 + 4 \\
&= 0
\end{align*}
\]
\[
\begin{align*}
\text{R.S.} &= 0
\end{align*}
\]
L.S. = R.S.

The roots are \( 2i \) and \( -2i \).

Example 7 Planning a Dining Room

A preliminary floor plan for a new house includes a rectangular dining room that measures 5 m by 4 m. The customers want a larger dining room and agree to an area of 25 m\(^2\). The architect decides to redesign the floor plan by adding a strip of floor of uniform width to two adjacent sides of the dining room, as shown in the diagram. How wide should the strip be, to the nearest thousandth of a metre?

Solution

Let the width of the strip be \( x \) metres.
Write and solve an equation to find \( x \).
The dimensions of the new dining room are \((5 + x)\) by \((4 + x)\).
The area of the new dining room is 25 m\(^2\).

Write the equation:
\[
(5 + x)(4 + x) = 25
\]
Expand the left side:
\[
20 + 9x + x^2 = 25
\]
Write in the form \( ax^2 + bx + c = 0 \):
\[
x^2 + 9x - 5 = 0
\]
For \( x^2 + 9x - 5 = 0 \), \( a = 1 \), \( b = 9 \), and \( c = -5 \).
Use the quadratic formula.
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{-9 \pm \sqrt{81 - 4(1)(-5)}}{2}
\]
\[
= \frac{-9 \pm \sqrt{101}}{2}
\]
The width cannot be negative, so the root −9.525 is inadmissible and is rejected.

The strip should be 0.525 m wide, to the nearest thousandth of a metre.

**Key Concepts**

- To solve a quadratic equation by completing the square, first complete the square, and then take the square root of both sides to find the roots.
- To solve a quadratic equation by factoring, write the equation in the form \(ax^2 + bx + c = 0\), factor \(ax^2 + bx + c\), use the zero product property, and then solve the two resulting equations to find the roots.
- To solve a quadratic equation using the quadratic formula, write the equation in the form \(ax^2 + bx + c = 0\), \(a \neq 0\), and substitute the values for \(a\), \(b\), and \(c\) into the formula \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\) to find the roots.

**Communicate Your Understanding**

1. Describe how you would solve \(x^2 + 6x + 7 = 0\) by completing the square.
2. Describe how you would solve \(2x^2 + 7x = -3\) by factoring.
3. Describe how you would use the quadratic formula to solve \(x^2 = 1 - x\).

**Practice**

**A**

1. State the value of \(k\) that makes each expression a perfect square trinomial. Then, write the trinomial as the square of a binomial.
   
   \(\begin{align*}
   a) \ x^2 + 10x + k & \quad b) \ w^2 - 14w + k \\
   c) \ x^2 + 7x + k & \quad d) \ p^2 - 5p + k \\
   e) \ x^2 + \frac{4}{3}x + k & \quad f) \ d^2 - \frac{2}{3}d + k \\
   g) \ x^2 + 1.4x + k & \quad h) \ x^2 - 0.06x + k
   \end{align*}\)

2. Solve.
   
   \(\begin{align*}
   a) \ (x + 3)^2 & = 9 \\
   b) \ (x - 10)^2 - 1 & = 0 \\
   c) \ (s - 1)^2 & = 4 \\
   d) \ (y - 4)^2 - 25 & = 0 \\
   e) \ 4 = \left(x + \frac{1}{2}\right)^2 & \quad f) \ \left(x - \frac{1}{3}\right)^2 = \frac{1}{9} \\
   g) \ \left(a + \frac{3}{4}\right)^2 = \frac{9}{16} & \quad h) \ (n - 0.5)^2 = 1.21 \\
   i) \ (x + 0.4)^2 - 0.01 & = 0
   \end{align*}\)
3. Solve by completing the square. Express solutions in simplest radical form.
   a) $x^2 + 6x + 4 = 0$
   b) $w^2 - 4w - 11 = 0$
   c) $t^2 + 8t - 7 = 0$
   d) $x^2 - 10x = 3$
   e) $d^2 = 7d - 9$
   f) $0 = x^2 - 5x + 2$
   g) $x - 3 = -x^2$
   h) $4 + y^2 = 20y$

4. Solve by completing the square. Express solutions in simplest radical form.
   a) $2x^2 + 8x + 5 = 0$
   b) $3x^2 - 6x + 2 = 0$
   c) $6x^2 + 3x - 2 = 0$
   d) $0 = 3w^2 - 5w - 2$
   e) $2x - 6 = -5x^2$
   f) $1 - 2z = 5z^2$
   g) $\frac{1}{2}x^2 + x - 13 = 0$
   h) $0.3y^2 - 0.2y = 0.3$

5. Solve by completing the square. Round solutions to the nearest hundredth.
   a) $x^2 + 2x - 1 = 0$
   b) $x^2 - 4x + 1 = 0$
   c) $d^2 + d = 7$
   d) $0 = 2r^2 - 8r + 3$
   e) $7x + 4 = -2x^2$
   f) $\frac{2}{3}x^2 - 2x - 3 = 0$
   g) $\frac{1}{4}n^2 + n = -1\frac{1}{8}$
   h) $1.2x^2 - 3x - 6 = 0$

6. Solve by completing the square.
   a) $x^2 + x + 6 = 0$
   b) $y^2 - 2y + 8 = 0$
   c) $x^2 + 6x = -17$
   d) $2x^2 - 3x + 6 = 0$
   e) $2n^2 + 4 = 5n$
   f) $3m^2 + 8m + 8 = 0$
   g) $\frac{1}{2}x^2 + x + 1 = 0$
   h) $0.1g^2 - 0.3g + 0.5 = 0$

7. Solve.
   a) $(x - 4)(x + 7) = 0$
   b) $(2y + 3)(y - 1) = 0$
   c) $(3z + 1)(4z + 3) = 0$
   d) $(2x - 5)(2x - 5) = 0$

8. Solve by factoring. Check solutions.
   a) $x^2 + 3x - 40 = 0$
   b) $x^2 - x = 12$
   c) $y^2 - 12y - 36$
   d) $z^2 - 30 = -z$
   e) $a^2 - 4 = 3a$
   f) $b^2 - 5b = 0$

   a) $4x^2 - 3 = 11x$
   b) $4y^2 - 17y = -4$
   c) $9z^2 = -24z - 16$
   d) $3x^2 = 4x + 15$
   e) $4x^2 = 25$
   f) $2m^2 + 9m = 5$
   g) $8t^2 = 1 - 2t$
   h) $y - 2 = -6y^2$
   i) $6x^2 + 7x + 2 = 0$
   j) $5z^2 + 44z = 60$
   k) $9r^2 - 16 = 0$
   l) $2x^2 = -18 - 12x$

10. Solve by factoring.
    a) $3p^2 = 15 - 4p$
    b) $3x^2 + 7x = 0$
    c) $4r^2 + 9 = 12r$
    d) $4y^2 - 11y - 3 = 0$
    e) $3t^2 + 13t = 10$
    f) $2x^2 - 5x = 0$
    g) $6n^2 = n + 5$
    h) $6t^2 + 7t = 3$
    i) $4m^2 - 12 = 13m$
    j) $9x^2 - 17x + 8 = 0$
    k) $\frac{x^2}{4} + x + 1 = 0$
    l) $\frac{x^2 + x - \frac{1}{3}}{2} = \frac{3}{6}$

11. Solve using the quadratic formula.
    a) $4x^2 - 12x + 5 = 0$
    b) $3y^2 + 5y = 28$
    c) $2m^2 = 15 + m$
    d) $2z^2 + 3 = 5z$
    e) $3r^2 - 20 = 7r$
    f) $4x^2 = 11x + 3$
    g) $5a^2 - a = 4$
    h) $15 + w - 6w^2 = 0$

12. Solve using the quadratic formula.
    Express answers as exact roots and as approximate roots, to the nearest hundredth.
    a) $3x^2 + 6x + 1 = 0$
    b) $2t^2 + 6t = -3$
    c) $4y^2 + 7 - 12y = 0$
    d) $m^2 + 4 = -6m$
    e) $2z^2 = 6z - 1$
    f) $2x^2 = 11$
    g) $3r^2 - 3r = 1$
    h) $3n - 1 + n^2 = 0$
    i) $3x^2 = 2 - 6x$
    j) $5 + 5t - t^2 = 0$
    k) $0.1 - 0.3m = 0.2m^2$
    l) $\frac{x + 1}{2} = \frac{7x^2}{2}$
    m) $y^2 + \frac{1}{6} = 0$
    n) $\frac{t^2}{5} = \frac{1}{2}$
### 13. Solve using the most appropriate method.

- **a)** \(x^2 + 2x + 2 = 0\)
- **b)** \(z^2 + 5z + 8 = 0\)
- **c)** \(x^2 - x + 7 = 0\)
- **d)** \(2x^2 + 3x + 3 = 0\)
- **e)** \(5x^2 + 5x + 2 = 0\)
- **f)** \(-y^2 + 3y - 9 = 0\)
- **g)** \(2x^2 + 3x + 3 = 0\)
- **h)** \(3m^2 - 4m = -2\)
- **i)** \(4y - 1 = 5y^2\)

### 14. Solve and check.

- **a)** \(x^2 + 9 = 0\)
- **b)** \(y^2 + 16 = 0\)
- **c)** \(n^2 - 3n + 3 = 0\)
- **d)** \(2k^2 + 50 = 0\)
- **e)** \(n^2 + 20 = 0\)
- **f)** \(6c^2 + 72 = 0\)
- **g)** \(-2x^2 - 16 = 0\)
- **h)** \(\frac{x^2}{2} - \frac{x^2}{3} = -1\)

### Apply, Solve, Communicate

#### 15. The Olympeion

The enormous temple Olympeion was constructed in Athens, Greece, in 174 B.C. The base of the temple is a rectangle with a perimeter of 300 m. The area of the base is 4400 m\(^2\). What are the dimensions of the base?

#### 16. Gardening

A rectangular lawn measures 7 m by 5 m. A uniform border of flowers is to be planted along two adjacent sides of the lawn, as shown in the diagram. If the flowers that have been purchased will cover an area of 6.25 m\(^2\), how wide is the border?

![Diagram of a rectangular lawn with a 5 m border](image)

#### 17. Measurement

The base of a triangle is 2 cm more than the height. The area of the triangle is 5 cm\(^2\). Find the base, to the nearest tenth of a centimetre.

#### 18. Measurement

The length of a rectangle is 2 m more than the width. The area of the rectangle is 20 m\(^2\). Find the dimensions of the rectangle, to the nearest tenth of a metre.

#### 19. Integers

The sum of an integer and its square is 210. Find the integer.

#### 20. Measurement

A square of side length \(x + 1\) has an area of 6 square units. Find the value of \(x\), to the nearest hundredth.

#### 21. Numbers

The sum of two numbers is 14, and their product is 37. What are the numbers

- **a)** in simplest radical form?
- **b)** to the nearest thousandth?

#### 22. Numbers

Subtracting a number from half its square gives a result of 13. Express the possible values of the number in simplest radical form.

#### 23. Application

Two positive integers are in the ratio 1:3. If their sum is added to their product, the result is 224. Find the integers.
24. **Whole numbers** Two whole numbers differ by 3. The sum of their squares is 89. What are the numbers?

25. **Architecture** Is it possible to design a building so that the rectangular ground floor has a perimeter of 50 m and an area of 160 m²? Explain.

26. **Building a fence** Is it possible to build a fence on three sides of a rectangular piece of land with an area of 100 m², so that the total length of the fence is as follows? If so, what are the dimensions of the piece of land?
   a) 30 m
   b) 25 m

27. **Measurement** Is it possible for a rectangle with a perimeter of 44 cm to have each of the following areas? If so, find the dimensions of the rectangle.
   a) 125 cm²
   b) 121 cm²
   c) 117 cm²

28. Solve. Express each solution in simplest radical form.
   a) \(x(x + 3) = 2x(x + 5) + 1\)
   b) \(3(n - 1)^2 = (n + 1)(2n + 1)\)
   c) \(\frac{1}{2}(r + 2)^2 = \frac{1}{3}(2r - 1)^2\)
   d) \((4x - 1)(3x + 7) = (5x - 1)(2x + 3) - 6\)

29. **Football** The function \(h = -5t^2 + 20t + 2\) gives the approximate height, \(h\) metres, of a thrown football as a function of the time, \(t\) seconds, since it was thrown. The ball hit the ground before a receiver could get near it.
   a) How long was the ball in the air, to the nearest tenth of a second?
   b) For how many seconds was the height of the ball at least 17 m?

30. **Measurement** The difference between the length of the hypotenuse and the length of the next longest side of a right triangle is 3 cm. The difference between the lengths of the two perpendicular sides is 3 cm. Find the three side lengths.

31. **Numbers** Two numbers have a sum of 31. Find the numbers if their product is as follows. Express radical solutions in simplest radical form.
   a) 240
   b) 230
   c) 250
   d) \(\frac{385}{4}\)

32. **Technology**
   a) Graph the equation \(y = x^2 - 2x + 2\) using a graphing calculator.
   b) Describe the results when you try to find the zeros of the function using the **Zero operation**.
   c) Repeat part b) for another quadratic function with imaginary roots.
33. **Inquiry/Problem Solving** For what values of \( k \) is \( x^2 + kx + \frac{49}{4} \) a perfect square trinomial?

34. **Airborne object** The height, \( h \) metres, of an object fired upward from the ground at 50 m/s is given approximately by the equation
\[
h = -5t^2 + 50t
\]
where \( t \) seconds is the time since the object was launched.

Does an object fired upward at 50 m/s reach each of the following heights? If so, after how many seconds is the object at the given height?

a) 45 m  
b) 125 m  
c) 150 m

35. **Communication** Is it possible for a quadratic equation to have one real root and one imaginary root? Explain using examples.

36. Solve and check.

a) \((3t + 2)^2 = (2t - 5)^2\)  
b) \((x - 3)(x + 3) = 7\)

c) \((y - 1)^2 = 2y - 3\)  
d) \((m - 2)(m + 2) - 3(m + 5) + 1 = 0\)

37. Solve. Express answers as exact roots and as approximate roots, to the nearest hundredth.

a) \((x + 5)(x - 1) = -2\)  
b) \((m + 2)^2 + 7(m + 2) = 3\)

c) \(2r(r + 2) - 3(1 - r) = 0\)  
d) \((2x - 5)^2 - (x + 1)^2 = 2\)

38. Solve.

a) \((x + 6)(2x + 5) + 8 = 0\)  
b) \((2n - 1)^2 = 5(n - 3)\)

c) \(\frac{(w + 1)^2}{3} = \frac{1}{4}\)  
d) \(2x(x + 3) - 6 = x(4 - x)\)

39. **Measurements of lengths and areas** A mural is to be painted on a wall that is 15 m long and 12 m high. A border of uniform width is to surround the mural. If the mural is to cover 75% of the area of the wall, how wide must the border be, to the nearest hundredth of a metre?

40. Solve each equation for \( x \) by completing the square.

a) \(x^2 + 2x = k\)  
b) \(kx^2 - 2x = k\)  
c) \(x^2 = kx + 1\)

41. Write a quadratic equation with the given roots in the form \(ax^2 + bx + c = 0\).

a) \(\sqrt{5} \text{ and } -\sqrt{5}\)  
b) \(2i \text{ and } -2i\)
42. Find the values of $k$ that make $x^2 + (k + 7)x + (7k + 1)$ a perfect square trinomial.

43. Solve $x^2 + bx + c = 0$ for $x$ by completing the square.

44. Solve for $x$ in terms of the other variables.
   a) $x^2 - t = 0$
   b) $ax^2 - b = 0$
   c) $rx^2 + tx = 0$
   d) $x^2 - mx - t = 0$

45. **Discriminant** In the quadratic formula, the discriminant is the quantity $b^2 - 4ac$ under the radical sign. What can you conclude about the value of $b^2 - 4ac$ if the equation has
   a) two equal real roots?
   b) two distinct real roots?
   c) two imaginary roots?

---

**Knowledge/Understanding**

Show that the equation $2x^2 - 5x + 2 = 0$ has roots that are reciprocals of each other. Under what conditions will a quadratic equation in the form $ax^2 + bx + c = 0$ have roots that are reciprocals of each other?

---

**CAREER CONNECTION  Publishing**

Many thousands of publications are produced each year in Canada, including books, magazines, and newspapers, and multimedia and online publications. The publishing industry employs tens of thousands of Canadians and contributes billions of dollars to the economy. Much of the Canadian publishing industry is based in Ontario.

1. **Book sales** A publishing company expects to sell 5000 copies of a new book from its web site, if the company charges $30 per book. The company expects that 500 more books would be sold for each price reduction of $2.
   a) For the company to make revenues of $156,000 from sales of the book, how many books must be sold and at what price?
   b) If its assumptions are correct, can the company make $160,000 in revenues from sales of the book? Explain.

2. **Research** Use your research skills to investigate a publishing career that interests you. Examples might include journalist, editor, designer, or marketer. Describe the education and training needed for the career, and outline the tasks that someone with this background might undertake.