Parallactic Displacement

Hold a pencil at arm's length and look at it with both eyes open. Now close one eye, open it, and then close the other eye. You will notice that the position of the pencil, relative to the background, appears to change. This effect is known as parallactic displacement, or parallax. Since parallactic displacement can be measured in degrees, it can be used to determine the distance to an object using trigonometry.

The first use of parallactic displacement to determine the distance from the Earth to a star is usually credited to the German astronomer Friedrich Wilhelm Bessel (1784−1846). He studied 61 Cygni, a star in the constellation Cygnus, the Swan. He compared the positions of the star, relative to its background, on different nights half a year apart. He announced his results in 1838.

In the diagram, S represents the sun, C represents the star 61 Cygni, and E₁ and E₂ represent the Earth at its opposite positions around the sun.

Bessel measured the angular shift in the star's position due to the Earth's motion around the sun, \( \angle E₁C E₂ \), as \( 1.7 \times 10^{-4} \) degrees.

1. Annual parallax is defined as half the angular shift, that is, as \( \angle SCE₁ \) or \( \angle SCE₂ \) in the diagram. Determine the degree measure of the annual parallax for 61 Cygni in standard scientific notation.

2. If \( r \), the average radius of the Earth's orbit around the sun, is \( 1.5 \times 10^8 \) km, calculate the distance \( d \), in kilometres. Express your answer in standard scientific notation, with the decimal part rounded to the nearest tenth.

3. Stellar distances are usually expressed in light-years, where one light-year is the distance light travels through space in one year. The speed of light is \( 3 \times 10^5 \) km/s. Express \( d \) to the nearest tenth of a light-year.

4. The accepted value for the distance from the Earth to 61 Cygni is now 11.1 light-years. Using this value, work backward and calculate a more accurate value for the angular shift.
Review of Prerequisite Skills

If you need help with any of the skills named in purple below, refer to Appendix A.

1. Trigonometric ratios For each right triangle, use the Pythagorean theorem to calculate the length of the third side. Then, find the sine, cosine, and tangent of each acute angle. Express answers as fractions in lowest terms.

   a) b) c) d) e) f)

2. Find each of the following, to three decimal places.
   a) \(\sin 27^\circ\) b) \(\cos 56^\circ\) c) \(\tan 78^\circ\)
   d) \(\cos 7^\circ\) e) \(\tan 40^\circ\) f) \(\sin 62^\circ\)

3. Find the size of each angle, to the nearest degree.
   a) \(\sin D = 0.602\) b) \(\cos A = 0.309\)
   c) \(\tan C = 0.445\) d) \(\tan R = 2.246\)
   e) \(\sin X = 0.978\) f) \(\cos W = 0.951\)

4. Finding a side length in a right triangle Calculate \(x\), to the nearest tenth of a unit.
   a) b) c) d) e) f)

5. Finding an angle in a right triangle Find \(\angle x\), to the nearest degree.
   a) b) c) d) e) f)

6. Solving proportions Solve for \(x\). Express answers as decimals. Round to the nearest hundredth, if necessary.
   a) \(\frac{x}{3.15} = 11.8\) b) \(\frac{x}{9.73} = \frac{7.65}{0.46}\)
   c) \(\frac{91.24}{83.56} = \frac{x}{71.77}\) d) \(\frac{12.56}{x} = \frac{19.83}{27.77}\)
   e) \(\frac{0.57}{0.81} = \frac{1.52}{x}\) f) \(\frac{1.38}{x} = \frac{5.72}{4.11}\)