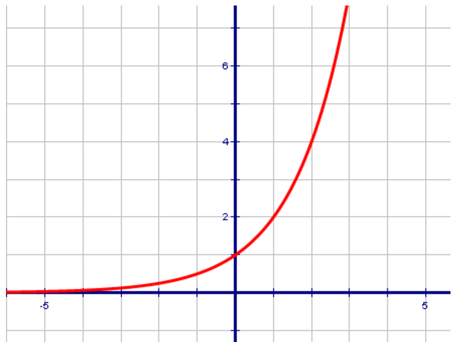
1.4A Applications of Exponential Functions

Exponential Growth:



Graph:
Increases slowly then quickly

Exponential Decay:

Graph:
Decreases quickly then slowly

Asymptote:
A line that a curve will continually approach but never touch

Asymptote:
- x axis
- eq'n: y=0

y will never be zero!

NOTE: for our purposes we will not be looking at a graph that has been translated up or down, that is why the x axis will be the asymptote

Equation:

Example:
$A = 250(1.04)^x$

Final Amount

$$A = a_o(b)^x$$

exponent is a variable

Initial value (stretch)

base

exp growth if $b > 1$
exp decay if $0 < b < 1$

Explore:

- y intercept is 1 if there is no stretch
  i.e. a = 1
- your base is the ratio of your 1^{st} differences
- from your graph you can see your base by finding your y value at x = 1

* watch for a stretch*

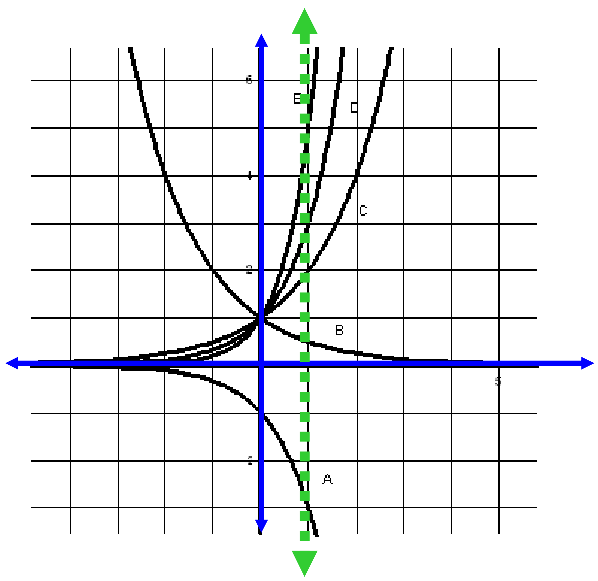
Explore gizmos:
Look fors:
The location of the x and y intercepts?
The location of the horizontal asymptote?

Don't write this ....let's try it together...

Match the Graph to the Exponential Equation

(Hint: To find the appropriate base find the value of y when $x = 1$)

1.



$y = 3^x$ D

$y = 2^x$ C

$y = \left(\frac{1}{2}\right)^x$ B

$y = \left(\frac{1}{3}\right)^x$ DNE

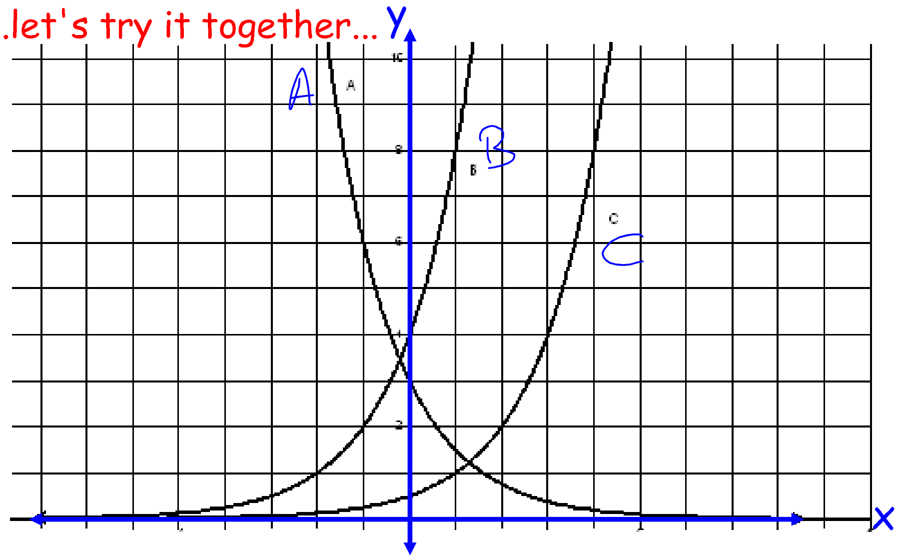
$y = 5^x$ E

$y = -3^x$ A

Nov 26-9:51 AM

Don't write this ....let's try it together...

2.



$y = 4(2)^x$ B

$y = \frac{1}{2}(2)^x$ C

$y = 3\left(\frac{1}{2}\right)^x$ A

$y = (-6)^x$ DNE

Neg base ???

Nov 26-9:52 AM

Don't write this ....let's try it together...

3. Discuss each of the following graphs:

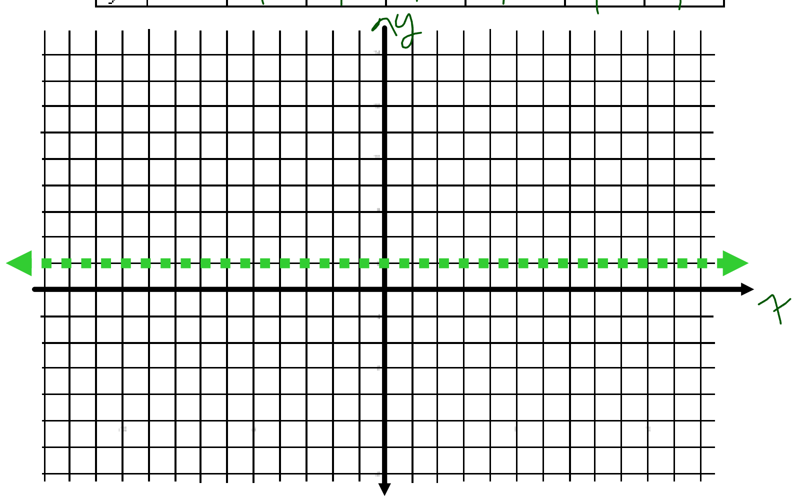
$y = (-2)^x$

x	- 3	- 2	- 1	0	1	2	3
y	$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{2}$	1	-2	4	-8

can't have neg base

Straight line $y = 1^x$

x	- 3	- 2	- 1	0	1	2	3
y	1	1	1	1	1	1	1

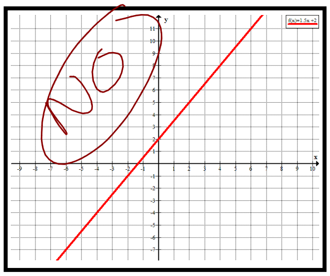


What do you notice:

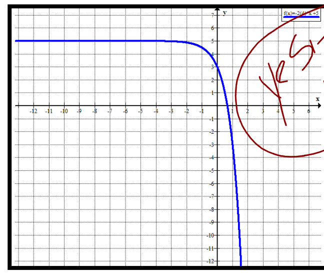
- Exponential functions need a positive base
- when the base is 1 the graph is a horizontal line (y=1)

Nov 27-10:19 AM

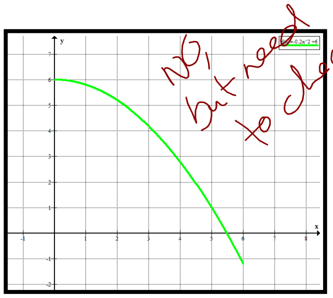
Ex. 1 Which could represent exponential growth or decay?



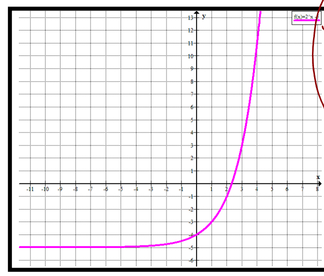
NO



Yes decay



NO, but need to check



Yes Growth

$y = 5x^2 + 7x - 3$

NO (quad)

$A = 400(0.76)^{t/4}$

yes decay

$P = 200(1.07)^t$

yes growth

$P = 7w + 5$

NO (linear)

x	y
0	5
1	13
2	37
3	109
4	325

8, 24, 72, 216

Ratio?

$\frac{24}{8} = 3$

$\frac{72}{24} = 3$

Yes! Ratio is the same.

$\therefore$ Exponential

Sep 12-12:24 PM

Ex. 2 Take a regular 8.5" x 11" piece of paper and fold it in half...then in half again...and so on. Record the # of rectangles created on the paper by each successive fold.

# folds	# rectangles
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

Myth Busters-7 Folds

a) Is this relationship exponential? Explain how you know.

yes, common ratio (doubling)

b) Does the relation model growth forever? Explain how the context of the questions limits the values of the domain and range.

No,

finite # of times an actual paper could be folded.

Sep 12-12:34 PM

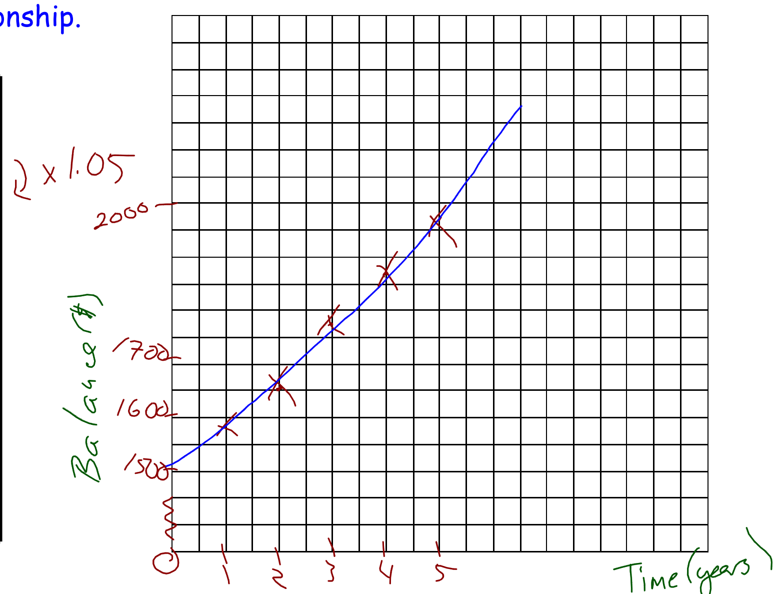
Ex. 3 Sarah invests $1500 into an account that pays 5% compound interest per year. This means that Sarah earns 5% of the balance in interest every year.

a) Complete the table to show the balance in the account at the end of each year.

b) Graph the relationship.

Year	Balance ($)
0	1500
1	1575
2	1653.75
3	1736.43
4	1823.25
5	1914.42
6	2010.14
7	2110.65
8	2216.18

$\times 1.05$



c) How long does it take for Sarah to have $2000 in her account?

6 years

d) Does this relationship model exponential growth or exponential decay? Give reasons.

Growth, because its growing 5%/year

e) Explain how the context of the question limits the values of the domain and range.

Domain -> Set of 'x' values $0 \le x \le$ lifespan

Range -> Set of 'y' values $y \ge 1500$

f) What would happen to the shape of the graph if she deposited more money in the account at the end of every year?

Steeper

Sep 12-12:35 PM

Ex. 4 Model each situation with an exponential equation.
Define "x" for each.

a) An initial population of 200 tent caterpillars grows by 15% each day

Let t be # of days
Let P be the population

$P = 200(1.15)^t$

b) A car worth $25 000, depreciates in value by 13% each year.

Let t be # yrs
Let V be value in $$$

$V = 25000(0.87)^t$

100% − 13% = 87%

c) 400 mg of radioactive material deteriorates by 5% every 4 hours.

Let t be # of "4 hours"
Let A be amount left

$A = 400(0.95)^t$

OR

Let t be # of hours

$A = 400(0.95)^{\frac{t}{4}}$

d) A rabbit population of 50 doubles every 6 weeks.

Let t be # of weeks
Let P be pop. of rabbits

$P = 50(2)^{\frac{t}{6}}$

Sep 12-12:36 PM

Ex. 5 The table below shows the amount of radioactive material remaining from a 300 g sample.

Time (hours)	Amount (g)	1st diff	ratio of 1st diff
0	300		
1	285	-15	
2	270.75	-14.25	0.95
3	257.21	-13.54	0.95
4	244.35	-12.86	0.95
5	232.13	-12.22	0.95

a) Write an exponential equation to model the situation.

Equation

Let t rep the time in hours
and A rep Amount in g

$$A = 300\,(0.95)^t$$

b) Determine an approximate growth/decay rate.

decay factor: 0.95

decay rate: 5%

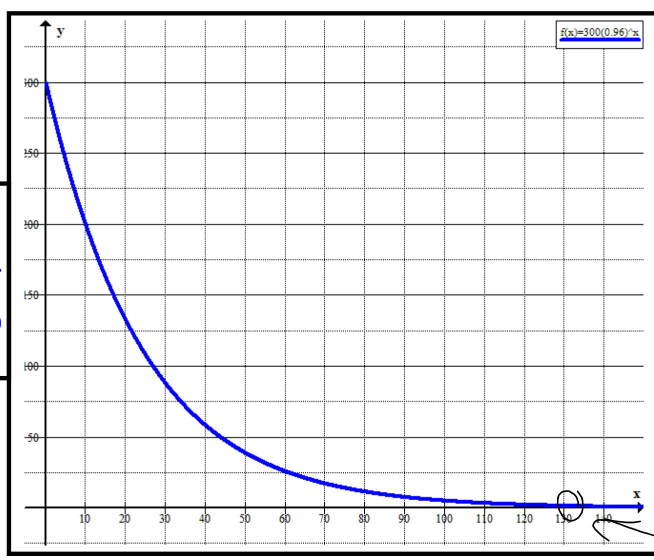
c) Use this equation to determine the amount that will remain after 12 hours.

$A = 300(0.95)^{12}$
$\doteq 162$

$\therefore$ There will be approx. 162g left.

d) Use your equation from b) to draw a graph the relation. How long will it take for there to be an indiscernible amount of radioactive material left (less than 0.5 g)?

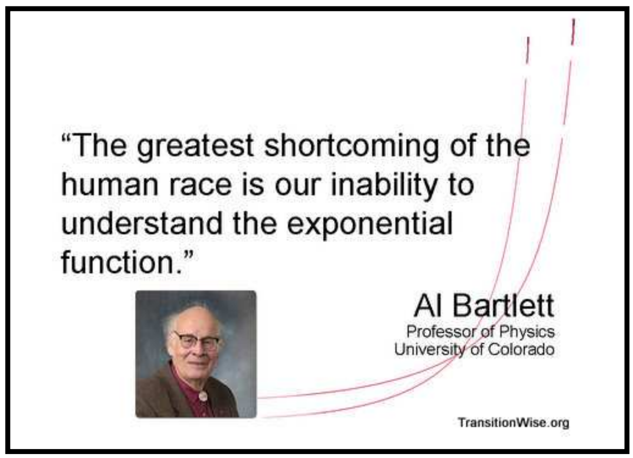
graph



Homework

Link to Handout

Handout "1.4 A Exponential Relations Handout"



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