3.6 Stretches of Functions

The clock in the Heritage Hall clock tower, in Vancouver, is known as Little Ben. It was built by the same company that built Big Ben in London, England. Little Ben is a mechanical clock with a pendulum.

On the Earth, the period of a pendulum is approximately represented by the function $T = \frac{2\sqrt{l}}{l}$, where $T$ is the period, in seconds, and $l$ is the length of the pendulum, in metres. Since the force of gravity varies from one location to another in the solar system, the function for the period of a pendulum also varies. On the moon, the function is $T = \frac{5\sqrt{l}}{l}$. On Pluto, the function is $T = \frac{8\sqrt{l}}{l}$.

**Investigate & Inquire**

1. Let $y = T$ and $x = l$, and graph the three functions above, plus the function $y = \sqrt{x}$, on the same set of axes or in the same viewing window of a graphing calculator.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = \sqrt{x}$</th>
<th>$y = 2\sqrt{x}$</th>
<th>$y = 5\sqrt{x}$</th>
<th>$y = 8\sqrt{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

2. For the functions $y = \sqrt{x}$ and $y = 2\sqrt{x}$, how do the $y$-coordinates of any two points that have the same non-zero $x$-coordinate compare? Explain why.

3. For the functions $y = \sqrt{x}$ and $y = 5\sqrt{x}$, how do the $y$-coordinates of any two points that have the same non-zero $x$-coordinate compare? Explain why.

4. For the functions $y = \sqrt{x}$ and $y = 8\sqrt{x}$, how do the $y$-coordinates of any two points that have the same non-zero $x$-coordinate compare? Explain why.
5. a) If Little Ben were placed on Pluto, the period of its pendulum would be 9.8 s. What is the length of the pendulum, to the nearest tenth of a metre?

b) If Little Ben were placed on the moon, what would the period of its pendulum be, to the nearest tenth of a second?

**Example 1 Vertical Stretching**

**a)** Copy the graph of \( y = f(x) \), as shown. On the same set of axes, graph \( y = 3f(x) \) and \( y = 0.5f(x) \).

**b)** Describe how the graphs of \( y = 3f(x) \) and \( y = 0.5f(x) \) are related to the graph of \( y = f(x) \).

**Solution**

**a)** Use the graph to complete a table of values for \( y = f(x) \). Then, complete tables of values for \( y = 3f(x) \) and \( y = 0.5f(x) \), and draw the graphs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-4</td>
<td>-2</td>
<td>-4</td>
<td>-6</td>
<td>-4</td>
<td>-1</td>
</tr>
</tbody>
</table>

**b)** Comparing \( y = f(x) \) and \( y = 3f(x) \), the point \((x, y)\) on the graph of \( y = f(x) \) is transformed to the point \((x, 3y)\) on the graph of \( y = 3f(x) \). The graph of \( y = 3f(x) \) is the graph of \( y = f(x) \) expanded vertically by a factor of 3.

Comparing \( y = f(x) \) and \( y = 0.5f(x) \), the point \((x, y)\) on the graph of \( y = f(x) \) is transformed to the point \((x, 0.5y)\) on the graph of \( y = 0.5f(x) \). The graph of \( y = 0.5f(x) \) is the graph of \( y = f(x) \) compressed vertically by a factor of 0.5.
In Example 1, the points (7, 0), (3, 0), and (−1, 0) are points on the graphs of all three functions. Recall that these points are said to be invariant, because they are unaltered by the transformations. The three functions have the same domain, $-4 \leq x \leq 7$, but different ranges. The range of $y = f(x)$ is $-2 \leq y \leq 3$, of $y = 3f(x)$ is $-6 \leq y \leq 9$, and of $y = 0.5f(x)$ is $-1 \leq y \leq 1.5$.

Note from Example 1 that a stretch may be an expansion or a compression. An expansion is a stretch by a factor greater than 1. A compression is a stretch by a factor less than 1.

**Example 2  Vertical Stretching of a Quadratic Function**

**a)** Graph $y = x^2$, $y = 2x^2$, and $y = \frac{2}{3}x^2$ on the same grid.

**b)** Describe how the graphs of $y = 2x^2$ and $y = \frac{2}{3}x^2$ are related to the graph of $y = x^2$.

**Solution**

**a)** Complete tables of values using convenient values of $x$, or use a graphing calculator.

\[
\begin{array}{|c|c|} 
\hline x & y \\
\hline 3 & 9 \\
2 & 4 \\
1 & 1 \\
0 & 0 \\
-1 & 1 \\
-2 & 4 \\
-3 & 9 \\
\hline
\end{array}
\quad
\begin{array}{|c|c|} 
\hline x & y \\
\hline 3 & 18 \\
2 & 8 \\
1 & 2 \\
0 & 0 \\
-1 & 2 \\
-2 & 8 \\
-3 & 18 \\
\hline
\end{array}
\quad
\begin{array}{|c|c|} 
\hline x & y \\
\hline 3 & 6 \\
2 & 3 \\
1 & 0 \\
0 & -3 \\
-1 & -6 \\
-2 & -24 \\
-3 & -18 \\
\hline
\end{array}
\]

**b)** Given $y = 2x^2$, then $y = 2(x^2)$. The point $(x, y)$ on the graph of the function $y = x^2$ is transformed to the point $(x, 2y)$ on the graph of $y = 2x^2$. The graph of $y = 2x^2$ is the graph of $y = x^2$ expanded vertically by a factor of 2.

Given $y = \frac{2}{3}x^2$, then $y = \frac{2}{3}(x^2)$. The point $(x, y)$ on the graph of the function $y = x^2$ is transformed to the point $(x, \frac{2}{3}y)$ on the graph of $y = \frac{2}{3}x^2$. The graph of $y = \frac{2}{3}x^2$ is the graph of $y = x^2$ compressed vertically by a factor of $\frac{2}{3}$.
In general, for any function \( y = f(x) \), the graph of the function \( y = af(x) \), where \( a \) is any real number, is obtained by multiplying the \( y \)-value at each point on the graph of \( y = f(x) \) by \( a \).

The point \((x, y)\) on the graph of the function \( y = f(x) \) is transformed into the point \((x, ay)\) on the graph of \( y = af(x) \).

If \( a > 1 \), the graph expands vertically by a factor of \( a \).

If \( 0 < a < 1 \), the graph is compressed vertically by a factor of \( a \).

**Example 3** Horizontal Stretching

Given the graph of \( y = f(x) \), compare it to the graphs of

a) \( y = f(2x) \)  

b) \( y = f\left(\frac{1}{2}x\right) \)

**Solution**

a) Use the given graph to complete a table of values for \( y = f(x) \). Then, complete a table of values for \( y = f(2x) \). Use convenient values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>( f(2 \times (-1)) = f(-2) = 0 )</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>-0.5</td>
<td>( f(2 \times (-0.5)) = f(-1) = 2 )</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td>( f(2 \times 0) = f(0) = 4 )</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.5</td>
<td>( f(2 \times 0.5) = f(1) = 3 )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>( f(2 \times 1) = f(2) = 2 )</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>( f(2 \times 1.5) = f(3) = 1 )</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>2</td>
<td>( f(2 \times 2) = f(4) = 0 )</td>
</tr>
</tbody>
</table>

Note that, in the table for \( y = f(2x) \), \( x \)-values such as \(-2 \) and \( 4 \) are not convenient, because \( f(2 \times (-2)) = f(-4) \) and \( f(2 \times 4) = f(8) \). Both \( f(-4) \) and \( f(8) \) are not defined for the function \( y = f(x) \).

For non-zero values of \( x \), each point on \( y = f(2x) \) is half as far from the \( y \)-axis as the equivalent point on \( y = f(x) \).

The point \((x, y)\) on the graph of the function \( y = f(x) \) is transformed to the point \(\left(\frac{x}{2}, y\right)\) on the graph of \( y = f(2x) \). The graph of \( y = f(2x) \) is a horizontal compression of the graph of \( y = f(x) \) by a factor of \( \frac{1}{2} \).
b) Use the table of values from part a) for \( y = f(x) \).

\[
\begin{array}{c|c}
 x & y \\
-2 & 0 \\
-1 & 2 \\
0 & 4 \\
1 & 3 \\
2 & 2 \\
3 & 1 \\
4 & 0 \\
\end{array}
\]

Then, complete a table of values for \( y = f\left(\frac{1}{2}x\right) \).

\[
\begin{array}{c|c}
 x & y \\
-4 & f\left(\frac{1}{2} \times (-4)\right) = f(-2) = 0 \\
-2 & f\left(\frac{1}{2} \times (-2)\right) = f(-1) = 2 \\
0 & f\left(\frac{1}{2} \times 0\right) = f(0) = 4 \\
2 & f\left(\frac{1}{2} \times 2\right) = f(1) = 3 \\
4 & f\left(\frac{1}{2} \times 4\right) = f(2) = 2 \\
6 & f\left(\frac{1}{2} \times 6\right) = f(3) = 1 \\
8 & f\left(\frac{1}{2} \times 8\right) = f(4) = 0 \\
\end{array}
\]

The functions have the same range, \( 0 \leq y \leq 4 \). The domain of \( y = f(x) \) is \(-2 \leq x \leq 4\). The domain of \( y = f\left(\frac{1}{2}x\right) \) is \(-4 \leq x \leq 8\).

For non-zero values of \( x \), each point on the graph of \( y = f\left(\frac{1}{2}x\right) \) is twice as far from the \( y \)-axis as the equivalent point on \( y = f(x) \). The point \((x, y)\) on the graph of the function \( y = f(x) \) is transformed to the point \((2x, y)\) on the graph of \( y = f\left(\frac{1}{2}x\right) \).

The graph of \( y = f\left(\frac{1}{2}x\right) \) is a horizontal expansion of the graph of \( y = f(x) \) by a factor of 2.

Note that the point \((0, 4)\) is invariant under both transformations in Example 3, because this point lies on the \( y \)-axis.
**Example 4  Horizontal Stretching of a Radical Function**

Compare the graphs of \( y = \sqrt{2x} \) and \( y = \frac{1}{\sqrt{2}}x \) to the graph of \( y = \sqrt{x} \).

**Solution**

Complete tables of values using convenient values for \( x \).

\[
\begin{array}{c|c} \hline x & y \hline \end{array} \quad \begin{array}{c|c} \hline x & y \hline \end{array} \quad \begin{array}{c|c} \hline x & y \hline \end{array}
\]

\[
\begin{array}{c|c} \hline 0 & 0 \hline \end{array} \quad \begin{array}{c|c} \hline 0 & 0 \hline \end{array} \quad \begin{array}{c|c} \hline 0 & 0 \hline \end{array}
\]

\[
\begin{array}{c|c} \hline 1 & 1 \hline \end{array} \quad \begin{array}{c|c} \hline 0.5 & 1 \hline \end{array} \quad \begin{array}{c|c} \hline 2 & 1 \hline \end{array}
\]

\[
\begin{array}{c|c} \hline 4 & 2 \hline \end{array} \quad \begin{array}{c|c} \hline 2 & 2 \hline \end{array} \quad \begin{array}{c|c} \hline 8 & 2 \hline \end{array}
\]

\[
\begin{array}{c|c} \hline 9 & 3 \hline \end{array} \quad \begin{array}{c|c} \hline 4.5 & 3 \hline \end{array} \quad \begin{array}{c|c} \hline 18 & 3 \hline \end{array}
\]

\[
\begin{array}{c|c} \hline 16 & 4 \hline \end{array} \quad \begin{array}{c|c} \hline 8 & 4 \hline \end{array} \quad \begin{array}{c|c} \hline 32 & 4 \hline \end{array}
\]

The graph of \( y = \sqrt{2x} \) is the graph of \( y = \sqrt{x} \) compressed horizontally by a factor of \( \frac{1}{2} \).

The graph of \( y = \frac{1}{\sqrt{2}}x \) is the graph of \( y = \sqrt{x} \) expanded horizontally by a factor of 2.

In general, for any function \( y = f(x) \), the graph of a function \( y = f(kx) \), where \( k \) is any real number, is obtained by dividing the \( x \)-value at each point on the graph of \( y = f(x) \) by \( k \).

The point \((x, y)\) on the graph of the function \( y = f(x) \) is transformed into the point \( \left( \frac{x}{k}, y \right) \) on the graph of \( y = f(kx) \).

If \( k > 1 \), the graph of \( y = f(x) \) is compressed horizontally by a factor of \( \frac{1}{k} \).

If \( k < 1 \), the graph of \( y = f(x) \) is expanded horizontally by a factor of \( \frac{1}{k} \).
**Example 5 Vertical and Horizontal Stretches**

a) Copy the graph of \( y = f(x) \), as shown. Compare the graphs of \( y = 3f(x) \), and \( y = f\left(\frac{1}{2}x\right) \) to the graph of \( y = f(x) \).

b) Compare the graph of \( y = 3f\left(\frac{1}{2}x\right) \) to the graph of \( y = f(x) \).

**Solution**

a) Complete tables of values using convenient values for \( x \).

\[
\begin{array}{c|c}
 x & y \\
\hline
 0 & 0 \\
 2 & 2 \\
 4 & 2 \\
 6 & 4 \\
 8 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
 x & y \\
\hline
 0 & 0 \\
 2 & 6 \\
 4 & 6 \\
 6 & 12 \\
 8 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
 x & y \\
\hline
 0 & 0 \\
 4 & 2 \\
 8 & 2 \\
 12 & 4 \\
 16 & 0 \\
\end{array}
\]

The graph of \( y = 3f(x) \) is the graph of \( y = f(x) \) expanded vertically by a factor of 3.

The graph of \( y = f\left(\frac{1}{2}x\right) \) is the graph of \( y = f(x) \) expanded horizontally by a factor of 2.

b) First, apply the transformation \( y = 3f(x) \) to the function \( y = f(x) \). Then, apply the transformation \( y = f\left(\frac{1}{2}x\right) \) to the function \( y = 3f(x) \) to give the transformation \( y = 3f\left(\frac{1}{2}x\right) \).

\[
\begin{array}{c|c}
 x & y \\
\hline
 0 & 0 \\
 2 & 2 \\
 4 & 2 \\
 6 & 4 \\
 8 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
 x & y \\
\hline
 0 & 0 \\
 2 & 6 \\
 4 & 6 \\
 6 & 12 \\
 8 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
 x & y \\
\hline
 0 & 0 \\
 4 & 2 \\
 8 & 2 \\
 12 & 4 \\
 16 & 0 \\
\end{array}
\]
The graph of \( y = 3f\left(\frac{1}{2}x\right) \) is the graph of \( y = f(x) \) expanded vertically by a factor of 3 and expanded horizontally by a factor of 2.

In Example 5b), note that the transformation \( y = f\left(\frac{1}{2}x\right) \) could have been applied to \( y = f(x) \) first, followed by the transformation \( y = 3f\left(\frac{1}{2}x\right) \), to give \( y = 3f\left(\frac{1}{2}x\right) \).

\[
\begin{align*}
y &= f(x) \\
y &= f\left(\frac{1}{2}x\right) \\
y &= 3f\left(\frac{1}{2}x\right)
\end{align*}
\]

\[
\begin{array}{|c|c|} 
\hline x & y \\
\hline 0 & 0 \\
2 & 2 \\
4 & 4 \\
6 & 4 \\
8 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|} 
\hline x & y \\
\hline 0 & 0 \\
4 & 2 \\
8 & 2 \\
12 & 4 \\
16 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|} 
\hline x & y \\
\hline 0 & 0 \\
4 & 6 \\
8 & 6 \\
12 & 12 \\
16 & 0 \\
\hline
\end{array}
\]

**Key Concepts**

- The table summarizes stretches of the function \( y = f(x) \).

<table>
<thead>
<tr>
<th>Stretch</th>
<th>Mathematical Form</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>( y = af(x) )</td>
<td>If ( a &gt; 1 ), then expand the graph vertically by a factor of ( a ). If ( 0 &lt; a &lt; 1 ), then compress the graph vertically by a factor of ( a ).</td>
</tr>
<tr>
<td>Horizontal</td>
<td>( y = f(kx) )</td>
<td>If ( k &gt; 1 ), then compress the graph horizontally by a factor of ( \frac{1}{k} ). If ( 0 &lt; k &lt; 1 ), then expand the graph horizontally by a factor of ( \frac{1}{k} ).</td>
</tr>
</tbody>
</table>

- When a function is stretched horizontally and stretched vertically, the stretches can be performed in either order to give the same image.

**Communicate Your Understanding**

1. If \( y = f(x) \), describe the difference in the graphs of \( y = 0.5f(x) \) and \( y = 2f(x) \).
2. If \( y = f(x) \), describe the difference in the graphs of \( y = f(2x) \) and \( y = f(0.5x) \).
3. The graph of \( y = f(x) \) is shown. Describe how the coordinates of the points on each of the following graphs compare with the coordinates of the points on \( y = f(x) \).
   - \( a) \ y = 2f(x) \)
   - \( b) \ y = f\left(\frac{1}{2}x\right) \)
   - \( c) \ y = 2f\left(\frac{1}{2}x\right) \)
Practise

A

1. The graph of function \( y = f(x) \) is shown. Sketch the graph of each of the following functions, and state the domain and range.

   a) \( y = 2f(x) \)
   b) \( y = \frac{1}{2}f(x) \)
   c) \( y = f(2x) \)
   d) \( y = f\left(\frac{1}{2}x\right) \)
   e) \( y = 2f\left(\frac{1}{2}x\right) \)
   f) \( y = \frac{1}{2}f\left(\frac{1}{2}x\right) \)
   g) \( y = 2f(2x) \)
   h) \( y = 0.5f(2x) \)

2. The graph of function \( y = f(x) \) is shown. Sketch the graph of each of the following functions, and state the domain and range.

   a) \( y = 2f(x) \)
   b) \( y = \frac{1}{3}f(x) \)
   c) \( y = f(3x) \)
   d) \( y = f\left(\frac{1}{3}x\right) \)
   e) \( y = f\left(\frac{1}{2}x\right) \)
   f) \( y = \frac{1}{3}f(3x) \)

3. The graph of \( y = f(x) \) is shown. Identify the transformations used to obtain the graphs of \( y = g(x) \) and \( y = h(x) \).

   a)
   b)

4. For each set of three functions,
   a) sketch them on the same grid, or graph them in the same viewing window of a graphing calculator
   b) describe how the graphs of the second and third functions are related to the graph of the first function
   c) identify any invariant points

   i) \( y = x, y = 2x, \) and \( y = \frac{1}{2}x \)
   ii) \( y = x^2, y = 3x^2, \) and \( y = \frac{1}{2}x^2 \)
   iii) \( y = \sqrt{x}, y = 3\sqrt{x}, \) and \( y = 1.5\sqrt{x} \)
   iv) \( y = x^2, y = (2x)^2, \) and \( y = \left(\frac{1}{2}x\right)^2 \)
5. Describe how the graphs of the following functions can be obtained from the graph of the function \( y = f(x) \).
   a) \( y = 3f(x) \)
   b) \( y = \frac{1}{2}f(x) \)
   c) \( y = 2f(x) \)
   d) \( y = \frac{1}{3}f(x) \)
   e) \( y = f(2x) \)
   f) \( y = f \left( \frac{1}{2}x \right) \)
   g) \( y = f(4x) \)

6. Describe how the graphs of the following functions can be obtained from the graph of the function \( y = f(x) \).
   a) \( y = 3f(2x) \)
   b) \( y = \frac{1}{2}f \left( \frac{1}{3}x \right) \)
   c) \( y = 4f \left( \frac{1}{2}x \right) \)
   d) \( y = \frac{1}{3}f(3x) \)
   e) \( y = 2f(4x) \)
   f) \( y = 5f \left( \frac{1}{2}x \right) \)

Apply, Solve, Communicate

8. Use transformations and the zeros of the quadratic function \( f(x) = (x + 4)(x - 2) \) to determine the zeros of each of the following functions.
   a) \( y = 3f(x) \)
   b) \( y = f \left( \frac{1}{2}x \right) \)
   c) \( y = f(2x) \)

9. **Stopping distances**  The distance required to stop a car is directly proportional to the square of the speed of the car. The stopping distance for a car on dry asphalt can be approximated using the function \( d(s) = 0.006s^2 \), where \( d(s) \) is the stopping distance, in metres, and \( s \) is the speed of the car, in kilometres per hour. The stopping distance for a car on wet asphalt can be
approximated using the function \( d(s) = 0.009s^2 \). The stopping distance for a car on black ice can be approximated using the function \( d(s) = 0.04s^2 \).

a) For each of the three surfaces, what is the stopping distance for a car travelling at 80 km/h?

b) Write a reasonable domain and range for each of the functions \( d(s) = 0.006s^2, \ d(s) = 0.009s^2, \text{ and } d(s) = 0.04s^2 \).

c) Let \( y = d(s) \) and \( x = s \). Graph the three functions, plus the function \( y = x^2 \), in the same viewing window of a graphing calculator.

d) Compare the graphs of \( y = 0.006x^2, \ y = 0.009x^2, \text{ and } y = 0.04x^2 \) to the graph of \( y = x^2 \).

10. The function \( y = f(x) \) has been transformed to \( y = af(kx) \). Determine the values of \( a \) and \( k \) for each of the following transformations.

a) a vertical expansion by a factor of 4

b) a horizontal compression by a factor of \( \frac{1}{3} \)

c) a vertical compression by a factor of \( \frac{1}{2} \) and a horizontal expansion by a factor of 3

d) a vertical expansion by a factor of 2 and a horizontal compression by a factor of \( \frac{1}{4} \)

11. The graph of \( f(x) = \sqrt{16 - x^2} \) is shown. Use transformations to sketch the graph of each of the following functions. State the domain and range of each function.

a) \( y = 3f(x) \)          b) \( y = \frac{1}{2}f(x) \)

c) \( y = f(2x) \)          d) \( y = f\left(\frac{1}{2}x\right) \)

e) \( y = 2f(x) \)          f) \( y = f(4x) \)

12. **Application** Use transformations and the zeros of the polynomial function \( f(x) = x(x + 3)(x - 6) \) to determine the zeros of each of the following functions.

a) \( y = f(3x) \)          b) \( y = 2f(x) \)

c) \( y = f\left(\frac{1}{2}x\right) \)         d) \( y = f(2x) \)
13. **Communication** Describe how the graphs of \( y = \frac{(x - 2)(x + 2)}{3} \) and \( y = x^2 - 4 \) are related. Explain.

14. **Falling objects** If an object is dropped from an initial height of \( x \) metres, its approximate height above the ground, \( h \) metres, after \( t \) seconds is given by \( h = -5t^2 + x \) on the Earth and by \( h = -0.8t^2 + x \) on the moon. So, for objects dropped from an initial height of 20 m, the functions are \( h(t) = -5t^2 + 20 \) and \( h(t) = -0.8t^2 + 20 \).

a) Graph \( h(t) \) versus \( t \) for \( h(t) = -5t^2 + 20 \) and \( h(t) = -0.8t^2 + 20 \) on the same axes or in the same viewing window.

b) **Inquiry/Problem Solving** Describe how the graph of \( h(t) = -0.8t^2 + 20 \) can be transformed onto the graph of \( h(t) = -5t^2 + 20 \). Justify your reasoning.

c) Explain the meaning of the point that the two graphs have in common.

d) State the domain and range of each function.

15. **Greatest integer function** Sketch the graph of \( y = [x] \). Then, use transformations to sketch the graphs of the following functions. Check your solutions using a graphing calculator.

a) \( y = [2x] \)  

b) \( y = [0.5x] \)  

c) \( y = 2[x] \)  

16. a) Describe the horizontal stretch that transforms \( y = \sqrt{x} \) to \( y = \sqrt{4x} \).

b) Describe the vertical stretch that transforms \( y = \sqrt{x} \) to \( y = 2\sqrt{x} \).

c) How do the graphs of \( y = \sqrt{4x} \) and \( y = 2\sqrt{x} \) compare?

d) How do the transformations in parts a) and b) compare? Explain why they compare in this way.

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**Logic Power**

Copy the diagram. Show two different ways to divide the shape along the lines into four congruent figures.