TECHNOLOGY EXTENSION

Using The Geometer’s Sketchpad® to Explore the SSA Case

Exploring the SSA Case

1. Use the following steps to construct \( \triangle ABC \) where \( \angle A = 50^\circ \), \( AC = 6 \text{ cm} \), and \( CB = 5 \text{ cm} \).
   a) Construct a base segment PQ. From the **Construct** menu, choose the **Point On Object** command. Highlight points A and Q, and measure the distance between them. If necessary, drag point A away from point Q, so that this distance is more than 8 cm.
   b) Construct line segment AC. Measure the line segment AC and \( \angle CAQ \). Drag point C so that AC = 6 cm and \( \angle CAQ = 50^\circ \).
   c) In an open area, construct line segment EF. Measure the length of EF and drag one endpoint so that EF = 5 cm.
   d) Highlight point C and line segment EF. From the **Construct** menu, choose the **Circle By Center And Radius** command. This will create a circle that intersects line segment PQ twice. Highlight line segment PQ and the circle. From the **Construct** menu, choose the **Point At Intersection** command. Label the two points of intersection B1 and B2.
e) Construct a line segment joining C to B1, and construct a line segment joining C to B2. Measure each segment. How many different possible triangles satisfy the given conditions that $\angle A = 50^\circ$, AC = 6 cm, and CB = 5 cm?

2. Use the following set of steps to construct $\triangle ABC$ where $\angle A = 120^\circ$ and AC = 6 cm.
   a) Start with the base line segment with endpoints labelled P and Q. From the Construct menu, choose the Point On Object command. Label this point A.
   b) Using the Point Tool, construct a point in the open area above line segment PQ. Label this point C. Construct line segment AC. Measure line segment AC and $\angle QAC$. Drag point C to a location where AC = 6 cm and $\angle QAC = 120^\circ$.
   c) To complete the triangle, a third point B must be placed on line segment AQ. In an open area of the screen, construct a line segment EF. Measure the length of EF. Highlight point C and line segment EF. From the Construct menu, choose the Circle By Center And Radius command. Dragging endpoint F, will change the radius of the circle.
d) Drag point F until the circle with centre C intersects line segment PQ in a point to the right of point A. Choose the Point At Intersection command from the Construct menu to create point B. Construct line segment CB. Measure the length of CB.

e) \( \triangle ABC \) now satisfies the given conditions that \( \angle A = 120^\circ \) and \( AC = 6 \) cm. Drag point F closer to point E to change the lengths of both EF and CB.
   i) For what values of CB will no triangle exist?
   ii) For what values of CB will one triangle exist?
   iii) Are there any values of CB for which two triangles will exist?

Making Generalizations

1. To make generalizations, investigate constructions that give no triangles, exactly one triangle, or two triangles.
   a) Start with a point P on line segment PQ and a point C above the line segment. Measure the length of AC and \( \angle A \). Show the label for line segment AC, and re-label this line segment as b. Construct a line segment EF, and measure the length of this line segment. Construct a circle with centre C and radius EF.
   b) Drag point F to determine a range of values for the length of EF such that the circle does not intersect line segment PQ. Record the range of values in your notebook. Drag point F to a point that allows the circle to intersect line segment PQ at two points. Label these points B1 and B2, as shown. Measure line segment CB1 and label it a1. Measure CB2 and label it a2. Measure the
angles \( \angle AB_1 C \) and \( \angle AB_2 C \). Drag point F and record the range of lengths that produces two triangles.

c) State the condition that must exist so that there is exactly one triangle in the construction. What angle is created at point B?

d) Use an appropriate trigonometric ratio to calculate the exact value for side CB in terms of \( \angle A \) and side b so that there is exactly one triangle. How is this value related to the ranges of values you recorded for no triangles and two triangles in part b)?

e) Make a generalization about the conditions on the length of side BC that will give no triangles, one triangle, and two triangles.

f) If you were given the measure of \( \angle A \) and the length of side AC, how would you determine if the length of a side BC would produce no triangles, one triangle, or two triangles.

2. As shown in the diagram, use the calculator built into The Geometer's Sketchpad® to complete the Sine Law calculation for your construction.

a) What is the relationship between the two angles \( \angle AB_1 C \) and \( \angle AB_2 C \)?

b) Explain why the values from the two calculations are equal.