The angles you have worked with so far have had measures ranging from 0° to 360°, or 0 to $2\pi$. Other measures are possible.

The measure of an angle in standard position is determined by the amount of rotation from the initial arm to the terminal arm. If the rotation is counterclockwise, the angle is positive. If the rotation is clockwise, the angle is negative.

If the terminal arm makes exactly one counterclockwise revolution, the angle has a measure of 360°. If the terminal arm makes exactly one clockwise revolution, the angle has a measure of $-360°$.

When the terminal arm rotates, it may make one or more revolutions.

If you draw a 420° angle and a 60° angle in standard position on the same set of axes, the terminal arm of the 420° angle and the terminal arm of the 60° angle are the same. When two angles in standard position have the same terminal arm, they are called coterminal angles.
Sine, Cosine, and Tangent of Positive Angles

To find the exact values of the sine, cosine, and tangent of 135°, first draw the angle in standard position. Then, construct a triangle by drawing a perpendicular from the terminal arm to the $x$-axis. The angle between the terminal arm and the $x$-axis is 45°.

The triangle formed is a 45°-45°-90° triangle. Choose point P on the terminal arm so that $r = \sqrt{2}$. It follows that $x = -1$ and $y = 1$.

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

\[
\sin 135° = \frac{1}{\sqrt{2}} \quad \cos 135° = -\frac{1}{\sqrt{2}} \quad \tan 135° = -1
\]

The same method can be used to find the exact values of the sine, cosine, and tangent of 510°. In this case, the angle between the terminal arm and the $x$-axis is 30°.

The triangle formed is a 30°-60°-90° triangle. Choose point P on the terminal arm so that $r = 2$. It follows that $x = -\sqrt{3}$ and $y = 1$.

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

\[
\sin 510° = \frac{1}{2} \quad \cos 510° = -\frac{\sqrt{3}}{2} \quad \tan 510° = -\frac{1}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}}{3}
\]

To find the values of the sine, cosine, and tangent of 630°, first draw the angle in standard position. Choose P(0, -1) on the terminal arm of the angle. Therefore, $x = 0$, $y = -1$, and $r = 1$.

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

\[
\sin 630° = \frac{-1}{1} \quad \cos 630° = \frac{0}{1} \quad \tan 630° = \frac{-1}{0}
\]

Since division by 0 is not defined, tan 630° is not defined.
1. Find the exact values of the sine, cosine, and tangent for each angle.
   a) 390°    b) 405°    c) 690°    d) 450°
   e) $\frac{7}{3}\pi$    f) $\frac{13}{4}\pi$    g) $3\pi$    h) $\frac{10}{3}\pi$

2. Find the exact value of each trigonometric ratio.
   a) $\cos 495°$    b) $\tan 480°$    c) $\sin 660°$
   d) $\cos 720°$    e) $\tan 570°$    f) $\sin 390°$

**Sine, Cosine, and Tangent of Negative Angles**

To find the exact values of the sine, cosine, and tangent of $-135°$, first draw the angle in standard position. Then, construct a triangle by drawing a perpendicular from the terminal arm to the $x$-axis. The angle between the terminal arm and the $x$-axis is 45°. The triangle formed is a 45°-45°-90° triangle. Choose point $P$ on the terminal arm so that $r = \sqrt{2}$. It follows that $x = -1$ and $y = -1$.

$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}, \quad \tan \theta = \frac{y}{x}$$

$$\sin (-135°) = -\frac{1}{\sqrt{2}} \quad \cos (-135°) = -\frac{1}{\sqrt{2}} \quad \tan (-135°) = 1$$

The same method can be used to find the exact values of the sine, cosine, and tangent of $-420°$. In this case, the angle between the terminal arm and the $x$-axis is 60°. The triangle formed is a 30°-60°-90° triangle. Choose point $P$ on the terminal arm so that $r = 2$. It follows that $x = 1$ and $y = -\sqrt{3}$.

$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}, \quad \tan \theta = \frac{y}{x}$$

$$\sin (-420°) = -\frac{\sqrt{3}}{2} \quad \cos (-420°) = \frac{1}{2} \quad \tan (-420°) = -\frac{\sqrt{3}}{1} \quad \text{or} \quad -\sqrt{3}$$
To find the values of the sine, cosine, and tangent for an angle that measures $-180^\circ$, first draw the angle in standard position. Choose $P(-1, 0)$ on the terminal arm of the angle. Therefore $x = -1$, $y = 0$, and $r = 1$.

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

\[
\sin (-180^\circ) = \frac{0}{-1} = 0 \quad \cos (-180^\circ) = \frac{-1}{1} = -1 \quad \tan (-180^\circ) = \frac{0}{-1} = 0
\]

3. Find the exact values of the sine, cosine, and tangent for each angle.
   a) $-60^\circ$  
   b) $-225^\circ$  
   c) $-90^\circ$  
   d) $-150^\circ$
   e) $-\frac{3}{2}\pi$  
   f) $-\frac{11}{6}\pi$  
   g) $-\frac{3}{4}\pi$  
   h) $-\pi$

4. Find the exact value of each trigonometric ratio.
   a) $\sin (-540^\circ)$  
   b) $\cos (-450^\circ)$
   c) $\sin (-630^\circ)$  
   d) $\cos (-510^\circ)$
   e) $\sin (-405^\circ)$  
   f) $\cos (-675^\circ)$