CHAPTER 1 Algebraic Tools for Operating with Functions

1.1 Reviewing the Exponent Laws

Practise

Section 1.1 Page 9 Question 1

a) $2^4 \times 2^3 = 2^{4+3}$
$= 2^7$

c) $(2^4)^3 = 2^{4\times3}$
$= 2^{12}$

e) $2^3 \times 2^m = 2^{3+m}$

g) $2^x \div 2^4 = 2^{x-4}$

i) $2^{-3} \times 2^4 = 2^{-3+4}$
$= 2^1$

k) $(2^3)^{-1} = 2^{3\times(-1)}$
$= 2^{-3}$

Section 1.1 Page 9 Question 2

a) $3^{-2} = \frac{1}{3^2}$
$= \frac{1}{9}$

c) $(-2)^{-4} = \frac{1}{(-2)^4}$
$= \frac{1}{16}$

e) $-(-3)^0 = -1$

g) $\frac{1}{(-4)^{-1}} = (-4)^1$
$= -4$

Section 1.1 Page 9 Question 3

a) $a^4 \times a^3 = a^{4+3}$
$= a^7$

c) $b^5 \times b^6 \times b = b^{5+6+1}$
$= b^{12}$

e) $(x^3)(y)(y^4)(x^5) = (x^{3+5})(y^{1+4})$
$= x^8y^5$

g) $m^{-4} \times m^{-5} = m^{-4+(-5)}$
$= m^{-9}$
$= \frac{1}{m^9}$

i) $a^5 \times a^0 = a^5 \times 1$
$= a^5$

Section 1.1 Page 9 Question 4

a) $x^6 \div x^3 = x^{6-3}$
$= x^3$

c) $t^4 \div t^{-2} = t^{4-(-2)}$
$= t^6$

e) $m^4 \div m^0 = m^4 \div 1$
$= m^4$

Section 1.1 Page 9 Question 5

a) $(x^3)^2 = x^{3(2)}$
$= x^6$

c) $(x^2)^{-1} = x^{2(-1)}$
$= x^{-2}$
$= \frac{1}{x^2}$

e) $(a^{-1}b^2)^{-2} = a^{-1(-2)}b^{2(-2)}$
$= a^2b^{-4}$
$= \frac{a^2}{b^4}$

Section 1.1 Page 9 Question 6

a) $\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3}$
$= \frac{x^3}{8}$

c) $\left(\frac{x^2}{y^3}\right)^5 = \frac{x^{2(5)}}{y^{3(5)}}$
$= \frac{x^{10}}{y^{15}}$

e) $\left(\frac{a^{-2}}{b^{-3}}\right)^{-2} = \frac{a^{-2(-2)}}{b^{-3(-2)}}$
$= \frac{a^4}{b^6}$

Section 1.1 Page 9 Question 7

a) $5m^4 \times 3m^2 = 15m^{4+2}$
$= 15m^6$

c) $5a(-2ab^2)(-3b^3) = 30a^{1+1}b^{2+3}$
$= 30a^2b^5$

e) $(7x^2)(6x^{-2}) = 42x^{2+(-2)}$
$= 42x^0$
$= 42$

g) $(-6a^{-1}b^2)(-a^{-3}b^{-4}) = 6a^{-1-3}b^{2-4}$
$= 6a^{-4}b^{-2}$
$= \frac{6}{a^4b^2}$

i) $\frac{45a^2b^4}{9ab^2} = 5a^{2-1}b^{4-2}$
$= 5ab^2$

k) $\frac{3ab^3 \times 10a^4b^2}{15a^2b^6} = \frac{30a^5b^5}{15a^2b^6}$
$= \frac{2a^3}{b}$

m) $(35x^5) \div (5x^{-3}) = 7x^{5-(-3)}$
$= 7x^8$

o) $(-6m^{-4}n^2) \div (2m^{-1}n^{-6}) = -3m^{-4-(-1)}n^{2-(-6)}$
$= -3m^{-3}n^8$
$= -\frac{3n^8}{m^3}$

Section 1.1 Page 9 Question 8

a) $(2m^3)^2 = 2^2m^{3(2)}$
$= 4m^6$

c) $(-3m^3n^2)^2 = (-3)^2m^{3(2)}n^{2(2)}$
$= 9m^6n^4$

e) $(2a^{-3}b^{-2})^{-3} = 2^{-3}a^{-3(-3)}b^{-2(-3)}$
$= \frac{a^9b^6}{8}$

g) $\left(\frac{4x}{3y}\right)^2 = \frac{16x^2}{9y^2}$

i) $\left(\frac{3a}{-b^4}\right)^4 = \frac{81a^4}{b^{16}}$

k) $\left(\frac{6ab^3}{2ab}\right)^3 = \frac{216a^3b^9}{8a^3b^3}$
$= 27b^6$

Section 1.1 Page 10 Question 9

a) $\frac{6}{x^0 + y^0} = \frac{6}{1+1}$
$= 3$

c) $4^{-1} + 2^{-3} = \frac{1}{4} + \frac{1}{8}$
$= \frac{2+1}{8}$
$= \frac{3}{8}$

e) $\frac{(6^4 + 4^6)^0}{3^{-1}} = \frac{1}{3^{-1}}$
$= 3$

Apply, Solve, Communicate

Section 1.1 Page 10 Question 10

Let t represent the age, in years, of the earliest human settlement in British Columbia.

$$t = \frac{5.4 \times 10^8}{4.5 \times 10^4}$$
$$= \frac{5.4}{4.5} \times 10^{8-4}$$
$$= 1.2 \times 10^4$$

The first human settlement in British Columbia was 12 000 years ago.

Section 1.1 Page 10 Question 11

Let the time taken to burn, in seconds, be represented by t and the temperature, in degrees Celsius, over the oven be T. Then,

$$t = 2^{\frac{600-T}{10}} \quad (1)$$

a) Substitute $T = 500°C$ into (1).

$$\begin{aligned} t &= 2^{\frac{600-500}{10}} \\ &= 2^{\frac{100}{10}} \\ &= 2^{10} \\ &= 1024 \end{aligned}$$

The wood would take 1024 s to burn.

b) Substitute $T = 650°C$ into (1).

$$\begin{aligned} t &= 2^{\frac{600-650}{10}} \\ &= 2^{\frac{-50}{10}} \\ &= 2^{-5} \\ &= \frac{1}{32} \end{aligned}$$

The wood would take $\frac{1}{32}$ s to burn.

Section 1.1 Page 10 Question 12

a)
$$\begin{aligned} 2^{-2} \times \left(2^2 + 2^2\right) - 2^0 &= 2^{-2} \times (4+4) - 2^0 \\ &= 2^{-2} \times 8 - 2^0 \\ &= 2^{-2} \times 2^3 - 2^0 \\ &= 2^1 - 2^0 \\ &= 2 - 1 \\ &= 1 \\ &= 2^0 \end{aligned}$$

b)
$$\begin{aligned} \left(3^{-4} - 3^{-2}\right) \div \left(3^0 - 3^2\right) &= \left(\frac{1}{81} - \frac{1}{9}\right) \div (1-9) \\ &= \left(\frac{1}{81} - \frac{9}{81}\right) \div (-8) \\ &= -\frac{8}{81} \div (-8) \\ &= \frac{1}{81} \\ &= \frac{1}{3^4} \\ &= 3^{-4} \end{aligned}$$

Section 1.1 Page 10 Question 13

$$\begin{aligned} 400^{40} &= (20^2)^{40} \\ &= 20^{80} \end{aligned}$$

$$20^{100} > 20^{80}$$

Thus, 20^{100} is greater.

Section 1.1 Page 10 Question 14

a)

$$\begin{aligned}\frac{6^1+6^{-1}}{6^1-6^{-1}} &= \frac{6+\frac{1}{6}}{6-\frac{1}{6}}\\ &= \frac{\frac{36}{6}+\frac{1}{6}}{\frac{36}{6}-\frac{1}{6}}\\ &= \frac{\frac{37}{6}}{\frac{35}{6}}\\ &= \frac{37}{35}\end{aligned}$$

b)

$$\begin{aligned}\frac{5^{-4}-5^{-6}}{5^{-3}+5^{-5}} &= \left(\frac{1}{5^4}-\frac{1}{5^6}\right)\div\left(\frac{1}{5^3}+\frac{1}{5^5}\right)\\ &= \left(\frac{5^2}{5^6}-\frac{1}{5^6}\right)\div\left(\frac{5^2}{5^5}+\frac{1}{5^5}\right)\\ &= \left(\frac{25-1}{5^6}\right)\div\left(\frac{25+1}{5^5}\right)\\ &= \frac{24}{5^6}\div\frac{26}{5^5}\\ &= \frac{24}{5^6}\times\frac{5^5}{26}\\ &= \frac{12}{5\times 13}\\ &= \frac{12}{65}\end{aligned}$$

c)

$$\begin{aligned}2^{-n}(2^n-2^{1+n}) &= 2^{-n}\times 2^n - 2^{-n}\times 2^{1+n}\\ &= 2^{-n+n}-2^{-n+1+n}\\ &= 2^0-2^1\\ &= 1-2\\ &= -1\end{aligned}$$

d)

$$\begin{aligned}3\left(3^{2x}-\frac{1}{3^{-2x}}\right) &= 3(3^{2x}-3^{2x})\\ &= 3(0)\\ &= 0\end{aligned}$$

Section 1.1 Page 10 Question 15

a) None. For non-zero real values of x, the expression $-x^{-4} < 0$ and the expression $(-x)^{-4} > 0$.

b) All.

$$\begin{aligned}(-x)^{-3} &= \frac{1}{(-x)^3}\\ &= \frac{1}{(-x)(-x)(-x)}\\ &= -\frac{1}{x^3}\\ &= -x^{-3},\ x\neq 0\end{aligned}$$

Section 1.1 Page 10 Question 16

a)

$$\begin{aligned}x^2\times x^3 &= 32\\ x^{2+3} &= 32\\ x^5 &= 32\\ x^5 &= 2^5\\ x &= 2\end{aligned}$$

b)

$$\begin{aligned}x^5\div x^2 &= 64\\ x^{5-2} &= 64\\ x^3 &= 64\\ x^3 &= 4^3\\ x &= 4\end{aligned}$$

c)

$$\begin{aligned}x^{-1}\times x^{-3} &= \frac{1}{81}\\ x^{-1+(-3)} &= \frac{1}{81}\\ x^{-4} &= \frac{1}{81}\\ x^{-4} &= \frac{1}{3^4}\text{ or }\frac{1}{(-3)^4}\\ x^{-4} &= 3^{-4}\text{ or }(-3)^{-4}\\ x &= 3\text{ or }-3\end{aligned}$$

d)

$$\begin{aligned}x^2\div x^5 &= \frac{1}{125}\\ x^{2-5} &= \frac{1}{125}\\ x^{-3} &= \frac{1}{125}\\ x^{-3} &= \frac{1}{5^3}\\ x^{-3} &= 5^{-3}\\ x &= 5\end{aligned}$$

Section 1.1 Page 10 Question 17

Division by 0 is undefined. If $x \neq 0$, the equation reduces to $1 = 1$, and thus is defined for all real numbers except 0.

1.2 Rational Exponents

Practise

Section 1.2 Page 16 Question 1

a) $2^{\frac{1}{3}} = \sqrt[3]{2}$

c) $x^{\frac{1}{2}} = \sqrt{x}$

e) $6^{\frac{4}{3}} = (\sqrt[3]{6})^4$

g) $7^{-\frac{1}{2}} = \frac{1}{7^{\frac{1}{2}}}$
$= \frac{1}{\sqrt{7}}$

i) $x^{-\frac{3}{7}} = \frac{1}{x^{\frac{3}{7}}}$
$= \frac{1}{\sqrt[7]{x^3}}$

k) $(3x)^{\frac{1}{2}} = \sqrt{3x}$

Section 1.2 Page 16 Question 2

a) $\sqrt{7} = 7^{\frac{1}{2}}$

c) $\sqrt[3]{-11} = (-11)^{\frac{1}{3}}$
$= -11^{\frac{1}{3}}$

e) $\sqrt[3]{6^4} = (6^4)^{\frac{1}{3}}$
$= 6^{\frac{4}{3}}$

g) $\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}}$
$= x^{-\frac{1}{2}}$

i) $\frac{1}{\sqrt[5]{x^4}} = \frac{1}{(x^4)^{\frac{1}{5}}}$
$= \frac{1}{x^{\frac{4}{5}}}$
$= x^{-\frac{4}{5}}$

k) $\sqrt{3x^5} = (3x^5)^{\frac{1}{2}}$
$= 3^{\frac{1}{2}}(x^5)^{\frac{1}{2}}$
$= 3^{\frac{1}{2}}x^{\frac{5}{2}}$

Section 1.2 Page 16 Question 3

a) $4^{\frac{1}{2}} = \sqrt{4}$
$= 2$

c) $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}}$
$= \frac{1}{\sqrt[4]{16}}$
$= \frac{1}{2}$

e) $25^{0.5} = 25^{\frac{1}{2}}$
$= \sqrt{25}$
$= 5$

g) $64^{-\frac{1}{6}} = \frac{1}{64^{\frac{1}{6}}}$
$= \frac{1}{\sqrt[6]{64}}$
$= \frac{1}{2}$

i) $81^{0.25} = 81^{\frac{1}{4}}$
$= \sqrt[4]{81}$
$= 3$

k) $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}}$
$= \frac{2}{3}$

Section 1.2 Page 16 Question 4

a)
$$\begin{aligned} 8^{\frac{2}{3}} &= (\sqrt[3]{8})^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

c)
$$\begin{aligned} 9^{2.5} &= 9^{\frac{5}{2}} \\ &= (\sqrt{9})^5 \\ &= 3^5 \\ &= 243 \end{aligned}$$

e)
$$\begin{aligned} 16^{-\frac{3}{4}} &= \frac{1}{16^{\frac{3}{4}}} \\ &= \frac{1}{(\sqrt[4]{16})^3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8} \end{aligned}$$

g)
$$\begin{aligned} (-8)^{-\frac{5}{3}} &= \frac{1}{(-8)^{\frac{5}{3}}} \\ &= \frac{1}{(\sqrt[3]{-8})^5} \\ &= \frac{1}{(-2)^5} \\ &= \frac{1}{-32} \\ &= -\frac{1}{32} \end{aligned}$$

i)
$$\begin{aligned} 1^{\frac{5}{3}} &= (\sqrt[3]{1})^5 \\ &= 1^5 \\ &= 1 \end{aligned}$$

k)
$$\begin{aligned} \left(\frac{100}{9}\right)^{\frac{3}{2}} &= \left(\sqrt{\frac{100}{9}}\right)^3 \\ &= \left(\frac{10}{3}\right)^3 \\ &= \frac{1000}{27} \end{aligned}$$

Section 1.2 Page 16 Question 5

a)
$$(-9)^{\frac{1}{2}} = \sqrt{-9}$$

Since $\sqrt{x}$ is defined only for $x \geq 0$, there are no real solutions.

c)
$$\begin{aligned} \left(\frac{27}{8}\right)^{\frac{2}{3}} &= \left(\frac{\sqrt[3]{27}}{\sqrt[3]{8}}\right)^2 \\ &= \left(\frac{3}{2}\right)^2 \\ &= \frac{9}{4} \end{aligned}$$

e)
$$\begin{aligned} -9^{\frac{1}{2}} &= -\sqrt{9} \\ &= -3 \end{aligned}$$

g)
$$\begin{aligned} -8^{\frac{5}{3}} &= -\left(\sqrt[3]{8}\right)^5 \\ &= -2^5 \\ &= -32 \end{aligned}$$

i)
$$\begin{aligned} (-1)^{-\frac{3}{2}} &= \frac{1}{(-1)^{\frac{3}{2}}} \\ &= \frac{1}{(\sqrt{-1})^3} \end{aligned}$$

Since $\sqrt{x}$ is defined only for $x \geq 0$, there are no real solutions.

k)
$$\begin{aligned} \left(\frac{36}{121}\right)^{-\frac{1}{2}} &= \frac{1}{\left(\frac{36}{121}\right)^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{\frac{36}{121}}} \\ &= \frac{1}{\frac{6}{11}} \\ &= \frac{11}{6} \end{aligned}$$

m)
$$(-0.0016)^{\frac{1}{4}} = \sqrt[4]{\frac{-16}{1000}}$$

Since $\sqrt[4]{x}$ is defined only for $x \geq 0$, there are no real solutions.

o)
$$\begin{aligned} (625^{-1})^{-\frac{1}{4}} &= 625^{\frac{1}{4}} \\ &= \sqrt[4]{625} \\ &= 5 \end{aligned}$$

q)
$$\begin{aligned} \left[\left(\sqrt{125}\right)^4\right]^{\frac{1}{6}} &= \left(\left(125^{\frac{1}{2}}\right)^4\right)^{\frac{1}{6}} \\ &= (125^2)^{\frac{1}{6}} \\ &= 125^{\frac{2}{6}} \\ &= 125^{\frac{1}{3}} \\ &= \sqrt[3]{125} \\ &= 5 \end{aligned}$$

s) $\sqrt{\sqrt[3]{729}} = \sqrt{9}$
$= 3$

Section 1.2 Page 16 Question 6

a) $\sqrt{\sqrt{x^4}} = \left(\sqrt{x^4}\right)^{\frac{1}{2}}$
$= \left(\left(x^4\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$
$= x^1$
$= x$

c) $\sqrt{\sqrt{3x^6}} = \left(\sqrt{3x^6}\right)^{\frac{1}{2}}$
$= \left(\left(3x^6\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$
$= \left(3x^6\right)^{\frac{1}{4}}$
$= 3^{\frac{1}{4}}x^{\frac{6}{4}}$
$= 3^{\frac{1}{4}}x^{\frac{3}{2}}$

e) $\sqrt{\sqrt{81x^8}} = \left(\sqrt{81x^8}\right)^{\frac{1}{2}}$
$= \left(\left(81x^8\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$
$= \left(81x^8\right)^{\frac{1}{4}}$
$= 81^{\frac{1}{4}}x^{\frac{8}{4}}$
$= 3x^2$

g) $\left(a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^{12} = \left(a^{\frac{1}{3}}\right)^{12}\left(b^{\frac{1}{4}}\right)^{12}$
$= a^4b^3$

i) $\left(81a^8b^4\right)^{\frac{1}{4}} = 81^{\frac{1}{4}}\left(a^8\right)^{\frac{1}{4}}\left(b^4\right)^{\frac{1}{4}}$
$= 3a^2b$

k) $\left(\sqrt{x^3}\right)\left(\sqrt[3]{x}\right) = \left(x^3\right)^{\frac{1}{2}}x^{\frac{1}{3}}$
$= x^{\frac{3}{2}}x^{\frac{1}{3}}$
$= x^{\frac{3}{2}+\frac{1}{3}}$
$= x^{\frac{9}{6}+\frac{2}{6}}$
$= x^{\frac{11}{6}}$

m) $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right) = \left(x^3\right)^{\frac{1}{5}}\left(x^2\right)^{\frac{1}{3}}$
$= x^{\frac{3}{5}}x^{\frac{2}{3}}$
$= x^{\frac{3}{5}+\frac{2}{3}}$
$= x^{\frac{9}{15}+\frac{10}{15}}$
$= x^{\frac{19}{15}}$

o) $\left(\sqrt[4]{a^3b^5}\right)^{\frac{1}{2}} = \left(\left(a^3b^5\right)^{\frac{1}{4}}\right)^{\frac{1}{2}}$
$= \left(a^3b^5\right)^{\frac{1}{8}}$
$= a^{\frac{3}{8}}b^{\frac{5}{8}}$

Section 1.2 Page 16 Question 7

a) $0 < 0.4 < 1$
$6^0 < 6^{0.4} < 6^1$
which leads to
$1 < 6^{0.4} < 6$

An estimate might be $6^{0.4} \doteq 3$.
Use a calculator to find $6^{0.4} \doteq 2.05$.

c) $-2 < -1.2 < -1$
$4^{-2} < 4^{-1.2} < 4^{-1}$
which leads to
$\frac{1}{16} < 4^{-1.2} < \frac{1}{4}$

An estimate might be $4^{-1.2} \doteq \frac{1}{5}$ or 0.2. Use a calculator to find $4^{-1.2} \doteq 0.19$.

e) $-1 < -\frac{3}{5} < 0$
$7^{-1} < 7^{-\frac{3}{5}} < 7^0$
which leads to
$\frac{1}{7} < 7^{-\frac{3}{5}} < 1$

An estimate might be $7^{-\frac{3}{5}} \doteq \frac{1}{4}$ or 0.25. Use a calculator to find $7^{-\frac{3}{5}} \doteq 0.31$.

Apply, Solve, Communicate

Section 1.2 Page 17 Question 8

Let S_m represent the speed of the model in metres per second,

S_r represent the speed of the real ship in metres per second,

L_m represent the length of the model in metres, and

L_r represent the length of the real ship in metres.

From the formula,
$$S_m = \frac{S_r \times L_m^{\frac{1}{2}}}{L_r^{\frac{1}{2}}}$$

Thus,
$$S_m = \frac{12 \times 15^{\frac{1}{2}}}{300^{\frac{1}{2}}}$$
$$\doteq 2.68$$

The model ship should be tested at a speed of approximately 2.7 m/s.

Section 1.2 Page 17 Question 9

a) Use the Pythagorean Theorem.
$$d^2 + r^2 = (r + h)^2$$
$$d^2 = r^2 + 2rh + h^2 - r^2$$
$$d^2 = 2rh + h^2$$

Since $d \geq 0$,
$$d = (2rh + h^2)^{\frac{1}{2}}$$

b) For the aircraft, use $r = 6370$, $h = 10$.
$$d = (2(6370)(10) + 10^2)^{\frac{1}{2}}$$
$$\doteq 357.07$$

The distance from the aircraft to the horizon is approximately 357 km.

For the spacecraft, use $r = 6370$, $h = 200$.
$$d = (2(6370)(200) + 200^2)^{\frac{1}{2}}$$
$$\doteq 1608.73$$

The distance from the spacecraft to the horizon is approximately 1609 km.

Section 1.2 Page 17 Question 10
$$D = 9.4t^{\frac{2}{3}}$$
$$= 9.4\sqrt[3]{18^2}$$
$$= 9.4\sqrt[3]{324}$$
$$\doteq 64.6$$

The diameter of the hurricane is approximately 65 km.

Section 1.2 Page 17 Question 11

a) The formula for the volume, V, of a cube gives an expression for the edge length, e, in terms of the volume.
$$e^3 = V$$
$$e = \sqrt[3]{V}$$

Thus,
$$e = \sqrt[3]{21\,000}$$
$$\doteq 27.6$$

The edge length is approximately 27.6 m.

b) Answers may vary.

Section 1.2 Page 18 Question 12

Note	Multiple of C	Frequency (Hz)
C	1	261.6
C#	$\sqrt[12]{2}$	277.2
D	$\left(\sqrt[12]{2}\right)^2$	293.6
D#	$\left(\sqrt[12]{2}\right)^3$	311.1
E	$\left(\sqrt[12]{2}\right)^4$	329.6
F	$\left(\sqrt[12]{2}\right)^5$	349.2
F#	$\left(\sqrt[12]{2}\right)^6$	370.0
G	$\left(\sqrt[12]{2}\right)^7$	392.0
G#	$\left(\sqrt[12]{2}\right)^8$	415.3
A	$\left(\sqrt[12]{2}\right)^9$	440.0
A#	$\left(\sqrt[12]{2}\right)^{10}$	466.1
B	$\left(\sqrt[12]{2}\right)^{11}$	493.8
C	$\left(\sqrt[12]{2}\right)^{12}$	523.2

Section 1.2 Page 18 Question 13

a)
$2^x = 32$
$2^x = 2^5$
$x = 5$

b)
$3^{x+1} = 81$
$3^{x+1} = 3^4$
$x + 1 = 4$
$x = 3$

c)
$(-1)^x = 1$
$(-1)^x = (-1)^{2n}$, n any integer
$x = \ldots, -6, -4, -2, 0, 2, 4, 6, \ldots$

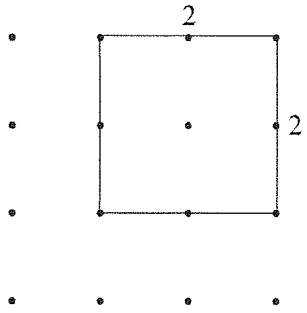
d)
$6^{x-2} = 36$
$6^{x-2} = 6^2$
$x - 2 = 2$
$x = 4$

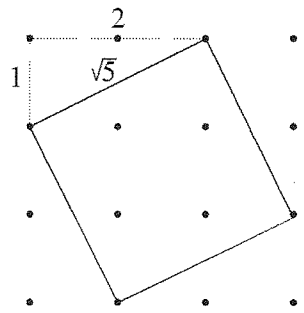
e)
$2^{2x} = 16$
$2^{2x} = 2^4$
$2x = 4$
$x = 2$

f)
$(-1)^x = -1$
$(-1)^x = (-1)^{2n-1}$, n any integer
$x = \ldots, -5, -3, -1, 1, 3, 5, \ldots$

Section 1.2 Page 18 Question 14

a)

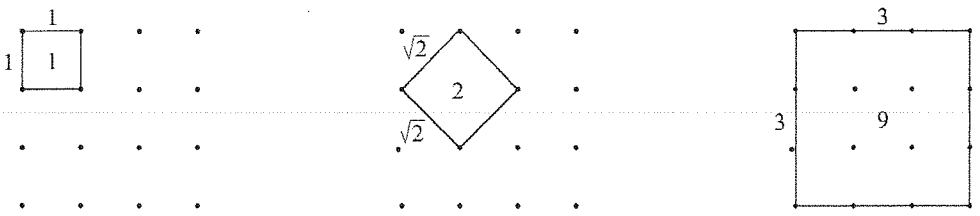




In the left diagram, the square has a side length of 2, yielding an area of 4 square units.

The square on the right has a side length of $\sqrt{1^2 + 2^2}$ or $\sqrt{5}$ units, yielding an area of $\left(\sqrt{5}\right)^2$ or 5 square units.

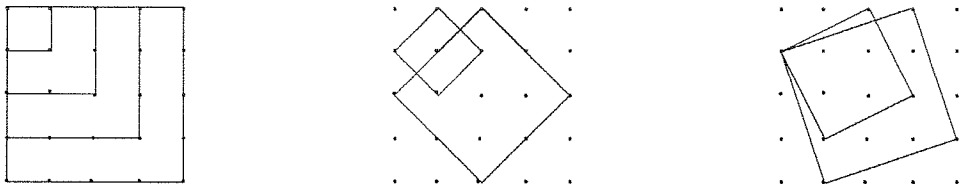
b)



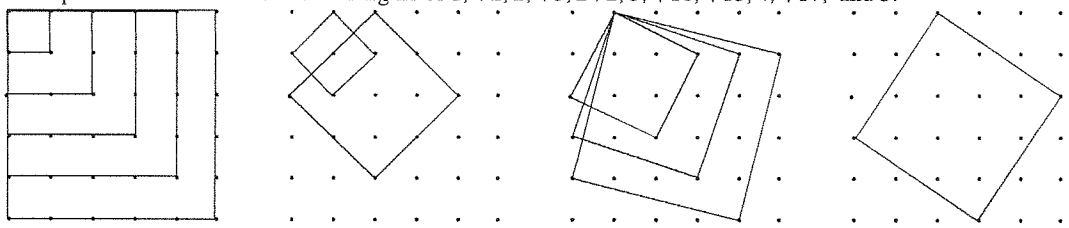
In the left diagram, the square has a side length of 1, yielding an area of 1 square unit. The square in the middle has a side length of $\sqrt{2}$ units, yielding an area of 2 square units. The square at the right has a side length of 3 units, yielding an area of 9 square units.

c) The squares have sides of $\sqrt{2}$ or $2^{\frac{1}{2}}$ and $\sqrt{5}$ or $5^{\frac{1}{2}}$.

d) The squares in the middle and right graphic have side lengths of $2^{\frac{1}{2}}, 8^{\frac{1}{2}}, 5^{\frac{1}{2}}, 10^{\frac{1}{2}}$, respectively.



e) 11 squares can be made with side lengths of $1, \sqrt{2}, 2, \sqrt{5}, 2\sqrt{2}, 3, \sqrt{10}, \sqrt{13}, 4, \sqrt{17}$, and 5.



f) The number of squares goes up by 3 each time.

1.3 Solving Exponential Equations

Practise

Section 1.3 Page 23 Question 1

a) $2^x = 16$
$2^x = 2^4$
$x = 4$

c) $2^x = 128$
$2^x = 2^7$
$x = 7$

e) $4^y = 256$
$4^y = 4^4$
$y = 4$

g) $(-3)^x = -27$
$(-3)^x = (-3)^3$
$x = 3$

i) $(-5)^a = 25$
$(-5)^a = (-5)^2$
$a = 2$

k) $-2^x = -16$
$-(2^x) = -(2^4)$
$x = 4$

m) $-5^x = -625$
$-(5^x) = -(5^4)$
$x = 4$

o) m is any odd integer

Section 1.3 Page 23 Question 2

a) $7^{w-2} = 49$
$7^{w-2} = 7^2$
$w - 2 = 2$
$w = 4$

c) $2^{1-x} = 128$
$2^{1-x} = 2^7$
$1 - x = 7$
$x = -6$

e) $5^{3x-1} = 25$
$5^{3x-1} = 5^2$
$3x - 1 = 2$
$3x = 3$
$x = 1$

g) $4^{x-1} = 1$
$4^{x-1} = 4^0$
$x - 1 = 0$
$x = 1$

i) $(-1)^{2x} = 1$

$\left((-1)^2\right)^x = 1$

$1^x = 1$

x is any real number.

Section 1.3 Page 23 Question 3

a) $6^{x+3} = 6^{2x}$

$x + 3 = 2x$

$x = 3$

Check $x = 3$.

L.S. $= 6^{3+3}$

$= 6^6$

R.S. $= 6^{2(3)}$

$= 6^6$

L.S. = R.S.

c) $3^{2y+3} = 3^{y+5}$

$2y + 3 = y + 5$

$y = 2$

Check $y = 2$.

L.S. $= 3^{2(2)+3}$

$= 3^7$

R.S. $= 3^{2+5}$

$= 3^7$

L.S. = R.S.

e) $7^{5d-1} = 7^{2d+5}$

$5d - 1 = 2d + 5$

$3d = 6$

$d = 2$

Check $d = 2$.

L.S. $= 7^{5(2)-1}$

$= 7^9$

R.S. $= 7^{2(2)+5}$

$= 7^9$

L.S. = R.S.

Section 1.3 Page 23 Question 4

a) $16^{2x} = 8^{3x}$

$\left(2^4\right)^{2x} = \left(2^3\right)^{3x}$

$2^{8x} = 2^{9x}$

$8x = 9x$

$x = 0$

c) $27^{x-1} = 9^{2x}$

$\left(3^3\right)^{x-1} = \left(3^2\right)^{2x}$

$3^{3x-3} = 3^{4x}$

$3x - 3 = 4x$

$x = -3$

e) $16^{2p+1} = 8^{3p+1}$

$\left(2^4\right)^{2p+1} = \left(2^3\right)^{3p+1}$

$2^{8p+4} = 2^{9p+3}$

$8p + 4 = 9p + 3$

$p = 1$

Section 1.3 Page 23 Question 5

a) $2^{x+5} = 4^{x+2}$

$2^{x+5} = \left(2^2\right)^{x+2}$

$2^{x+5} = 2^{2x+4}$

$x + 5 = 2x + 4$

$x = 1$

Check $x = 1$.

L.S. $= 2^{1+5}$

$= 2^6$

$= 64$

R.S. $= 4^{1+2}$

$= 4^3$

$= 64$

L.S. = R.S.

c) $9^{2q-6} = 3^{q+6}$

$\left(3^2\right)^{2q-6} = 3^{q+6}$

$3^{4q-12} = 3^{q+6}$

$4q - 12 = q + 6$

$3q = 18$

$q = 6$

Check $q = 6$.

L.S. $= 9^{2(6)-6}$

$= 9^6$

$= 531\,441$

R.S. $= 3^{6+6}$

$= 3^{12}$

$= 531\,441$

L.S. = R.S.

e) $27^{y-1} = 9^{2y-4}$

$\left(3^3\right)^{y-1} = \left(3^2\right)^{2y-4}$

$3^{3y-3} = 3^{4y-8}$

$3y - 3 = 4y - 8$

$y = 5$

Check $y = 5$.

L.S. $= 27^{5-1}$

$= 27^4$

$= 531\,441$

R.S. $= 9^{2(5)-4}$

$= 9^6$

$= 531\,441$

L.S. = R.S.

Section 1.3 Page 23 Question 6

a)
$$5^{4-x} = \frac{1}{5}$$
$$5^{4-x} = 5^{-1}$$
$$4 - x = -1$$
$$x = 5$$

Check $x = 5$.

$$\text{L.S.} = 5^{4-5}$$
$$= 5^{-1}$$
$$= \frac{1}{5}$$
$$= \text{R.S.}$$

c)
$$6^{3x-7} = \frac{1}{6}$$
$$6^{3x-7} = 6^{-1}$$
$$3x - 7 = -1$$
$$3x = 6$$
$$x = 2$$

Check $x = 2$.

$$\text{L.S.} = 6^{3(2)-7}$$
$$= 6^{-1}$$
$$= \frac{1}{6}$$
$$= \text{R.S.}$$

e)
$$5^{2n+1} = \frac{1}{125}$$
$$5^{2n+1} = 5^{-3}$$
$$2n + 1 = -3$$
$$2n = -4$$
$$n = -2$$

Check $n = -2$.

$$\text{L.S.} = 5^{2(-2)+1}$$
$$= 5^{-3}$$
$$= \frac{1}{125}$$
$$= \text{R.S.}$$

Section 1.3 Page 24 Question 7

a)
$$4^x = 8$$
$$\left(2^2\right)^x = 2^3$$
$$2^{2x} = 2^3$$
$$2x = 3$$
$$x = \frac{3}{2}$$

Check $x = \frac{3}{2}$.

$$\text{L.S.} = 4^{\frac{3}{2}}$$
$$= \sqrt{4^3}$$
$$= \sqrt{64}$$
$$= 8$$
$$= \text{R.S.}$$

c)
$$(-8)^y = -2$$
$$\left((-2)^3\right)^y = -2$$
$$(-2)^{3y} = (-2)^1$$
$$3y = 1$$
$$y = \frac{1}{3}$$

Check $y = \frac{1}{3}$.

$$\text{L.S.} = (-8)^{\frac{1}{3}}$$
$$= \sqrt[3]{-8}$$
$$= -2$$
$$= \text{R.S.}$$

e)
$$2^{9x} = \frac{1}{8}$$
$$2^{9x} = 2^{-3}$$
$$9x = -3$$
$$x = -\frac{1}{3}$$

Check $x = -\frac{1}{3}$.

$$\text{L.S.} = 2^{9\left(-\frac{1}{3}\right)}$$
$$= 2^{-3}$$
$$= \frac{1}{8}$$
$$= \text{R.S.}$$

g)
$$2^x = 16^4$$
$$2^x = \left(2^4\right)^4$$
$$2^x = 2^{16}$$
$$x = 16$$

Check $x = 16$.

$$\text{L.S.} = 2^{16}$$
$$= 65\,536$$
$$= 16^4$$
$$= \text{R.S.}$$

i)
$$9^{2s+1} = 27$$
$$\left(3^2\right)^{2s+1} = 3^3$$
$$3^{4s+2} = 3^3$$
$$4s + 2 = 3$$
$$4s = 1$$
$$s = \frac{1}{4}$$

Check $s = \frac{1}{4}$.

$$\text{L.S.} = 9^{2\left(\frac{1}{4}\right)+1}$$
$$= 9^{\frac{3}{2}}$$
$$= \left(\sqrt{9}\right)^3$$
$$= 3^3$$
$$= 27$$
$$= \text{R.S.}$$

Section 1.3 Page 24 Question 8

a)
$$9^{x+1} = 27^{2x}$$
$$(3^2)^{x+1} = (3^3)^{2x}$$
$$3^{2x+2} = 3^{6x}$$
$$2x + 2 = 6x$$
$$4x = 2$$
$$x = \frac{1}{2}$$

Check $x = \frac{1}{2}$.

$$\text{L.S.} = 9^{\frac{1}{2}+1}$$
$$= 9^{\frac{3}{2}}$$
$$= \left(\sqrt{9}\right)^3$$
$$= 3^3$$
$$= 27$$
$$\text{R.S.} = 27^{2\left(\frac{1}{2}\right)}$$
$$= 27$$
$$= \text{L.S.}$$

c)
$$36^{t-2} = 216^{-2t}$$
$$(6^2)^{t-2} = (6^3)^{-2t}$$
$$6^{2t-4} = 6^{-6t}$$
$$2t - 4 = -6t$$
$$8t = 4$$
$$t = \frac{1}{2}$$

Check $t = \frac{1}{2}$.

$$\text{L.S.} = 36^{\frac{1}{2}-2}$$
$$= 36^{-\frac{3}{2}}$$
$$= \left(\sqrt{36}\right)^{-3}$$
$$= 6^{-3}$$
$$= \frac{1}{216}$$
$$\text{R.S.} = 216^{-2\left(\frac{1}{2}\right)}$$
$$= 216^{-1}$$
$$= \frac{1}{216}$$
$$= \text{L.S.}$$

e)
$$25^{1-3x} = 125^{-x}$$
$$(5^2)^{1-3x} = (5^3)^{-x}$$
$$5^{2-6x} = 5^{-3x}$$
$$2 - 6x = -3x$$
$$3x = 2$$
$$x = \frac{2}{3}$$

Check $x = \frac{2}{3}$.

$$\text{L.S.} = 25^{1-3\left(\frac{2}{3}\right)}$$
$$= 25^{-1}$$
$$= \frac{1}{25}$$
$$\text{R.S.} = 125^{-\frac{2}{3}}$$
$$= \left(\sqrt[3]{125}\right)^{-2}$$
$$= 5^{-2}$$
$$= \frac{1}{25}$$
$$= \text{L.S.}$$

Section 1.3 Page 24 Question 9

a)
$$5 = 25^{\frac{x}{2}}$$
$$5 = (5^2)^{\frac{x}{2}}$$
$$5^1 = 5^x$$
$$x = 1$$

Check $x = 1$.

$$\text{R.S.} = 25^{\frac{1}{2}}$$
$$= 5$$
$$= \text{L.S.}$$

c)
$$9^{\frac{y}{5}} = 27$$
$$(3^2)^{\frac{y}{5}} = 3^3$$
$$3^{\frac{2y}{5}} = 3^3$$
$$\frac{2y}{5} = 3$$
$$y = \frac{15}{2}$$

Check $y = \frac{15}{2}$.

$$\text{L.S.} = 9^{\frac{\frac{15}{2}}{5}}$$
$$= 9^{\frac{3}{2}}$$
$$= (3^2)^{\frac{3}{2}}$$
$$= 3^3$$
$$= 27$$
$$= \text{R.S.}$$

e)
$$4^{\frac{x}{4}} = \frac{1}{8}$$
$$(2^2)^{\frac{x}{4}} = 2^{-3}$$
$$2^{\frac{x}{2}} = 2^{-3}$$
$$\frac{x}{2} = -3$$
$$x = -6$$

Check $x = -6$.

$$\text{L.S.} = 4^{\frac{-6}{4}}$$
$$= 4^{-\frac{3}{2}}$$
$$= (2^2)^{\frac{-3}{2}}$$
$$= 2^{-3}$$
$$= \frac{1}{8}$$
$$= \text{R.S.}$$

Section 1.3 Page 24 Question 10

a)
$$\begin{aligned} 3\left(5^{x+1}\right) &= 15 \\ 5^{x+1} &= 5^1 \\ x+1 &= 1 \\ x &= 0 \end{aligned}$$

c)
$$\begin{aligned} 5\left(4^x\right) &= 10 \\ 4^x &= 2 \\ \left(2^2\right)^x &= 2 \\ 2^{2x} &= 2^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

e)
$$\begin{aligned} 2 &= 6\left(3^{4f-2}\right) \\ \frac{1}{3} &= 3^{4f-2} \\ 3^{-1} &= 3^{4f-2} \\ -1 &= 4f-2 \\ 4f &= 1 \\ f &= \frac{1}{4} \end{aligned}$$

Section 1.3 Page 24 Question 11

a)
$$\begin{aligned} 2^{x+2} - 2^x &= 48 \\ 2^x(3) &= 48 \\ 2^x(2^2-1) &= 48 \\ 2^x &= 16 \\ x &= 4 \end{aligned}$$

Check $x = 4$.

$$\begin{aligned} \textbf{L.S.} &= 2^{4+2} - 2^4 \\ &= 2^6 - 2^4 \\ &= 64 - 16 \\ &= 48 \\ &= \textbf{R.S.} \end{aligned}$$

c)
$$\begin{aligned} 2^{a+5} + 2^a &= 1056 \\ 2^a(2^5+1) &= 1056 \\ 2^a(33) &= 1056 \\ 2^a &= 32 \\ a &= 5 \end{aligned}$$

Check $a = 5$.

$$\begin{aligned} \textbf{L.S.} &= 2^{5+5} + 2^5 \\ &= 2^{10} + 2^5 \\ &= 1024 + 32 \\ &= 1056 \\ &= \textbf{R.S.} \end{aligned}$$

e)
$$\begin{aligned} 3^{x+3} - 3^{x+1} &= 648 \\ 3^x(3^3 - 3^1) &= 648 \\ 3^x(24) &= 648 \\ 3^x &= 27 \\ x &= 3 \end{aligned}$$

Check $x = 3$.

$$\begin{aligned} \textbf{L.S.} &= 3^{3+3} - 3^{3+1} \\ &= 3^6 - 3^4 \\ &= 729 - 81 \\ &= 648 \\ &= \textbf{R.S.} \end{aligned}$$

g)
$$\begin{aligned} 2^{x+2} - 2^{x+5} &= -7 \\ 2^x(2^2 - 2^5) &= -7 \\ 2^x(-28) &= -7 \\ 2^x &= -\frac{1}{4} \\ x &= -2 \end{aligned}$$

Check $x = -2$.

$$\begin{aligned} \textbf{L.S.} &= 2^{-2+2} - 2^{-2+5} \\ &= 2^0 - 2^3 \\ &= 1 - 8 \\ &= -7 \\ &= \textbf{R.S.} \end{aligned}$$

i)
$$\begin{aligned} 5^{n+2} - 5^{n+3} &= -2500 \\ 5^n(5^2 - 5^3) &= -2500 \\ 5^n(-100) &= -2500 \\ 5^n &= 25 \\ n &= 2 \end{aligned}$$

Check $n = 2$.

$$\begin{aligned} \textbf{L.S.} &= 5^{2+2} - 5^{2+3} \\ &= 5^4 - 5^5 \\ &= 625 - 3125 \\ &= -2500 \\ &= \textbf{R.S.} \end{aligned}$$

Apply, Solve, Communicate

Section 1.3 Page 24 Question 12

$$\begin{aligned} 4^{3x+3} &= 8^{2x+2} \\ \left(2^2\right)^{3x+3} &= \left(2^3\right)^{2x+2} \\ 2^{6x+6} &= 2^{6x+6} \end{aligned}$$

Since both sides of the equation reduce to the same expression, the equation is true for all real numbers.

Section 1.3 Page 24 Question 13

At $t = 0$, the original mass is 128 mg, so $A_0 = 128$.

$$\begin{aligned} A_L &= 2 \\ 128\left(\frac{1}{2}\right)^t &= 2 \\ \left(\frac{1}{2}\right)^t &= \frac{1}{64} \\ \left(\frac{1}{2}\right)^t &= \left(\frac{1}{2}\right)^6 \\ t &= 6 \end{aligned}$$

The elapsed time is 6 years.

Section 1.3 Page 24 Question 14

The decay of strontium-90 can be defined by the exponential equation $A_L = A_0\left(\frac{1}{2}\right)^{\frac{t}{28}}$.

a)

$$\begin{aligned} A_L &= \frac{1}{4}A_0 \\ A_0\left(\frac{1}{2}\right)^{\frac{t}{28}} &= \left(\frac{1}{4}\right)A_0 \\ \left(\frac{1}{2}\right)^{\frac{t}{28}} &= \left(\frac{1}{2}\right)^2 \\ \frac{t}{28} &= 2 \\ t &= 56 \end{aligned}$$

The elapsed time is 56 years.

b)

$$\begin{aligned} A_L &= \frac{1}{8}A_0 \\ A_0\left(\frac{1}{2}\right)^{\frac{t}{28}} &= \left(\frac{1}{8}\right)A_0 \\ \left(\frac{1}{2}\right)^{\frac{t}{28}} &= \left(\frac{1}{2}\right)^3 \\ \frac{t}{28} &= 3 \\ t &= 84 \end{aligned}$$

The elapsed time is 84 years.

c)

$$\begin{aligned} A_L &= \frac{1}{32}A_0 \\ A_0\left(\frac{1}{2}\right)^{\frac{t}{28}} &= \left(\frac{1}{32}\right)A_0 \\ \left(\frac{1}{2}\right)^{\frac{t}{28}} &= \left(\frac{1}{2}\right)^5 \\ \frac{t}{28} &= 5 \\ t &= 140 \end{aligned}$$

The elapsed time is 140 years.

Section 1.3 Page 25 Question 15

The decay of thyroid hormone T4 can be defined by the exponential equation $A_L = A_0\left(\frac{1}{2}\right)^{\frac{t}{6.5}}$, where t is measured in days.

a)

$$\begin{aligned} A_L(19.5) &= A_0\left(\frac{1}{2}\right)^{\frac{19.5}{6.5}} \\ &= A_0\left(\frac{1}{2}\right)^3 \\ &= \left(\frac{1}{8}\right)A_0 \end{aligned}$$

After 19.5 days, $\frac{1}{8}$ of the dose remains.

b)

$A_L = 0.0625A_0$ In fact they told you.

$$\begin{aligned} A_0\left(\frac{1}{2}\right)^{\frac{t}{6.5}} &= \left(\frac{1}{16}\right)A_0 \\ \left(\frac{1}{2}\right)^{\frac{t}{6.5}} &= \left(\frac{1}{2}\right)^4 \\ \frac{t}{6.5} &= 4 \\ t &= 26 \end{aligned}$$

After 26 days, only 6.25% of the dose remains.

Section 1.3 Page 25 Question 16

a)

$$\begin{aligned} s &= 64\% \\ (0.8)^d \times 100\% &= 64\% \\ (0.8)^d &= 0.64 \\ (0.8)^d &= (0.8)^2 \\ d &= 2 \end{aligned}$$

At a depth of 2 m, 64% of the sunlight remains.

b)

$$\begin{aligned} s &= (0.8)^{10} \times 100\% \\ &\doteq 0.107 \times 100\% \\ &= 10.7\% \end{aligned}$$

At a depth of 10 m, approximately 11% of the sunlight reaches the diver.

Section 1.3 Page 25 Question 17

The decay of each isotope can be defined by the exponential equation $A_L = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where h is the half-life.

a)

$$A_L(30) = \frac{1}{64}A_0$$
$$A_0\left(\frac{1}{2}\right)^{\frac{30}{h}} = \left(\frac{1}{64}\right)A_0$$
$$\left(\frac{1}{2}\right)^{\frac{30}{h}} = \left(\frac{1}{2}\right)^6$$
$$\frac{30}{h} = 6$$
$$h = 5$$

The half-life is 5 h.

b)

$$A_L(40.8) = 0.25A_0$$
$$A_0\left(\frac{1}{2}\right)^{\frac{40.8}{h}} = \left(\frac{1}{4}\right)A_0$$
$$\left(\frac{1}{2}\right)^{\frac{40.8}{h}} = \left(\frac{1}{2}\right)^2$$
$$\frac{40.8}{h} = 2$$
$$h = 20.4$$

The half-life is 20.4 years.

c)

$$A_L(2) = 0.0625A_0$$
$$A_0\left(\frac{1}{2}\right)^{\frac{2}{h}} = \left(\frac{1}{16}\right)A_0$$
$$\left(\frac{1}{2}\right)^{\frac{2}{h}} = \left(\frac{1}{2}\right)^4$$
$$\frac{2}{h} = 4$$
$$t = 0.5$$

The half-life is 0.5 min or 30 s.

Section 1.3 Page 25 Question 18

$$A_L = 2.5$$
$$40(0.5)^{\frac{t}{14.9}} = 2.5$$
$$(0.5)^{\frac{t}{14.9}} = 0.625$$
$$(0.5)^{\frac{t}{14.9}} = 0.5^4$$
$$\frac{t}{14.9} = 4$$
$$t = 59.6$$

After 59.6 h only 2.5 mg will remain.

Section 1.3 Page 25 Question 19

a)

$$\frac{27^x}{9^{2x-1}} = 3^{x+4}$$
$$\frac{(3^3)^x}{(3^2)^{2x-1}} = 3^{x+4}$$
$$\frac{3^{3x}}{3^{4x-2}} = 3^{x+4}$$
$$3^{3x-(4x-2)} = 3^{x+4}$$
$$3^{2-x} = 3^{x+4}$$
$$2 - x = x + 4$$
$$2x = -2$$
$$x = -1$$

b)

$$27^x\left(9^{2x-1}\right) = 3^{x+4}$$
$$(3^3)^x(3^2)^{2x-1} = 3^{x+4}$$
$$3^{3x}3^{4x-2} = 3^{x+4}$$
$$3^{3x+(4x-2)} = 3^{x+4}$$
$$3^{7x-2} = 3^{x+4}$$
$$7x - 2 = x + 4$$
$$6x = 6$$
$$x = 1$$

c)

$$27^{x+1} = \left(\frac{1}{9}\right)^{2x-5}$$
$$(3^3)^{x+1} = (3^{-2})^{2x-5}$$
$$3^{3x+3} = 3^{-4x+10}$$
$$3x + 3 = -4x + 10$$
$$7x = 7$$
$$x = 1$$

Section 1.3 Page 25 Question 20

a)

$$2^{x^2+2x} = 2^{x+6}$$
$$x^2 + 2x = x + 6$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3 \text{ or } 2$$

b)

$$3^{x^2-2x} = 3^{x-2}$$
$$x^2 - 2x = x - 2$$
$$x^2 - 3x + 2 = 0$$
$$(x - 1)(x - 2) = 0$$
$$x = 1 \text{ or } 2$$

c)

$$2^{2x^2-3x} = 2^{x^2-2x+12}$$
$$2x^2 - 3x = x^2 - 2x + 12$$
$$x^2 - x - 12 = 0$$
$$(x + 3)(x - 4) = 0$$
$$x = -3 \text{ or } 4$$

Section 1.3 Page 25 Question 21

The decay of vanadium-48 can be defined by the exponential equation $A_L = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where h is the half-life.

$$A_L(8) = \frac{1}{\sqrt{2}}A_0$$
$$A_0\left(\frac{1}{2}\right)^{\frac{8}{h}} = \left(\frac{1}{2}\right)^{\frac{1}{2}}A_0$$
$$\left(\frac{1}{2}\right)^{\frac{8}{h}} = \left(\frac{1}{2}\right)^{\frac{1}{2}}$$
$$\frac{8}{h} = \frac{1}{2}$$
$$h = 16$$

The half-life of vanadium-48 is 16 days.

Section 1.3 Page 25 Question 22

a)

$$\frac{2^{2x+1}}{2^{x-3}} = 4$$
$$2^{2x+1-(x-3)} = 2^2$$
$$2^{x+4} = 2^2$$
$$x + 4 = 2$$
$$x = -2$$

Check $x = -2$.

$$\text{L.S.} = \frac{2^{2(-2)+1}}{2^{-2-3}}$$
$$= \frac{2^{-3}}{2^{-5}}$$
$$= 2^2$$
$$= 4$$
$$= \text{R.S.}$$

b)

$$\frac{9^{x+4}}{27^{x-1}} = 81$$
$$\frac{\left(3^2\right)^{x+4}}{\left(3^3\right)^{x-1}} = 3^4$$
$$3^{2x+8-3(x-1)} = 3^4$$
$$3^{11-x} = 3^4$$
$$11 - x = 4$$
$$x = 7$$

Check $x = 7$.

$$\text{L.S.} = \frac{9^{7+4}}{27^{7-1}}$$
$$= \frac{9^{11}}{27^6}$$
$$= \frac{3^{22}}{3^{18}}$$
$$= 3^4$$
$$= 81$$
$$= \text{R.S.}$$

c)

$$\frac{8^{x+2}}{4^{x+3}} = 16^{x-3}$$
$$\frac{\left(2^3\right)^{x+2}}{\left(2^2\right)^{x+3}} = \left(2^4\right)^{x-3}$$
$$2^{3x+6-(2x+6)} = 2^{4x-12}$$
$$x = 4x - 12$$
$$3x = 12$$
$$x = 4$$

Check $x = 4$.

$$\text{L.S.} = \frac{8^{4+2}}{4^{4+3}}$$
$$= \frac{8^6}{4^7}$$
$$= \frac{2^{18}}{2^{14}}$$
$$= 2^4$$
$$= 16$$
$$\text{R.S.} = 16^{4-3}$$
$$= 16$$
$$= \text{L.S.}$$

Section 1.3 Page 25 Question 23

$$\frac{16^{x+2y}}{8^{x-y}} = 32$$
$$\frac{(2^4)^{x+2y}}{(2^3)^{x-y}} = 2^5$$
$$\frac{2^{4(x+2y)}}{2^{3(x-y)}} = 2^5$$
$$2^{4(x+2y)-3(x-y)} = 2^5$$
$$4x + 8y - 3x + 3y = 5$$
$$x + 11y = 5 \quad (1)$$

$$\frac{32^{x+3y}}{16^{x+2y}} = \frac{1}{8}$$
$$\frac{(2^5)^{x+3y}}{(2^4)^{x+2y}} = 2^{-3}$$
$$\frac{2^{5(x+3y)}}{2^{4(x+2y)}} = 2^{-3}$$
$$2^{5(x+3y)-4(x+2y)} = 2^{-3}$$
$$5x + 15y - 4x - 8y = -3$$
$$x + 7y = -3 \quad (2)$$

Subtracting (2) from (1) yields $4y = 8$ or $y = 2$. Substitution of $y = 2$ into (1) yields $x = -17$. A check in (2) confirms the solution $(x, y) = (-17, 2)$.

Section 1.3 Page 26 Number Power

Any natural number can be expressed as a sum of powers of 2. For example,

$$\begin{aligned} 89 &= 1 + 8 + 16 + 64 \\ &= 2^0 + 2^3 + 2^4 + 2^6 \end{aligned}$$

At most, p powers of 2 are required to sum to any natural number, n, where 2^p is the smallest power of 2 greater than or equal to n. Since $1023 < 1024 (= 2^{10})$, at most 10 powers of 2 are required to sum to 1023.

Number the bags 0 to 9 and fill each bag with 2^n coins, where n is the bag number. A request for 89 coins therefore can be satisfied by providing bags 0, 3, 4, and 6.

1.4 Review: Adding, Subtracting, and Multiplying Polynomials

Practise

Section 1.4 Page 29 Question 1

a)
$$\begin{aligned} (3x^2 - x + 2) + (4x^2 + 3x - 1) &= 3x^2 - x + 2 + 4x^2 + 3x - 1 \\ &= 7x^2 + 2x + 1 \end{aligned}$$

c)
$$\begin{aligned} (7m^2 - mn - 8n^2) + (6m^2 + 9mn + 11n^2) &= 7m^2 - mn - 8n^2 + 6m^2 + 9mn + 11n^2 \\ &= 13m^2 + 8mn + 3n^2 \end{aligned}$$

e)
$$\begin{aligned} (3xy - 2x + 7) + (6xy + 5x - 3) &= 3xy - 2x + 7 + 6xy + 5x - 3 \\ &= 9xy + 3x + 4 \end{aligned}$$

Section 1.4 Page 29 Question 2

a)
$$\begin{aligned} (3x^2 - 7x + 3) - (x^2 + 5x - 2) &= 3x^2 - 7x + 3 - x^2 - 5x + 2 \\ &= 2x^2 - 12x + 5 \end{aligned}$$

c)
$$\begin{aligned} (9x^2 - 4xy - y^2) - (6y^2 + 3xy + 10x^2) &= 9x^2 - 4xy - y^2 - 6y^2 - 3xy - 10x^2 \\ &= -x^2 - 7xy - 7y^2 \end{aligned}$$

e)
$$\begin{aligned} (3x + 4y - 5z) - (x - y - z) &= 3x + 4y - 5z - x + y + z \\ &= 2x + 5y - 4z \end{aligned}$$

Section 1.4 Page 29 Question 3

$$\begin{aligned} ((3x^2 - 6x + 5) + (-3x^2 + 6)) + (-x^2 - x - 1) &= 3x^2 - 6x + 5 - 3x^2 + 6 - x^2 - x - 1 \\ &= -x^2 - 7x + 10 \end{aligned}$$

Section 1.4 Page 29 Question 4

$$\begin{aligned} (4x + 2y - 7) + ((-2x + 3y - 2) + (3x + y - 4)) &= 4x + 2y - 7 + (-2x + 3y - 2 + 3x + y - 4) \\ &= 4x + 2y - 7 + x + 4y - 6 \\ &= 5x + 6y - 13 \end{aligned}$$

Section 1.4 Page 29 Question 5

$$\begin{aligned} ((2t^2 - 5t + 3) + (4t^2 + 2t + 3)) - (3t^2 + 4t - 7) &= (2t^2 - 5t + 3 + 4t^2 + 2t + 3) - (3t^2 + 4t - 7) \\ &= 6t^2 - 3t + 6 - 3t^2 - 4t + 7 \\ &= 3t^2 - 7t + 13 \end{aligned}$$

Section 1.4 Page 29 Question 6

$$\begin{aligned} (3m^2 + 4m - 1) - ((m^2 + 2m - 3) + (4m^2 - m + 2)) &= 3m^2 + 4m - 1 - (m^2 + 2m - 3 + 4m^2 - m + 2) \\ &= 3m^2 + 4m - 1 - (5m^2 + m - 1) \\ &= 3m^2 + 4m - 1 - 5m^2 - m + 1 \\ &= -2m^2 + 3m \end{aligned}$$

Section 1.4 Page 29 Question 7

Let l be the length of the third side.

$$\begin{aligned} l &= 5x - 2y + 3z - ((3y + z) + (4x - y + z)) \\ &= 5x - 2y + 3z - (3y + z + 4x - y + z) \\ &= 5x - 2y + 3z - (4x + 2y + 2z) \\ &= 5x - 2y + 3z - 4x - 2y - 2z \\ &= x - 4y + z \end{aligned}$$

The length of the third side is $x - 4y + z$.

Section 1.4 Page 31 Question 8

a) $2(3x + 4) = 6x + 8$

c) $4y(2y - 3) = 8y^2 - 12y$

e) $2t(4s - 5t) = 8st - 10t^2$

g) $-2(q^2 - 5b - 4) = -2q^2 + 10b + 8$

i) $-4g(1 + 3g - 3g^2) = -4g - 12g^2 + 12g^3$

Section 1.4 Page 31 Question 9

a)
$$\begin{aligned} 2(x - 4) - 3(x - 5) &= 2x - 8 - 3x + 15 \\ &= -x + 7 \end{aligned}$$

c)
$$\begin{aligned} 5(3x - 4y) - (2x - 5y) + 7 &= 15x - 20y - 2x + 5y + 7 \\ &= 13x - 15y + 7 \end{aligned}$$

e)
$$\begin{aligned} 3(2x - 9) - 3 - (4x + 1) + 2 &= 6x - 27 - 3 - 4x - 1 + 2 \\ &= 2x - 29 \end{aligned}$$

g)
$$\begin{aligned} 2(1 - 3s + 2s^2) - (1 - 4s + 5s^2) &= 2 - 6s + 4s^2 - 1 + 4s - 5s^2 \\ &= -s^2 - 2s + 1 \end{aligned}$$

i)
$$\begin{aligned} 3a(2a + 3) + 5a(a - 4) - a(4a + 1) &= 6a^2 + 9a + 5a^2 - 20a - 4a^2 - a \\ &= 7a^2 - 12a \end{aligned}$$

k)
$$\begin{aligned} -4x(2x - 1) - x(1 - 2x) + 2x(x + 4) &= -8x^2 + 4x - x + 2x^2 + 2x^2 + 8x \\ &= -4x^2 + 11x \end{aligned}$$

Section 1.4 Page 31 Question 10

a)
$$\begin{aligned} 3[5 + 4(x - 7)] &= 3(5 + 4x - 28) \\ &= 3(4x - 23) \\ &= 12x - 69 \end{aligned}$$

c)
$$\begin{aligned} 2[3(2t - 4) + 5(t + 3)] &= 2(6t - 12 + 5t + 15) \\ &= 2(11t + 3) \\ &= 22t + 6 \end{aligned}$$

e)
$$\begin{aligned} 2x[x + 2(x - 3)] - x(3x - 4) &= 2x(x + 2x - 6) - 3x^2 + 4x \\ &= 2x(3x - 6) - 3x^2 + 4x \\ &= 6x^2 - 12x - 3x^2 + 4x \\ &= 3x^2 - 8x \end{aligned}$$

Section 1.4 Page 33 Question 11

a)
$$\begin{aligned} (x - 7)(x + 6) &= x^2 + 6x - 7x - 42 \\ &= x^2 - x - 42 \end{aligned}$$

c)
$$\begin{aligned} (y - 3)(y - 9) &= y^2 - 9y - 3y + 27 \\ &= y^2 - 12y + 27 \end{aligned}$$

e)
$$\begin{aligned} (4x + 3)(2x + 7) &= 8x^2 + 28x + 6x + 21 \\ &= 8x^2 + 34x + 21 \end{aligned}$$

g)
$$\begin{aligned} 2(8 - x)(5x + 2) &= 2(40x + 16 - 5x^2 - 2x) \\ &= 2(-5x^2 + 38x + 16) \\ &= -10x^2 + 76x + 32 \end{aligned}$$

i) $-(5x-6)(5x+6) = -((5x)^2 - 6^2)$
$= -(25x^2 - 36)$
$= -25x^2 + 36$

Section 1.4 Page 33 Question 12

a) $(7x+2y)(8x-7y) = 56x^2 - 49xy + 16xy - 14y^2$
$= 56x^2 - 33xy - 14y^2$

c) $(4x-5y)(3x-10y) = 12x^2 - 40xy - 15xy + 50y^2$
$= 12x^2 - 55xy + 50y^2$

e) $(5x^2-4x)(3x^2+2x) = 15x^4 + 10x^3 - 12x^3 - 8x^2$
$= 15x^4 - 2x^3 - 8x^2$

g) $(3x-4y)^2 = (3x)^2 + 2(3x)(-4y) + (-4y)^2$
$= 9x^2 - 24xy + 16y^2$

i) $5(1-xy)(1+xy) = 5[1^2 - (xy)^2]$
$= 5[1 - x^2y^2]$
$= 5 - 5x^2y^2$

Section 1.4 Page 33 Question 13

a) $(x-7)(x+1) + (x+6)(x+2) = x^2 - 6x - 7 + x^2 + 8x + 12$
$= 2x^2 + 2x + 5$

c) $2(x-4)(x+3) + 5(2x-1)(x+6) = 2(x^2 - x - 12) + 5(2x^2 + 11x - 6)$
$= 2x^2 - 2x - 24 + 10x^2 + 55x - 30$
$= 12x^2 + 53x - 54$

e) $2(m-3)(m-4) - 3(m+5)^2 = 2(m^2 - 7m + 12) - 3(m^2 + 10m + 25)$
$= 2m^2 - 14m + 24 - 3m^2 - 30m - 75$
$= -m^2 - 44m - 51$

g) $5(2y-5)(2y+5) - 4(y-2)(y+3) - (2y+1)^2 = 5(4y^2 - 25) - 4(y^2 + y - 6) - (4y^2 + 4y + 1)$
$= 20y^2 - 125 - 4y^2 - 4y + 24 - 4y^2 - 4y - 1$
$= 12y^2 - 8y - 102$

i) $4(x^2 - 3xy) - (x+y)^2 - 2(x-y)(x+y) + 5 = 4x^2 - 12xy - (x^2 + 2xy + y^2) - 2(x^2 - y^2) + 5$
$= 4x^2 - 12xy - x^2 - 2xy - y^2 - 2x^2 + 2y^2 + 5$
$= x^2 - 14xy + y^2 + 5$

Apply, Solve, Communicate

Section 1.4 Page 33 Question 14

a) The area of the rectangle of dimensions $2x + 1$ by $x + 2y + 3$ is found by adding the areas of the constituent rectangles with dimensions given by the constituent terms of the polynomials.

b) $(2x+1)(x+2y+3) = 2x^2 + 4xy + 6x + x + 2y + 3$
$= 2x^2 + 4xy + 7x + 2y + 3$

Section 1.4 Page 34 Question 15

a) $(x+3)(x^2+2x+4) = x^3 + 2x^2 + 4x + 3x^2 + 6x + 12$
$= x^3 + 5x^2 + 10x + 12$

b) $(y-2)(y^2 - y - 5) = y^3 - y^2 - 5y - 2y^2 + 2y + 10$
$= y^3 - 3y^2 - 3y + 10$

c) $(3m+2)(2m^2 + 3m - 4) = 6m^3 + 9m^2 - 12m + 4m^2 + 6m - 8$
$= 6m^3 + 13m^2 - 6m - 8$

d) $$\begin{aligned}(t^2 - 5t - 7)(2t + 1) &= 2t^3 + t^2 - 10t^2 - 5t - 14t - 7\\ &= 2t^3 - 9t^2 - 19t - 7\end{aligned}$$

e) $$\begin{aligned}(x^2 + 2x - 1)(x^2 - x - 4) &= x^4 - x^3 - 4x^2 + 2x^3 - 2x^2 - 8x - x^2 + x + 4\\ &= x^4 + x^3 - 7x^2 - 7x + 4\end{aligned}$$

f) $$\begin{aligned}(y - 2)(y^3 - 2y^2 + 3y - 1) &= y^4 - 2y^3 + 3y^2 - y - 2y^3 + 4y^2 - 6y + 2\\ &= y^4 - 4y^3 + 7y^2 - 7y + 2\end{aligned}$$

g) $$\begin{aligned}(3a^2 - 4a + 2)(a^2 - a - 5) &= 3a^4 - 3a^3 - 15a^2 - 4a^3 + 4a^2 + 20a + 2a^2 - 2a - 10\\ &= 3a^4 - 7a^3 - 9a^2 + 18a - 10\end{aligned}$$

h) $$\begin{aligned}(x^3 - 7)(3x^3 + 7) &= 3x^6 + 7x^3 - 21x^3 - 49\\ &= 3x^6 - 14x^3 - 49\end{aligned}$$

i) $$\begin{aligned}(x^2 - 4x + 1)^2 &= (x^2 - 4x + 1)(x^2 - 4x + 1)\\ &= x^4 - 4x^3 + x^2 - 4x^3 + 16x^2 - 4x + x^2 - 4x + 1\\ &= x^4 - 8x^3 + 18x^2 - 8x + 1\end{aligned}$$

j) $$\begin{aligned}(2n^2 - n - 1)^2 &= (2n^2 - n - 1)(2n^2 - n - 1)\\ &= 4n^4 - 2n^3 - 2n^2 - 2n^3 + n^2 + n - 2n^2 + n + 1\\ &= 4n^4 - 4n^3 - 3n^2 + 2n + 1\end{aligned}$$

k) $$\begin{aligned}(2a - b + 3c)^2 &= (2a - b + 3c)(2a - b + 3c)\\ &= 4a^2 - 2ab + 6ac - 2ab + b^2 - 3bc + 6ac - 3bc + 9c^2\\ &= 4a^2 + b^2 + 9c^2 - 4ab + 12ac - 6bc\end{aligned}$$

l) $$\begin{aligned}(2x - 1)(x^3 - 2x^2 + 5x - 3) &= 2x^4 - 4x^3 + 10x^2 - 6x - x^3 + 2x^2 - 5x + 3\\ &= 2x^4 - 5x^3 + 12x^2 - 11x + 3\end{aligned}$$

m) Let R be the result.

$$\begin{aligned}R &= 2(x - 1)(x^2 - 3x + 2) - (2x^2 - 3x + 4)(2x + 3)\\ &= 2(x^3 - 3x^2 + 2x - x^2 + 3x - 2) - (4x^3 + 6x^2 - 6x^2 - 9x + 8x + 12)\\ &= 2(x^3 - 4x^2 + 5x - 2) - (4x^3 - x + 12)\\ &= 2x^3 - 8x^2 + 10x - 4 - 4x^3 + x - 12\\ &= -2x^3 - 8x^2 + 11x - 16\end{aligned}$$

n) Let R be the result.

$$\begin{aligned}R &= 4(x - y + z)(x - 2y - 3z) - (x + y + z)^2 - (x - y - 2z)\\ &= 4(x^2 - 2xy - 3xz - xy + 2y^2 + 3yz + xz - 2yz - 3z^2)\\ &\quad - (x^2 + xy + xz + xy + y^2 + yz + xz + yz + z^2) - x + y + 2z\\ &= 4(x^2 + 2y^2 - 3z^2 - 3xy + yz - 2xz) - (x^2 + y^2 + z^2 + 2xy + 2yz + 2xz) - x + y + 2z\\ &= 4x^2 + 8y^2 - 12z^2 - 12xy + 4yz - 8xz - x^2 - y^2 - z^2 - 2xy - 2yz - 2xz - x + y + 2z\\ &= 3x^2 + 7y^2 - 13z^2 - 14xy + 2yz - 10xz - x + y + 2z\end{aligned}$$

o) $$\begin{aligned}(3x - 5)[3 + (2x + 4)(x - 1)] &= (3x - 5)(3 + 2x^2 + 2x - 4)\\ &= (3x - 5)(2x^2 + 2x - 1)\\ &= 6x^3 + 6x^2 - 3x - 10x^2 - 10x + 5\\ &= 6x^3 - 4x^2 - 13x + 5\end{aligned}$$

Section 1.4 Page 34 Question 16

a)
$$\begin{aligned}(x+1)(x+2)(x-3) &= (x^2+2x+x+2)(x-3)\\ &= (x^2+3x+2)(x-3)\\ &= x^3-3x^2+3x^2-9x+2x-6\\ &= x^3-7x-6\end{aligned}$$

b)
$$\begin{aligned}(x+1)(x-3)(x+2) &= (x^2-3x+x-3)(x+2)\\ &= (x^2-2x-3)(x+2)\\ &= x^3+2x^2-2x^2-4x-3x-6\\ &= x^3-7x-6\end{aligned}$$

c)
$$\begin{aligned}(x-3)(x+2)(x+1) &= (x^2+2x-3x-6)(x+1)\\ &= (x^2-x-6)(x+1)\\ &= x^3+x^2-x^2-x-6x-6\\ &= x^3-7x-6\end{aligned}$$

d) No.

Section 1.4 Page 34 Question 17

a)
$$\begin{aligned}(2x+1)(x-3)(4x-5) &= (2x+1)(4x^2-17x+15)\\ &= 8x^3-34x^2+30x+4x^2-17x+15\\ &= 8x^3-30x^2+13x+15\end{aligned}$$

b)
$$\begin{aligned}(x+2y)(x-3y)(2x-y) &= (x+2y)(2x^2-xy-6xy+3y^2)\\ &= (x+2y)(2x^2-7xy+3y^2)\\ &= 2x^3-7x^2y+3xy^2+4x^2y-14xy^2+6y^3\\ &= 2x^3-3x^2y-11xy^2+6y^3\end{aligned}$$

c)
$$\begin{aligned}(a+b+c+d)^2 &= (a+b+c+d)(a+b+c+d)\\ &= a^2+ab+ac+ad+ba+b^2+bc+bd+ca+cb+c^2+cd+da+db+dc+d^2\\ &= a^2+2ab+2ac+2ad+b^2+2bc+2bd+c^2+2cd+d^2\end{aligned}$$

Section 1.4 Page 34 Question 18

a) The surface area, A, of the rectangular prism is given by

$$\begin{aligned}A &= 2(lh)+2(lw)+2(wh)\\ &= 2(x-3)(x+4)+2(x-3)(2x+3)+2(2x+3)(x+4)\\ &= 2(x^2+x-12)+2(2x^2-3x-9)+2(2x^2+11x+12)\\ &= 2x^2+2x-24+4x^2-6x-18+4x^2+22x+24\\ &= 10x^2+18x-18 \qquad (1)\end{aligned}$$

The surface area is $10x^2+18x-18$ square units.

b) The volume, V, of the rectangular prism is given by

$$\begin{aligned}V &= lwh\\ &= (x-3)(2x+3)(x+4)\\ &= (x-3)(2x^2+11x+12)\\ &= (2x^3+11x^2+12x-6x^2-33x-36)\\ &= 2x^3+5x^2-21x-36 \qquad (2)\end{aligned}$$

The volume is $2x^3+5x^2-21x-6$ cubic units.

c) Given $x = 7$ cm, (1) becomes

$$\begin{aligned} A &= 10(7)^2 + 18(7) - 18 \\ &= 490 + 126 - 18 \\ &= 598 \end{aligned}$$

The surface area is 598 cm^2.

Given $x = 7$ cm, (2) becomes

$$\begin{aligned} V &= 2(7)^3 + 5(7)^2 - 21(7) - 36 \\ &= 686 + 245 - 147 - 36 \\ &= 748 \end{aligned}$$

The volume is 748 cm^3.

Section 1.4 Page 34 Question 19

The area of the green shaded region is found as follows:

$$\begin{aligned} A &= A_{\text{outer}} - A_{\text{inner}} \\ &= (y+1)(y+x-2) - (y-x)(y) \\ &= y^2 + xy - 2y + y + x - 2 - (y^2 - xy) \\ &= y^2 + xy - y + x - 2 - y^2 + xy \\ &= 2xy + x - y - 2 \end{aligned}$$

The area of the green shaded region is $2xy + x - y - 2$ square units.

Section 1.4 Page 34 Question 20

No. For example, the product $(x + y)(x - y) = x^2 - y^2$ is not a trinomial.

Section 1.4 Page 34 Question 21

a) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = x^2 - 1 + 1 - \frac{1}{x^2}$

$= x^2 - \frac{1}{x^2}$

b) $\left(y - \frac{2}{y}\right)\left(y + \frac{3}{y}\right) = y^2 + 3 - 2 - \frac{6}{y^2}$

$= y^2 - \frac{6}{y^2} + 1$

1.5 Simplifying Rational Expressions

Practise

Section 1.5 Page 40 Question 1

a) $\frac{3t^3 + 6t^2 - 15t}{3t} = \frac{3t(t^2 + 2t - 5)}{3t}$

$= t^2 + 2t - 5,\ t \neq 0$

c) $\frac{10y^4 + 5y^3 - 15y^2}{5y} = \frac{5y^2(2y^2 + y - 3)}{5y}$

$= y(2y^2 + y - 3),\ y \neq 0$

e) $\frac{4m^2 - 8mn}{4mn} = \frac{4m(m - 2n)}{4mn}$

$= \frac{m - 2n}{n},\ m, n \neq 0$

g) $\frac{16a^2bc}{4a^2b^2c^2} = \frac{4a^2bc(4)}{4a^2bc(bc)}$

$= \frac{4}{bc},\ a, b, c \neq 0$

i) $\frac{21m(m-4)}{7m^2} = \frac{7m(3)(m-4)}{7m(m)}$

$= \frac{3(m-4)}{m},\ m \neq 0$

Section 1.5 Page 40 Question 2

a) $\dfrac{5x}{5(x+4)} = \dfrac{x}{x+4},\ x \neq -4$

c) $\dfrac{7x(x-3)}{14x^2(x-3)} = \dfrac{1}{2x},\ x \neq 0, 3$

e) $\dfrac{2x}{2x+8} = \dfrac{2x}{2(x+4)}$
$= \dfrac{x}{x+4},\ x \neq -4$

g) $\dfrac{10x}{5x^2-15x} = \dfrac{5x(2)}{5x(x-3)}$
$= \dfrac{2}{x-3},\ x \neq 0, 3$

i) $\dfrac{3xy}{6x^2y-12xy^2} = \dfrac{3xy}{3xy(2x-4y)}$
$= \dfrac{1}{2x-4y},\ x \neq 0, 2y; y \neq 0$

Section 1.5 Page 40 Question 3

a) $\dfrac{6t-36}{t-6} = \dfrac{6(t-6)}{t-6}$
$= 6,\ t \neq 6$

c) $\dfrac{5x-10}{3x-6} = \dfrac{5(x-2)}{3(x-2)}$
$= \dfrac{5}{3},\ x \neq 2$

e) $\dfrac{8x^2+4x}{6x^2+3x} = \dfrac{4x(2x+1)}{3x(2x+1)}$
$= \dfrac{4}{3},\ x \neq 0, -\dfrac{1}{2}$

g) $\dfrac{4x+4y}{5x+5y} = \dfrac{4(x+y)}{5(x+y)}$
$= \dfrac{4}{5},\ x \neq -y$

i) $\dfrac{5xy+10x}{2y^2+4y} = \dfrac{5x(y+2)}{2y(y+2)}$
$= \dfrac{5x}{2y},\ y \neq 0, -2$

Section 1.5 Page 40 Question 4

a) $\dfrac{m-2}{m^2-5m+6} = \dfrac{m-2}{(m-2)(m-3)}$
$= \dfrac{1}{m-3},\ m \neq 2, 3$

c) $\dfrac{2x+6}{x^2-6x-27} = \dfrac{2(x+3)}{(x-9)(x+3)}$
$= \dfrac{2}{x-9},\ x \neq -3, 9$

e) $\dfrac{a^2+a}{a^2+2a+1} = \dfrac{a(a+1)}{(a+1)^2}$
$= \dfrac{a}{a+1},\ a \neq -1$

g) $\dfrac{2w+2}{2w^2+3w+1} = \dfrac{2(w+1)}{(2w+1)(w+1)}$
$= \dfrac{2}{2w+1},\ w \neq -\dfrac{1}{2}, -1$

i) $\dfrac{8z+6z^2}{9z^2-16} = \dfrac{2z(4+3z)}{(3z-4)(3z+4)}$
$= \dfrac{2z}{3z-4},\ z \neq \pm\dfrac{4}{3}$

Section 1.5 Page 41 Question 5

a) $\dfrac{y-2}{2-y} = \dfrac{y-2}{-(y-2)}$
$= -1,\ y \neq 2$

c) $\dfrac{2t-1}{4-8t} = \dfrac{2t-1}{-4(2t-1)}$
$= -\dfrac{1}{4},\ t \neq \dfrac{1}{2}$

e) $\dfrac{x^2-1}{1-x^2} = \dfrac{x^2-1}{-(x^2-1)}$
$= -1,\ x \neq \pm 1$

Section 1.5 Page 41 Question 6

a) $\dfrac{x^2+4x+4}{x^2+5x+6} = \dfrac{(x+2)^2}{(x+2)(x+3)}$
$= \dfrac{x+2}{x+3},\ x \neq -2, -3$

c) $\dfrac{m^2-5m+6}{m^2+2m-15} = \dfrac{(m-2)(m-3)}{(m+5)(m-3)}$
$= \dfrac{m-2}{m+5},\ m \neq -5, 3$

e) $\dfrac{x^2-10x+24}{x^2-12x+36} = \dfrac{(x-6)(x-4)}{(x-6)^2}$
$= \dfrac{x-4}{x-6},\ x \neq 6$

g) $\dfrac{p^2+8p+16}{p^2-16} = \dfrac{(p+4)^2}{(p-4)(p+4)}$
$= \dfrac{p+4}{p-4},\ p \neq \pm 4$

i) $\dfrac{6v^2+11v+3}{4v^2+8v+3} = \dfrac{(2v+3)(3v+1)}{(2v+3)(2v+1)}$
$= \dfrac{3v+1}{2v+1},\ v \neq -\dfrac{3}{2}, -\dfrac{1}{2}$

k) $\dfrac{3z^2-7z+2}{9z^2-6z+1} = \dfrac{(3z-1)(z-2)}{(3z-1)^2}$
$= \dfrac{z-2}{3z-1},\ z \neq \dfrac{1}{3}$

Apply, Solve, Communicate

Section 1.5 Page 41 Question 7

a) $\text{length} = \dfrac{\text{area}}{\text{width}}$
$= \dfrac{x^2+3x+2}{x+1}$

The length of the flag is $\dfrac{x^2+3x+2}{x+1}$ units.

b) $\text{length} = \dfrac{x^2+3x+2}{x+1}$
$= \dfrac{(x+2)(x+1)}{x+1}$
$= x+2,\ x \neq -1$

In simplest form, the length of the flag is $x+2$ units.

c) $\text{length} : \text{width} = (x+2) : (x+1)$

Substitute $x = 1$.

$\text{length} : \text{width} = (1+2) : (1+1)$
$= 3 : 2$

For a Saskatchewan flag, length : width = 3 : 2.

Section 1.5 Page 41 Question 8

a) $\dfrac{1-x}{x-1} = \dfrac{-(x-1)}{(x-1)}$
$= -1,\ x \neq 1$

b) $\dfrac{x-1}{x+1}$ cannot be simplified

c) $\dfrac{y^2+1}{y^2-1}$ cannot be simplified

d) $\dfrac{3t-7}{3t-7} = 1,\ t \neq \dfrac{7}{3}$

e)
$$\begin{aligned}\frac{t^2-s^2}{(s+t)^2} &= \frac{(t-s)(t+s)}{(s+t)(s+t)}\\ &= \frac{t-s}{s+t},\ s \neq -t\end{aligned}$$

f)
$$\begin{aligned}\frac{x^3-2x^2+3x}{2x^2-4x+6} &= \frac{x(x^2-2x+3)}{2(x^2-2x+3)}\\ &= \frac{x}{2}\end{aligned}$$

Section 1.5 Page 41 Question 9

The rational expressions are not defined for denominators of 0.

a)
$$\begin{aligned}x-y &= 0\\ x &= y\end{aligned}$$

The expression is not defined for $x = y$.

b)
$$\begin{aligned}3x+y &= 0\\ 3x &= -y\\ x &= -\frac{y}{3}\end{aligned}$$

The expression is not defined for $x = -\frac{y}{3}$.

c)
$$\begin{aligned}x^3 &= 0\\ x &= 0\end{aligned}$$

The expression is not defined for $x = 0$.

d)
$$\begin{aligned}x^3-8 &= 0\\ x^3 &= 8\\ x &= 2\end{aligned}$$

The expression is not defined for $x = 2$.

e)
$$\begin{aligned}x^2-1 &= 0\\ x^2 &= 1\\ x &= \pm 1\end{aligned}$$

The expression is not defined for $x = \pm 1$.

f)
$$\begin{aligned}4x^2-9y^2 &= 0\\ 4x^2 &= 9y^2\\ x^2 &= \frac{9y^2}{4}\\ x &= \pm\frac{3y}{2}\end{aligned}$$

The expression is not defined for $x = \pm\frac{3y}{2}$.

Section 1.5 Page 42 Question 10

a) No. The values differ when $x = 3$, for example.
b) No. The second expression is not defined when $x = 0$, whereas the first expression is.
c) Yes.
$$\begin{aligned}\frac{4+4x}{4x-4} &= \frac{4(1+x)}{4(x-1)}\\ &= \frac{x+1}{x-1}\end{aligned}$$
d) No. The values differ when $x = 2$, for example.
e) No. The values differ when $x = 2$, for example.
f) No. The expressions differ by a factor of -1.

Section 1.5 Page 42 Question 11

$$\begin{aligned}\frac{\text{volume}}{\text{surface area}} &= \frac{x^3}{6x^2}\\ &= \frac{x}{6},\ x > 0\end{aligned}$$

For a cube of edge length x, the ratio of volume to surface area is $\frac{x}{6}$.

Section 1.5 Page 42 Question 12

$$\begin{aligned}\frac{\text{volume}}{\text{surface area}} &= \frac{\frac{4}{3}\pi r^3}{4\pi r^2}\\ &= \frac{r}{3},\ r > 0\end{aligned}$$

For a sphere of radius r, the ratio of volume to surface area is $\frac{r}{3}$.

Section 1.5 Page 42 Question 13

a) There are $n + 1$ asterisks in pattern 1.

b)

$$(n + \triangle)(n + \blacksquare) = (n + 1)(n + 3) \quad (1)$$

Thus, $\triangle = 1$ and $\blacksquare = 3$.

c)

$$\frac{(n + 1)(n + 3)}{n + 1} = n + 3 \quad (2)$$

d) Substitute $n = 10$ into (2) to obtain 13.

e) First, determine n.

$$n + 1 = 20$$
$$n = 19 \quad (3)$$

Substitute (3) into (1).

$$(19 + 1)(19 + 3) = 20(22)$$
$$= 440$$

There are 440 asterisks in the corresponding diagram of pattern 2.

f) First, determine n.

$$(n + 1)(n + 3) = 1295$$
$$n^2 + 4n + 3 = 1295$$
$$n^2 + 4n - 1292 = 0$$
$$(n + 38)(n - 34) = 0$$
$$n = -38 \text{ or } 34$$

Since $n > 0$, $n = 34$. Thus, the corresponding diagram of pattern 1 has $34 + 1$ or 35 asterisks.

Section 1.5 Page 43 Question 14

$$\frac{A_{\text{square}}}{A_{\text{trapezoid}}} = \frac{(10x)^2}{\frac{1}{2}(10x + 10x + 16)(10x)}$$
$$= \frac{10x}{10x + 8}$$
$$= \frac{5x}{5x + 4}, \ x > 0$$

The ratio of the area of the square to the area of the trapezoid is $\frac{5x}{5x + 4}$.

Section 1.5 Page 43 Question 15

$$\frac{\text{volume}}{\text{surface area}} = \frac{(x + 4)^2(2x - 2)}{2(x + 4)^2 + 4(x + 4)(2x - 2)}$$
$$= \frac{(x + 4)(2x - 2)}{2(x + 4) + 4(2x - 2)}$$
$$= \frac{2(x + 4)(x - 1)}{2[(x + 4) + 2(2x - 2)]}$$
$$= \frac{(x + 4)(x - 1)}{(x + 4) + 2(2x - 2)}$$
$$= \frac{x^2 + 3x - 4}{x + 4 + 4x - 4}$$
$$= \frac{x^2 + 3x - 4}{5x}$$
$$= \frac{(x + 4)(x - 1)}{5x}, \ x > 1$$

The ratio of volume to surface area is $\frac{(x + 4)(x - 1)}{5x}$.

Section 1.5 Page 43 Question 16

Answers may vary.

a) If $x \neq 1$, then $x - 1$ is a factor of the denominator. Thus, an example is $\frac{x^2 - 1}{x - 1}$.

b) If $y \neq 0, -3$, then y and $y + 3$ are factors of the denominator. Thus, an example is $\frac{y^2 - 9}{y^2 - 3y}$.

c) If $a \neq \frac{1}{2}, -\frac{3}{4}$, then $2a - 1$ and $4a + 3$ are factors of the denominator. Both factors have to be in the denominator, and each can occur more than once. A possible rational expression is

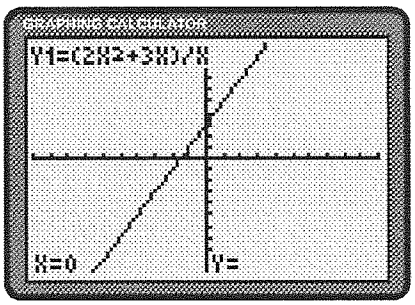
$$\frac{5a^3 + 2a}{(2a - 1)^2(4a + 3)}$$

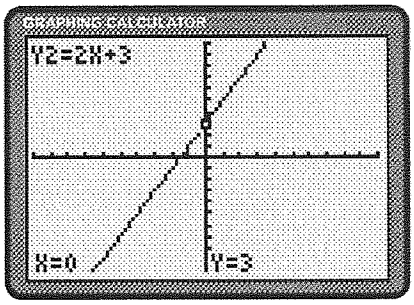
d) If $t \neq 1, \pm\sqrt{3}$, then $t + 1, t + \sqrt{3}$, and $t - \sqrt{3}$ are factors in the denominator. All three factors have to be in the denominator. A possible rational expression is

$$\frac{7t^3 + 5t + 1}{(t + 1)(t + \sqrt{3})(t - \sqrt{3})} = \frac{7t^3 + 5t + 1}{(t + 1)(t^2 - 3)}$$

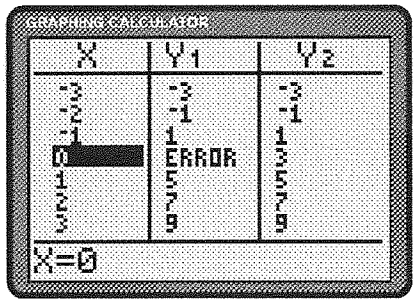
Section 1.5 Page 43 Question 17

a) The functions appear to define the same linear graph.





b) The table confirms that the function $y_1 = \frac{2x^2 + 3x}{x}$ is not defined for $x = 0$.



Section 1.5 Page 43 Question 18

a)

$$\frac{\text{volume}}{\text{surface area}} = \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 + \pi r s}$$
$$= \frac{rh}{3(r + s)}$$

b) Values of $r = 6$, $h = 8$, and $s = 10$ reduce the ratio in part a) to 1.

Section 1.5 Page 43 Word Power

Answers may vary. One such transformation is RING, SING, SINK, SILK, SILL, BILL, BELL.

1.6 Multiplying and Dividing Rational Expressions

Practise

Section 1.6 Page 50 Question 1

a)
$$\frac{y^2}{3} \times \frac{8}{y} = \frac{8y^2}{3y}$$
$$= \frac{8y}{3},\ y \neq 0$$

c)
$$\frac{-5n^2}{12} \times \frac{4}{-15n^5} = \frac{-20n^2}{-180n^5}$$
$$= \frac{1}{9n^3},\ n \neq 0$$

Section 1.6 Page 50 Question 2

a)
$$\frac{3}{x} \div \frac{12}{x^2} = \frac{3}{x} \times \frac{x^2}{12}$$
$$= \frac{3x^2}{12x}$$
$$= \frac{x}{4},\ x \neq 0$$

c)
$$\frac{-15}{2m^2} \div \frac{10}{3m^4} = \frac{-15}{2m^2} \times \frac{3m^4}{10}$$
$$= \frac{-45m^4}{20m^2}$$
$$= -\frac{9m^2}{4},\ m \neq 0$$

e)
$$\frac{20}{3x^5} \div \frac{-15}{8x^2} = \frac{20}{3x^5} \times \frac{8x^2}{-15}$$
$$= \frac{160x^2}{-45x^5}$$
$$= -\frac{32}{9x^3},\ x \neq 0$$

Section 1.6 Page 50 Question 3

a)
$$\frac{3x^3}{2y} \times \frac{8y^2}{9x} = \frac{24x^3y^2}{18xy}$$
$$= \frac{4x^2y}{3},\ x, y \neq 0$$

c)
$$\frac{21xy}{4t^2} \times \frac{12}{7x^2y} = \frac{252xy}{28x^2yt^2}$$
$$= \frac{9}{xt^2},\ x, y, t \neq 0$$

e)
$$\frac{12m}{-5t} \div \frac{8m^2}{-15} = \frac{12m}{-5t} \times \frac{-15}{8m^2}$$
$$= \frac{-180m}{-40m^2t}$$
$$= \frac{9}{2mt},\ m, t \neq 0$$

Section 1.6 Page 50 Question 4

a)
$$\frac{16ab}{9x^4y^2} \times \frac{3x^5y^4}{8a^2b^2} = \frac{16ab}{8a^2b^2} \times \frac{3x^5y^4}{9x^4y^2}$$
$$= \frac{2}{ab} \times \frac{xy^2}{3}$$
$$= \frac{2xy^2}{3ab}, \ a, b, x, y \neq 0$$

c)
$$\frac{5xy}{6x^2y} \div \frac{10xy^2}{9x^3y^2} = \frac{5xy}{6x^2y} \times \frac{9x^3y^2}{10xy^2}$$
$$= \frac{45x^4y^3}{60x^3y^3}$$
$$= \frac{3x}{4}, \ x, y \neq 0$$

e)
$$6x^3y^4 \div \frac{2xy}{-3} = 6x^3y^4 \times \frac{-3}{2xy}$$
$$= \frac{-18x^3y^4}{2xy}$$
$$= -9x^2y^3, \ x, y \neq 0$$

Section 1.6 Page 50 Question 5

a)
$$\frac{3}{x-4} \times \frac{x-4}{6} = \frac{3(x-4)}{6(x-4)}$$
$$= \frac{1}{2}, \ x \neq 4$$

c)
$$\frac{5(y-2)}{y+1} \times \frac{y+1}{10} = \frac{5(y-2)(y+1)}{10(y+1)}$$
$$= \frac{y-2}{2}, \ y \neq -1$$

e)
$$\frac{4a^2b}{3(a+b)} \div \frac{-8ab^2}{a+b} = \frac{4a^2b}{3(a+b)} \times \frac{a+b}{-8ab^2}$$
$$= \frac{4a^2b(a+b)}{-24ab^2(a+b)}$$
$$= -\frac{a}{6b}, \ a \neq -b, \ a, b \neq 0$$

Section 1.6 Page 50 Question 6

a)
$$\frac{4x+4}{3x-3} \times \frac{6x-6}{5x+5} = \frac{4(x+1)}{3(x-1)} \times \frac{6(x-1)}{5(x+1)}$$
$$= \frac{24(x+1)(x-1)}{15(x-1)(x+1)}$$
$$= \frac{8}{5}, \ x \neq \pm 1$$

c)
$$\frac{3a+6}{9a^2} \div \frac{a+2}{-3a} = \frac{3a+6}{9a^2} \times \frac{-3a}{a+2}$$
$$= \frac{-9a(a+2)}{9a^2(a+2)}$$
$$= -\frac{1}{a}, \ a \neq 0, -2$$

e)
$$\frac{7y^2}{y^2-9} \times \frac{4y+12}{14y^3} = \frac{28y^2(y+3)}{14y^3(y-3)(y+3)}$$
$$= \frac{2}{y(y-3)}, \ y \neq 0, \pm 3$$

g)
$$\frac{4x-6}{8x^2y} \times \frac{4xy}{6x-9} = \frac{8xy(2x-3)}{24x^2y(2x-3)}$$
$$= \frac{1}{3x}, \ x \neq 0, \frac{3}{2}, \ y \neq 0$$

Section 1.6 Page 51 Question 7

a)
$$\begin{aligned}\frac{x^2+5x+6}{x^2-6x+5} \times \frac{x^2+x-30}{x^2+9x+18} &= \frac{(x+2)(x+3)(x+6)(x-5)}{(x-5)(x-1)(x+6)(x+3)} \\ &= \frac{x+2}{x-1},\ x \neq -6, -3, 1, 5\end{aligned}$$

c)
$$\begin{aligned}\frac{m^2-3m-4}{m^2+5m} \div \frac{m^2-7m+12}{m^2+2m-15} &= \frac{m^2-3m-4}{m^2+5m} \times \frac{m^2+2m-15}{m^2-7m+12} \\ &= \frac{(m-4)(m+1)(m+5)(m-3)}{m(m+5)(m-4)(m-3)} \\ &= \frac{m+1}{m},\ m \neq -5, 0, 3, 4\end{aligned}$$

e)
$$\begin{aligned}\frac{2x^2-5x-3}{2x^2-11x+15} \times \frac{4x^2-8x-5}{4x^2+4x+1} &= \frac{(2x+1)(x-3)}{(2x-5)(x-3)} \times \frac{(2x+1)(2x-5)}{(2x+1)(2x+1)} \\ &= \frac{(2x+1)(x-3)(2x+1)(2x-5)}{(2x-5)(x-3)(2x+1)(2x+1)} \\ &= 1,\ x \neq -\frac{1}{2}, \frac{5}{2}, 3\end{aligned}$$

Section 1.6 Page 51 Question 8

a)
$$\begin{aligned}\frac{x^2-xy-20y^2}{x^2+8xy+15y^2} \div \frac{x^2-2xy-8y^2}{x^2-xy-6y^2} &= \frac{x^2-xy-20y^2}{x^2-8xy+15y^2} \times \frac{x^2-xy-6y^2}{x^2+2xy-8y^2} \\ &= \frac{(x-5y)(x+4y)}{(x-5y)(x-3y)} \times \frac{(x+2y)(x-3y)}{(x-2y)(x+4y)} \\ &= \frac{x+2y}{x-2y},\ x \neq 5y, 3y, \pm 2y, -4y\end{aligned}$$

c)
$$\begin{aligned}\frac{a^2+15ab+56b^2}{a^2-3ab-54b^2} \div \frac{a^2+6ab-16b^2}{a^2+4ab-12b^2} &= \frac{a^2+15ab+56b^2}{a^2-3ab-54b^2} \times \frac{a^2+4ab-12b^2}{a^2+6ab-16b^2} \\ &= \frac{(a+7b)(a+8b)}{(a-9b)(a+6b)} \times \frac{(a+6b)(a-2b)}{(a-2b)(a+8b)} \\ &= \frac{a+7b}{a-9b},\ a \neq 9b, 2b, -6b, -8b\end{aligned}$$

Apply, Solve, Communicate

Section 1.6 Page 51 Question 9

a)

$$A_g = \frac{10x}{9} \times \frac{x}{3}$$
$$= \frac{10x^2}{27}$$

b)

$$A_p = \frac{22x}{9} \times x$$
$$= \frac{22x^2}{9}$$

c)

$$A_p \div A_g = \frac{22x^2}{9} \div \frac{10x^2}{27}$$
$$= \frac{22x^2}{9} \times \frac{27}{10x^2}$$
$$= 11 \times \frac{3}{5}$$
$$= \frac{33}{5}$$

The area of the penalty box is $\frac{33}{5}$ times greater than the area of the goal box.

d) No. The answer in c) is independent of x.

Section 1.6 Page 51 Question 10

$$A = \frac{x^2 - 9}{x} \times \frac{x^3 - 4x}{x^2 + 5x + 6}$$
$$= \frac{(x-3)(x+3)}{x} \times \frac{x(x-2)(x+2)}{(x+2)(x+3)}$$
$$= (x-3)(x-2)$$
$$= x^2 - 5x + 6$$

The area of the rectangle is $x^2 - 5x + 6$ square units.

Section 1.6 Page 51 Question 11

The formula for the area, A, of a trapezoid with side lengths S_1 and S_2 and height h is

$$A = \frac{h}{2}(S_1 + S_2) \quad (1)$$

Formula (1) gives

$$A = 6y^2 - 5y - 6$$
$$\frac{h}{2}[(3y-2) + (y-4)] = 6y^2 - 5y - 6$$
$$\frac{h}{2}(4y-6) = 6y^2 - 5y - 6$$
$$h = \frac{2(6y^2 - 5y - 6)}{4y - 6}$$
$$= \frac{2(2y-3)(3y+2)}{2(2y-3)}$$
$$= 3y + 2$$

The height of the trapezoid is $3y + 2$ units.

Section 1.6 Page 52 Question 12

Recall the formula for the area, A, of a triangle.

$$A = \frac{bh}{2} \quad (1)$$

a) Formula (1) gives

$$|\triangle ABC| = \frac{(6x-9)(2x+4)}{2} \quad (2)$$

b) Formula (1) gives

$$|\triangle DEF| = \frac{(2x-3)(3x+6)}{2} \quad (3)$$

c) The ratio of the areas is given by

$$\begin{aligned} |\triangle ABC| \div |\triangle DEF| &= \frac{(6x-9)(2x+4)}{2} \div \frac{(2x-3)(3x+6)}{2} \\ &= \frac{(6x-9)(2x+4)}{2} \times \frac{2}{(2x-3)(3x+6)} \\ &= \frac{3(2x-3)(2)(x+2)}{2} \times \frac{2}{(2x-3)(3)(x+2)} \\ &= 2 \end{aligned}$$

The ratio of the areas is 2.

Section 1.6 Page 52 Question 13

$$\begin{aligned} A_{\text{inner}} \div A_{\text{outer}} &= \left(\frac{x-2}{3}\right)\left(\frac{x+1}{4}\right) \div \left(\frac{x-3}{2}\right)\left(\frac{x+1}{2}\right) \\ &= \frac{(x-2)(x+1)}{(3)(4)} \times \frac{4}{(x-3)(x+1)} \\ &= \frac{x-2}{3(x-3)} \end{aligned}$$

Section 1.6 Page 52 Question 14

From

$$\frac{a}{b} \div \frac{c}{d} \quad (1)$$

$b, d \neq 0$, since division by 0 is not defined. Simplifying (1) gives

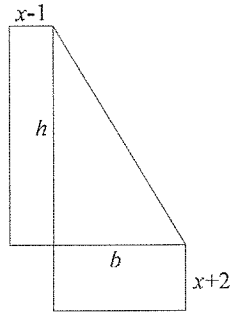
$$\frac{a}{b} \times \frac{d}{c}$$

requiring the further restriction, $c \neq 0$.

Section 1.6 Page 52 Question 15

Let A be the area of the triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times \frac{2x^2-3x+1}{x+2} \times \frac{2x^2+10x+12}{x-1} \\ &= \frac{1}{2} \times \frac{(2x-1)(x-1)}{x+2} \times \frac{2(x+2)(x+3)}{x-1} \\ &= (2x-1)(x+3), \; x \neq -2, 1 \end{aligned}$$



The area of the triangle is $(2x-1)(x+3)$ square units.

Section 1.6 Page 52 Question 16

a)
$$\begin{aligned}\frac{15y^2-2y-1}{6y^2+7y-3}\times\frac{2y-1}{10y^2-3y-1} &= \frac{(3y-1)(5y+1)}{(3y-1)(2y+3)}\times\frac{2y-1}{(5y+1)(2y-1)}\\ &= \frac{(3y-1)(5y+1)(2y-1)}{(3y-1)(2y+3)(5y+1)(2y-1)}\\ &= \frac{1}{2y+3},\ y\neq\frac{1}{3},-\frac{3}{2},-\frac{1}{5},\frac{1}{2}\end{aligned}$$

As y increases, the denominator increases and so the value of the expression decreases.

b)
$$\begin{aligned}\frac{8y^2+2y-1}{6y^2-y-2}\div\frac{8y-2}{9y-6} &= \frac{(4y-1)(2y+1)}{(3y-2)(2y+1)}\times\frac{9y-6}{8y-2}\\ &= \frac{(4y-1)(2y+1)}{(3y-2)(2y+1)}\times\frac{3(3y-2)}{2(4y-1)}\\ &= \frac{3(4y-1)(2y+1)(3y-2)}{2(3y-2)(2y+1)(4y-1)}\\ &= \frac{3}{2},\ y\neq\frac{2}{3},-\frac{1}{2},\frac{1}{4}\end{aligned}$$

The value of the expression is not dependent on y; it has a constant value of $\frac{3}{2}$.

Section 1.6 Page 52 Question 17

Answers may vary.

$$\begin{aligned}\frac{3x^2+7xy+2y^2}{x^2-y^2} &= \frac{(3x+y)(x+2y)}{(x-y)(x+y)}\\ &= \frac{3x+y}{x-y}\times\frac{x+2y}{x+y} &(1)\\ &= \frac{3x+y}{x+y}\times\frac{x+2y}{x-y} &(2)\end{aligned}$$

Section 1.6 Page 52 Question 18

Answers may vary.

$$\begin{aligned}\frac{4x^2-8x+4}{2x^2+5x-3} &= \frac{4(x^2-2x+1)}{(2x-1)(x+3)}\\ &= \frac{4(x-1)(x-1)}{(2x-1)(x+3)}\\ &= \frac{4}{2x-1}\times\frac{(x-1)^2}{x+3} &(1)\\ &= \frac{4(x-1)}{2x-1}\times\frac{x-1}{x+3} &(2)\\ &= \frac{4(x-1)^2}{2x-1}\times\frac{1}{x+3} &(3)\\ &= \frac{4(x-1)}{x+3}\times\frac{x-1}{2x-1} &(4)\end{aligned}$$

1.7 Adding and Subtracting Rational Expressions, I

Practise

Section 1.7 Page 58 Question 1

a)
$$\begin{aligned}\frac{2}{y}+\frac{4}{y}-\frac{5}{y} &= \frac{2+4-5}{y}\\ &= \frac{1}{y},\ y\neq 0\end{aligned}$$

c)
$$\begin{aligned}\frac{4}{x+3}+\frac{5}{x+3} &= \frac{4+5}{x+3}\\ &= \frac{9}{x+3},\ x\neq -3\end{aligned}$$

Section 1.7 Page 58 Question 2

a)
$$\begin{aligned}\frac{x+7}{2}+\frac{x+4}{2} &= \frac{x+7+x+4}{2}\\ &= \frac{2x+11}{2}\end{aligned}$$

c)
$$\begin{aligned}\frac{3a-1}{a}-\frac{4a+2}{a} &= \frac{3a-1-(4a+2)}{a}\\ &= \frac{3a-1-4a-2}{a}\\ &= \frac{-a-3}{a},\ a\neq 0\end{aligned}$$

e)
$$\begin{aligned}\frac{x^2+4}{x+1}+\frac{2x^2}{x+1} &= \frac{x^2+4+2x^2}{x+1}\\ &= \frac{3x^2+4}{x+1},\ x\neq -1\end{aligned}$$

g)
$$\begin{aligned}\frac{5z}{2z-1}-\frac{z-3}{2z-1} &= \frac{5z-(z-3)}{2z-1}\\ &= \frac{5z-z+3}{2z-1}\\ &= \frac{4z+3}{2z-1},\ z\neq \frac{1}{2}\end{aligned}$$

i)
$$\begin{aligned}\frac{4x+1}{x^2+5x+6}+\frac{3x+2}{x^2+5x+6} &= \frac{4x+1+3x+2}{x^2+5x+6}\\ &= \frac{7x+3}{x^2+5x+6},\ x\neq -2,-3\end{aligned}$$

Section 1.7 Page 58 Question 3

a)
$$\begin{aligned}4 &= 2\times 2\\ 5 &= 5\\ 6 &= 2\times 3\end{aligned}$$

The LCM is $2\times 2\times 5\times 3 = 60$.

c)
$$\begin{aligned}8 &= 2\times 2\times 2\\ 10 &= 2\times 5\\ 12 &= 2\times 2\times 3\end{aligned}$$

The LCM is $2\times 2\times 2\times 5\times 3 = 120$.

Section 1.7 Page 58 Question 4

a)
$$\begin{aligned}\frac{2x}{2}+\frac{x}{3} &= \frac{x}{1}+\frac{x}{3}\\ &= \frac{3x}{3(1)}+\frac{x}{3}\\ &= \frac{3x}{3}+\frac{x}{3}\\ &= \frac{4x}{3}\end{aligned}$$

c)
$$\begin{aligned}\frac{x}{5}-\frac{y}{2}+\frac{7}{10} &= \frac{2x}{2(5)}-\frac{5y}{5(2)}+\frac{7}{10}\\ &= \frac{2x}{10}-\frac{5y}{10}+\frac{7}{10}\\ &= \frac{2x-5y+7}{10}\end{aligned}$$

Section 1.7 Page 58 Question 5

a) $\frac{2m+3}{2}+\frac{3m+4}{7}=\frac{7(2m+3)}{7(2)}+\frac{2(3m+4)}{2(7)}$

$=\frac{7(2m+3)}{14}+\frac{2(3m+4)}{14}$

$=\frac{14m+21+6m+8}{14}$

$=\frac{20m+29}{14}$

c) $\frac{y-5}{6}-\frac{2y-3}{4}=\frac{2(y-5)}{2(6)}-\frac{3(2y-3)}{3(4)}$

$=\frac{2(y-5)}{12}-\frac{3(2y-3)}{12}$

$=\frac{2y-10-6y+9}{12}$

$=\frac{-4y-1}{12}$

e) $\frac{4t-1}{6}+\frac{3t+2}{2}-\frac{2t+1}{3}=\frac{4t-1}{6}+\frac{3(3t+2)}{3(2)}-\frac{2(2t+1)}{2(3)}$

$=\frac{4t-1}{6}+\frac{3(3t+2)}{6}-\frac{2(2t+1)}{6}$

$=\frac{4t-1+9t+6-4t-2}{6}$

$=\frac{9t+3}{6}$

$=\frac{3(3t+1)}{6}$

$=\frac{3t+1}{2}$

g) $\frac{5x-1}{5}+1-\frac{4x-3}{6}=\frac{6(5x-1)}{6(5)}+\frac{30(1)}{30}-\frac{5(4x-3)}{5(6)}$

$=\frac{6(5x-1)}{30}+\frac{30}{30}-\frac{5(4x-3)}{30}$

$=\frac{30x-6+30-20x+15}{30}$

$=\frac{10x+39}{30}$

Section 1.7 Page 59 Question 6

a) $\frac{3}{2-x}+\frac{2}{x-2}=\frac{3}{-(x-2)}+\frac{2}{x-2}$

$=\frac{-3}{x-2}+\frac{2}{x-2}$

$=\frac{-1}{x-2},\ x\neq 2$

c) $\frac{a-2}{2a-3}+\frac{a+3}{3-2a}=\frac{a-2}{2a-3}+\frac{a+3}{-(2a-3)}$

$=\frac{a-2}{2a-3}+\frac{-(a+3)}{2a-3}$

$=\frac{a-2-a-3}{2a-3}$

$=\frac{-5}{2a-3},\ a\neq\frac{3}{2}$

e) $\frac{5x-5}{x^2-9}-\frac{3x+1}{9-x^2}=\frac{5x-5}{x^2-9}-\frac{3x+1}{-(x^2-9)}$

$=\frac{5x-5}{x^2-9}-\frac{-(3x+1)}{x^2-9}$

$=\frac{5x-5+3x+1}{x^2-9}$

$=\frac{8x-4}{x^2-9}$

$=\frac{4(x-2)}{(x-3)(x+3)},\ x\neq\pm 3$

Apply, Solve, Communicate

Section 1.7 Page 59 Question 7

a) Let t_a be the time, in hours, to fly from Winnipeg to Calgary. $t_a = \frac{1191}{s}$.

b) Let t_b be the time, in hours, to fly from Calgary to Vancouver. $t_b = \frac{685}{s}$.

c) Let T be the total time, in hours, to fly from Winnipeg to Vancouver.

$$\begin{aligned} T &= t_a + t_b \\ &= \frac{1191}{s} + \frac{685}{s} \\ &= \frac{1876}{s} \end{aligned}$$

d) If $s = 700$ km/h, then

$$\begin{aligned} T &= \frac{1876}{700} \\ &= 2.68 \end{aligned}$$

The total flying time from Winnipeg to Vancouver is 2.68 h.

Section 1.7 Page 59 Question 8

a)

$$\begin{aligned} w &= \frac{x^2 + 8x}{x} \\ &= \frac{x(x+8)}{x} \\ &= x + 8 \end{aligned}$$

b)

$$\begin{aligned} \ell &= 2x + \frac{x}{5} \\ &= \frac{10x}{5} + \frac{x}{5} \\ &= \frac{11x}{5} \end{aligned}$$

c) Each table has dimensions $x + 8$ by x. If $x = 15$ cm, the dimensions are 23 cm by 15 cm. The whole board has dimensions $x + 8$ by $\frac{11x}{5}$. If $x = 15$ cm, the dimensions are 23 cm by 33 cm.

Section 1.7 Page 60 Question 9

Let A be the the total area of the triangles.

$$\begin{aligned} A &= A_1 + A_2 \\ &= \frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2 \\ &= \frac{x(x+1) + x(x+3)}{2} \\ &= \frac{x^2 + x + x^2 + 3x}{2} \\ &= \frac{2x^2 + 4x}{2} \\ &= x^2 + 2x \\ &= x(x+2) \end{aligned}$$

The total area of the triangles is $x(x + 2)$ square units.

Section 1.7 Page 60 Question 10

a)

$$w_A = \frac{6x^2 + 5x + 1}{2x + 1}$$

b)

$$w_B = \frac{4x^2 - 4x - 3}{2x + 1}$$

c) $$\begin{aligned} w_B - w_A &= \frac{4x^2 - 4x - 3}{2x + 1} - \frac{6x^2 + 5x + 1}{2x + 1} \\ &= \frac{4x^2 - 4x - 3 - (6x^2 + 5x + 1)}{2x + 1} \\ &= \frac{4x^2 - 4x - 3 - 6x^2 - 5x - 1}{2x + 1} \\ &= \frac{-2x^2 - 9x - 4}{2x + 1} \end{aligned}$$

d) $$\begin{aligned} w_A - w_B &= \frac{6x^2 + 5x + 1}{2x + 1} - \frac{4x^2 - 4x - 3}{2x + 1} \\ &= \frac{6x^2 + 5x + 1 - (4x^2 - 4x - 3)}{2x + 1} \\ &= \frac{6x^2 + 5x + 1 - 4x^2 + 4x + 3}{2x + 1} \\ &= \frac{2x^2 + 9x + 4}{2x + 1} \end{aligned}$$

f) The results are additive inverses of each other, since $w_B - w_A = -(w_A - w_B)$.

Section 1.7 Page 60 Question 11

a) The formula for the area, A, of a circle with radius r, $A = \pi r^2$, where $r = \frac{d}{2}$, gives

$$\begin{aligned} A_{\text{inner}} &= \pi \left(\frac{d}{2}\right)^2 \\ &= \frac{\pi d^2}{4} \end{aligned}$$

b) $$\begin{aligned} A_{\text{outer}} &= \pi \left(\frac{d + 1}{2}\right)^2 \\ &= \frac{\pi(d + 1)^2}{4} \end{aligned}$$

c) $$\begin{aligned} A &= A_{\text{outer}} - A_{\text{inner}} \\ &= \frac{\pi(d + 1)^2}{4} - \frac{\pi d^2}{4} \\ &= \frac{\pi(d + 1)^2 - \pi d^2}{4} \\ &= \frac{\pi[(d + 1)^2 - d^2]}{4} \\ &= \frac{\pi[(d + 1 - d)(d + 1 + d)]}{4} \\ &= \frac{\pi(2d + 1)}{4} \end{aligned} \quad (1)$$

d) If $d = 10$ cm, (1) becomes

$$\begin{aligned} A &= \frac{\pi(2(10) + 1)}{4} \\ &= \frac{21\pi}{4} \\ &\approx 16.5 \end{aligned}$$

The area is approximately 16.5 cm^2.

Section 1.7 Page 60 Question 12

a) $$\begin{aligned} |\triangle ADE| &= \frac{1}{2}\left(\frac{2x + 1}{4}\right)\left(\frac{x - 3}{2}\right) \\ &= \frac{(2x + 1)(x - 3)}{16} \end{aligned}$$

b) $$\begin{aligned} |ABCE| &= \left(\frac{x - 1}{2}\right)\left(\frac{x - 3}{2}\right) \\ &= \frac{(x - 1)(x - 3)}{4} \end{aligned}$$

c) $$\begin{aligned} |ABCD| &= |\triangle ADE| + |ABCE| \\ &= \frac{(2x + 1)(x - 3)}{16} + \frac{(x - 1)(x - 3)}{4} \\ &= \frac{(2x + 1)(x - 3)}{16} + \frac{4(x - 1)(x - 3)}{4(4)} \\ &= \frac{(2x + 1)(x - 3) + 4(x - 1)(x - 3)}{16} \\ &= \frac{(x - 3)[2x + 1 + 4(x - 1)]}{16} \\ &= \frac{(x - 3)(2x + 1 + 4x - 4)}{16} \\ &= \frac{(x - 3)(6x - 3)}{16} \\ &= \frac{3(x - 3)(2x - 1)}{16} \end{aligned}$$

d) Let b represent the length of the longer base.

$$\begin{aligned} b &= \frac{x - 1}{2} + \frac{2x + 1}{4} \\ &= \frac{2(x - 1) + 2x + 1}{4} \\ &= \frac{2x - 2 + 2x + 1}{4} \\ &= \frac{4x - 1}{4} \end{aligned}$$

e) Let a represent the length of the shorter base.

$$\begin{aligned}|ABCD| &= \frac{1}{2}h(a+b)\\ &= \frac{1}{2}\left(\frac{x-3}{2}\right)\left[\frac{x-1}{2}+\frac{4x-1}{4}\right]\\ &= \frac{x-3}{4}\left[\frac{x-1}{2}+\frac{4x-1}{4}\right]\\ &= \frac{x-3}{4}\left[\frac{2(x-1)+4x-1}{4}\right]\\ &= \frac{x-3}{4}\left(\frac{6x-3}{4}\right)\\ &= \frac{3(x-3)(2x-1)}{16}\end{aligned}$$

f) They are equal.

Section 1.7 Page 61 Question 13

a) Let t_n represent the nth triangular number. $t_n = \dfrac{n(n+1)}{2}$.

b)

$$\begin{aligned}t_5 &= \frac{5(5+1)}{2}\\ &= \frac{5(6)}{2}\\ &= 15\end{aligned}$$

$$\begin{aligned}t_6 &= \frac{6(6+1)}{2}\\ &= \frac{6(7)}{2}\\ &= 21\end{aligned}$$

$$\begin{aligned}t_7 &= \frac{7(7+1)}{2}\\ &= \frac{7(8)}{2}\\ &= 28\end{aligned}$$

$$\begin{aligned}t_8 &= \frac{8(8+1)}{2}\\ &= \frac{8(9)}{2}\\ &= 36\end{aligned}$$

$$\begin{aligned}t_9 &= \frac{9(9+1)}{2}\\ &= \frac{9(10)}{2}\\ &= 45\end{aligned}$$

c)

$$\begin{aligned}1+3&=4\\ 3+6&=9\\ 6+10&=16\\ 10+15&=25\end{aligned}$$

A perfect square results.

d)

$$\begin{aligned}t_{n+1} &= \frac{(n+1)(n+1+1)}{2}\\ &= \frac{(n+1)(n+2)}{2}\end{aligned}$$

e)

$$\begin{aligned}t_n + t_{n+1} &= \frac{n(n+1)}{2}+\frac{(n+1)(n+2)}{2}\\ &= \frac{n(n+1)+(n+1)(n+2)}{2}\\ &= \frac{(n+1)(n+n+2)}{2}\\ &= \frac{(n+1)(2n+2)}{2}\\ &= \frac{2(n+1)(n+1)}{2}\\ &= (n+1)^2\end{aligned}$$

f) $(n+1)^2$ is a perfect square.

1.8 Adding and Subtracting Rational Expressions, II

Practise

Section 1.8 Page 67 Question 1

a)
$$\frac{2}{xy} = \frac{12xy}{12xy} \times \frac{2}{xy}$$
$$= \frac{24xy}{12x^2y^2}, \; x, y \neq 0$$

c)
$$\frac{5}{3xy^2} = \frac{4x}{4x} \times \frac{5}{3xy^2}$$
$$= \frac{20x}{12x^2y^2}, \; x, y \neq 0$$

Section 1.8 Page 67 Question 2

a)
$$10a^2b = 2 \times 5 \times a \times a \times b$$
$$4ab^3 = 2 \times 2 \times a \times b \times b \times b$$

The LCM is $2 \times 2 \times 5 \times a \times a \times b \times b \times b$ or $20a^2b^3$.

c)
$$2x^3 = 2 \times x \times x \times x \times x$$
$$6xy^2 = 2 \times 3 \times x \times x \times y \times y$$
$$4y = 2 \times 2 \times y$$

The LCM is $2 \times 2 \times 3 \times x \times x \times x \times x \times y \times y$ or $12x^3y^2$.

Section 1.8 Page 67 Question 3

a)
$$\frac{3}{2x} + \frac{4}{5x} = \frac{5(3)}{5(2x)} + \frac{2(4)}{2(5x)}$$
$$= \frac{15}{10x} + \frac{8}{10x}$$
$$= \frac{23}{10x}, \; x \neq 0$$

c)
$$\frac{1}{2x^2} + \frac{3}{3x} - \frac{2}{x^3} = \frac{1}{2x^2} + \frac{1}{x} - \frac{2}{x^3}$$
$$= \frac{x}{2x^3} + \frac{2x^2}{2x^3} - \frac{4}{2x^3}$$
$$= \frac{2x^2 + x - 4}{2x^3}, \; x \neq 0$$

e)
$$x - \frac{2}{x} + 5 = \frac{x}{x} \times \frac{x}{1} - \frac{2}{x} + \frac{x}{x} \times \frac{5}{1}$$
$$= \frac{x^2}{x} - \frac{2}{x} + \frac{5x}{x}$$
$$= \frac{x^2 - 2 + 5x}{x}$$
$$= \frac{x^2 + 5x - 2}{x}, \; x \neq 0$$

g)
$$\frac{4x-1}{3x^2} - \frac{2x+3}{x} + \frac{5x+2}{5x^2} = \frac{5(4x-1)}{5(3x^2)} - \frac{15x(2x+3)}{15x(x)} + \frac{3(5x+2)}{3(5x^2)}$$
$$= \frac{5(4x-1)}{15x^2} - \frac{15x(2x+3)}{15x^2} + \frac{3(5x+2)}{15x^2}$$
$$= \frac{20x - 5 - 30x^2 - 45x + 15x + 6}{15x^2}$$
$$= \frac{-30x^2 - 10x + 1}{15x^2}, \; x \neq 0$$

Section 1.8 Page 67 Question 4

a)
$$3m + 6 = 3(m + 2)$$
$$2m + 4 = 2(m + 2)$$

The LCM is $2(3)(m + 2)$ or $6(m + 2)$.

c)
$$4m - 8 = 2 \times 2 \times (m - 2)$$
$$6m - 18 = 2 \times 3 \times (m - 3)$$

The LCM is $2(2)(3)(m - 2)(m - 3)$ or $12(m - 2)(m - 3)$.

i)
$$\begin{aligned}\frac{4}{2s-12}-\frac{s}{5s-5} &= \frac{5(s-1)}{5(s-1)}\times\frac{4}{2(s-6)}+\frac{2(s-6)}{2(s-6)}\times\frac{s}{5(s-1)}\\ &= \frac{20(s-1)}{10(s-1)(s-6)}-\frac{2s(s-6)}{10(s-1)(s-6)}\\ &= \frac{20(s-1)-2s(s-6)}{10(s-1)(s-6)}\\ &= \frac{20s-20-2s^2+12s}{10(s-1)(s-6)}\\ &= \frac{-2s^2+32s-20}{10(s-1)(s-6)}\\ &= \frac{2(-s^2+16s-10)}{2(5)(s-1)(s-6)}\\ &= \frac{-s^2+16s-10}{5(s-1)(s-6)},\ s\neq 1,6\end{aligned}$$

Section 1.8 Page 67 Question 7

a)
$$\begin{aligned}x+2&=x+2\\ x^2+4x+4&=(x+2)^2\end{aligned}$$

The LCM is $(x+2)^2$.

c)
$$\begin{aligned}t^2-t-12&=(t-4)(t+3)\\ t^2-3t-4&=(t-4)(t+1)\end{aligned}$$

The LCM is $(t+3)(t-4)(t+1)$.

e)
$$\begin{aligned}m^2+6m+9&=(m+3)^2\\ m^2-2m-15&=(m+3)(m-5)\end{aligned}$$

The LCM is $(m+3)^2(m-5)$.

Section 1.8 Page 67 Question 8

a)
$$\begin{aligned}\frac{2}{x+3}+\frac{3}{x^2+5x+6} &= \frac{2(x+2)}{(x+3)(x+2)}+\frac{3}{(x+3)(x+2)}\\ &= \frac{2x+4+3}{(x+3)(x+2)}\\ &= \frac{2x+7}{(x+3)(x+2)},\ x\neq -2,-3\end{aligned}$$

c)
$$\begin{aligned}\frac{3x}{x-5}+\frac{2x}{x^2-4x-5} &= \frac{3x(x+1)}{(x-5)(x+1)}+\frac{2x}{(x-5)(x+1)}\\ &= \frac{3x^2+3x+2x}{(x-5)(x+1)}\\ &= \frac{3x^2+5x}{(x-5)(x+1)},\ x\neq -1,5\end{aligned}$$

e)
$$\begin{aligned}\frac{4}{2x^2+3x+1}+\frac{2}{2x+1} &= \frac{4}{(2x+1)(x+1)}+\frac{2(x+1)}{(2x+1)(x+1)}\\ &= \frac{4+2x+2}{(2x+1)(x+1)}\\ &= \frac{2x+6}{(2x+1)(x+1)},\ x\neq -\frac{1}{2},-1\end{aligned}$$

Section 1.8 Page 68 Question 9

a)
$$\begin{aligned}\frac{2}{m^2+4m+3}+\frac{1}{m^2+7m+12} &= \frac{2}{(m+3)(m+1)}+\frac{1}{(m+3)(m+4)}\\ &= \frac{2(m+4)+1(m+1)}{(m+3)(m+1)(m+4)}\\ &= \frac{2m+8+m+1}{(m+3)(m+1)(m+4)}\\ &= \frac{3m+9}{(m+3)(m+1)(m+4)}\\ &= \frac{3(m+3)}{(m+3)(m+1)(m+4)}\\ &= \frac{3}{(m+1)(m+4)},\ m\neq -1,-3,-4\end{aligned}$$

c)
$$\begin{aligned}\frac{a}{a^2-25}-\frac{2}{a^2-9a+20} &= \frac{a}{(a-5)(a+5)}-\frac{2}{(a-5)(a-4)}\\ &= \frac{a(a-4)-2(a+5)}{(a-5)(a+5)(a-4)}\\ &= \frac{a^2-4a-2a-10}{(a-5)(a+5)(a-4)}\\ &= \frac{a^2-6a-10}{(a-5)(a+5)(a-4)},\ a\neq \pm 5,4\end{aligned}$$

e)
$$\begin{aligned}\frac{5}{3x^2+4x+1}+\frac{2}{3x^2-2x-1} &= \frac{5}{(3x+1)(x+1)}+\frac{2}{(3x+1)(x-1)}\\ &= \frac{5(x-1)+2(x+1)}{(3x+1)(x+1)(x-1)}\\ &= \frac{5x-5+2x+2}{(3x+1)(x+1)(x-1)}\\ &= \frac{7x-3}{(3x+1)(x+1)(x-1)},\ x\neq \pm 1,-\frac{1}{3}\end{aligned}$$

Section 1.8 Page 68 Question 10

a)
$$\begin{aligned}\frac{t+1}{t-1}+\frac{2}{t^2-5t+4} &= \frac{(t+1)(t-4)}{(t-1)(t-4)}+\frac{2}{(t-1)(t-4)}\\ &= \frac{t^2-3t-4+2}{(t-1)(t-4)}\\ &= \frac{t^2-3t-2}{(t-1)(t-4)},\ t\neq 1,4\end{aligned}$$

c)
$$\begin{aligned}\frac{x-2}{x^2+4x+3}-\frac{2x+1}{x+3} &= \frac{x-2}{(x+3)(x+1)}-\frac{(2x+1)(x+1)}{(x+3)(x+1)}\\ &= \frac{x-2-(2x^2+3x+1)}{(x+3)(x+1)}\\ &= \frac{x-2-2x^2-3x-1}{(x+3)(x+1)}\\ &= \frac{-2x^2-2x-3}{(x+3)(x+1)}\\ &= \frac{-(2x^2+2x+3)}{(x+3)(x+1)},\ x\neq -1,-3\end{aligned}$$

e)
$$\begin{aligned}\frac{m+4}{m^2-m-12}-\frac{m}{m^2-5m+4} &= \frac{m+4}{(m-4)(m+3)}-\frac{m}{(m-4)(m-1)}\\ &= \frac{(m+4)(m-1)}{(m-4)(m+3)(m-1)}-\frac{m(m+3)}{(m-4)(m-1)(m+3)}\\ &= \frac{m^2+3m-4-m^2-3m}{(m-4)(m+3)(m-1)}\\ &= \frac{-4}{(m-4)(m+3)(m-1)},\ m\neq -3,1,4\end{aligned}$$

g)
$$\begin{aligned}\frac{3w-4}{w^2+5w+4}+\frac{2w-3}{w^2+2w-8} &= \frac{3w-4}{(w+1)(w+4)}+\frac{2w-3}{(w-2)(w+4)}\\ &= \frac{(3w-4)(w-2)+(2w-3)(w+1)}{(w+1)(w+4)(w-2)}\\ &= \frac{3w^2-10w+8+2w^2-w-3}{(w+1)(w+4)(w-2)}\\ &= \frac{5w^2-11w+5}{(w+1)(w+4)(w-2)},\ w\neq -4,-1,2\end{aligned}$$

i)
$$\begin{aligned}\frac{2z-1}{4z^2-25}-\frac{2z+5}{4z^2-8z-5} &= \frac{2z-1}{(2z-5)(2z+5)}-\frac{2z+5}{(2z-5)(2z+1)}\\ &= \frac{(2z-1)(2z+1)}{(2z-5)(2z+5)(2z+1)}-\frac{(2z+5)(2z+5)}{(2z-5)(2z+1)(2z+5)}\\ &= \frac{4z^2-1-4z^2-20z-25}{(2z-5)(2z+5)(2z+1)}\\ &= \frac{-20z-26}{(2z-5)(2z+5)(2z+1)},\ z\neq \pm\frac{5}{2},-\frac{1}{2}\end{aligned}$$

Apply, Solve, Communicate

Section 1.8 Page 68 Question 11

a)

Expressions	Product	LCM	GCF	LCM × GCF
$3x, 5x$	$15x^2$	$15x$	x	$15x^2$
$12, 8$	96	24	4	96
$15y^2, 9y$	$135y^3$	$45y^2$	$3y$	$135y^3$
$a+1, a-1$	a^2-1	a^2-1 for even a, $\frac{a^2-1}{2}$ for odd a	1 for even a, 2 for odd a	a^2-1
$2t-2, 3t-3$	$6(t-1)^2$	$6(t-1)$	$t-1$	$6(t-1)^2$

b) The expressions are equal.

c) Consider the prime factorization of 8 and 12.
$$8\times 12 = \underbrace{2\times 2}_{GCF}\times\underbrace{2\times 2\times 2\times 3}_{LCM}$$

Section 1.8 Page 69 Question 12

a) Let t_o be the time out of Goderich, in hours.
$$t_o = \frac{45}{s}$$

b) Let t_r be the time taken to return to Goderich, in hours.
$$t_r = \frac{45}{2s}$$

Section 1.8 Page 67 Question 5

a) $$\frac{4}{x+3}+\frac{5}{4x+12}=\frac{4(4)}{4(x+3)}+\frac{5}{4(x+3)}$$
$$=\frac{16+5}{4(x+3)}$$
$$=\frac{21}{4(x+3)},\ x\neq -3$$

c) $$\frac{t}{t-4}-\frac{2t}{3t-12}=\frac{3(t)}{3(t-4)}-\frac{2t}{3(t-4)}$$
$$=\frac{3t-2t}{3(t-4)}$$
$$=\frac{t}{3(t-4)},\ t\neq 4$$

e) $$\frac{3}{4y-8}-\frac{2}{3y-6}=\frac{3(3)}{3(4)(y-2)}-\frac{4(2)}{4(3)(y-2)}$$
$$=\frac{9}{12(y-2)}-\frac{8}{12(y-2)}$$
$$=\frac{1}{12(y-2)},\ y\neq 2$$

Section 1.8 Page 67 Question 6

a) $$\frac{2}{x+1}+\frac{3}{x+2}=\frac{x+2}{x+2}\times\frac{2}{x+1}+\frac{x+1}{x+1}\times\frac{3}{x+2}$$
$$=\frac{2(x+2)}{(x+1)(x+2)}+\frac{3(x+1)}{(x+1)(x+2)}$$
$$=\frac{2(x+2)+3(x+1)}{(x+1)(x+2)}$$
$$=\frac{2x+4+3x+3}{(x+1)(x+2)}$$
$$=\frac{5x+7}{(x+1)(x+2)},\ x\neq -1,-2$$

c) $$\frac{3}{x}+\frac{5}{x-1}=\frac{x-1}{x-1}\times\frac{3}{x}+\frac{x}{x}\times\frac{5}{x-1}$$
$$=\frac{3(x-1)}{x(x-1)}+\frac{5x}{x(x-1)}$$
$$=\frac{3x-3+5x}{x(x-1)}$$
$$=\frac{8x-3}{x(x-1)},\ x\neq 0,1$$

e) $$\frac{2x}{x-2}-\frac{3x}{x+2}=\frac{x+2}{x+2}\times\frac{2x}{x-2}-\frac{x-2}{x-2}\times\frac{3x}{x+2}$$
$$=\frac{2x(x+2)}{(x+2)(x-2)}-\frac{3x(x-2)}{(x+2)(x-2)}$$
$$=\frac{2x(x+2)-3x(x-2)}{(x+2)(x-2)}$$
$$=\frac{2x^2+4x-3x^2+6x}{(x+2)(x-2)}$$
$$=\frac{10x-x^2}{(x+2)(x-2)},\ x\neq \pm 2$$

g) $$\frac{1}{2x-2}+\frac{3}{4x-8}=\frac{2(x-2)}{2(x-2)}\times\frac{1}{2(x-1)}+\frac{x-1}{x-1}\times\frac{3}{4(x-2)}$$
$$=\frac{2(x-2)}{4(x-1)(x-2)}+\frac{3(x-1)}{4(x-1)(x-2)}$$
$$=\frac{2(x-2)+3(x-1)}{4(x-1)(x-2)}$$
$$=\frac{2x-4+3x-3}{4(x-1)(x-2)}$$
$$=\frac{5x-7}{4(x-1)(x-2)},\ x\neq 1,2$$

c) Let T be the total time the boat was on the lake, in hours.

$$\begin{aligned} T &= t_o + t_r \\ &= \frac{45}{s} + \frac{45}{2s} \\ &= \frac{2}{2} \times \frac{45}{s} + \frac{45}{2s} \\ &= \frac{90}{2s} + \frac{45}{2s} \\ &= \frac{135}{2s} \end{aligned} \qquad (1)$$

d) If $s = 10$ km/h, (1) becomes

$$\begin{aligned} T &= \frac{135}{2(10)} \\ &= \frac{135}{20} \\ &= 6.75 \end{aligned}$$

The total travel time was 6.75 h.

Section 1.8 Page 69 Question 13

Answers may vary.

Section 1.8 Page 69 Question 14

a) $\dfrac{m+3}{m+2} \times \dfrac{m+2}{m+1} + \dfrac{5m}{m+1} = \dfrac{m+3}{m+1} + \dfrac{5m}{m+1}$

$$= \frac{m+3+5m}{m+1}$$

$$= \frac{6m+3}{m+1}, \; m \neq -1, -2$$

b) $\dfrac{2x^2-2x}{x^2+4x-5} - \dfrac{4x}{x+5} = \dfrac{2x(x-1)}{(x-1)(x+5)} - \dfrac{4x}{x+5}$

$$= \frac{2x}{x+5} - \frac{4x}{x+5}$$

$$= \frac{-2x}{x+5}, \; x \neq 1, -5$$

c)

$$\begin{aligned} \frac{x^2+2x-15}{x^2-7x+12} + \frac{x^2-6x-7}{x^2-3x-4} &= \frac{(x-3)(x+5)}{(x-3)(x-4)} + \frac{(x+1)(x-7)}{(x+1)(x-4)} \\ &= \frac{x+5}{x-4} + \frac{x-7}{x-4} \\ &= \frac{2x-2}{x-4}, \; x \neq -1, 3, 4 \end{aligned}$$

d)

$$\begin{aligned} \frac{3y-1}{y-4} - \frac{y^2+4y-12}{y^2-6y+8} &= \frac{3y-1}{y-4} - \frac{(y-2)(y+6)}{(y-2)(y-4)} \\ &= \frac{3y-1}{y-4} - \frac{y+6}{y-4} \\ &= \frac{2y-7}{y-4}, \; y \neq 2, 4 \end{aligned}$$

e)

$$\begin{aligned} \frac{2z^2-5z+3}{z^2-1} + \frac{4z^2-9}{4z+6} &= \frac{(2z-3)(z-1)}{(z+1)(z-1)} + \frac{(2z-3)(2z+3)}{2(2z+3)} \\ &= \frac{2z-3}{z+1} + \frac{2z-3}{2} \\ &= \frac{2(2z-3)+(2z-3)(z+1)}{2(z+1)} \\ &= \frac{4z-6+2z^2-z-3}{2(z+1)} \\ &= \frac{2z^2+3z-9}{2(z+1)}, \; z \neq \pm 1, -\frac{3}{2} \end{aligned}$$

f)
$$\begin{aligned}\frac{x^2+5x+6}{x^2-3x+2} \div \frac{x+3}{x-1} - \frac{6}{x+3} &= \frac{(x+2)(x+3)}{(x-1)(x-2)} \times \frac{x-1}{x+3} - \frac{6}{x+3} \\ &= \frac{x+2}{x-2} - \frac{6}{x+3} \\ &= \frac{(x+2)(x+3)-6(x-2)}{(x+3)(x-2)} \\ &= \frac{x^2+5x+6-6x+12}{(x+3)(x-2)} \\ &= \frac{x^2-x+18}{(x+3)(x-2)}, \ x \neq -3, 1, 2\end{aligned}$$

Section 1.8 Page 69 Question 15

Answers may vary.

a)
$$\begin{aligned}\frac{5x+8}{(x+1)(x+2)} &= \frac{3x+6}{(x+1)(x+2)} + \frac{2x+2}{(x+1)(x+2)} \\ &= \frac{3(x+2)}{(x+1)(x+2)} + \frac{2(x+1)}{(x+1)(x+2)} \\ &= \frac{3}{x+1} + \frac{2}{x+2}\end{aligned}$$

b)
$$\begin{aligned}\frac{5x-5}{6x^2-13x+6} &= \frac{5x-5}{(2x-3)(3x-2)} \\ &= \frac{2x-3}{(2x-3)(3x-2)} + \frac{3x-2}{(2x-3)(3x-2)} \\ &= \frac{1}{3x-2} + \frac{1}{2x-3}\end{aligned}$$

c)
$$\begin{aligned}\frac{x^2-3}{(x-1)(x-3)} &= \frac{x^2-x}{(x-1)(x-3)} + \frac{x-3}{(x-1)(x-3)} \\ &= \frac{x}{x-3} + \frac{1}{x-1}\end{aligned}$$

d)
$$\begin{aligned}\frac{4x^2}{4x^2-9} &= \frac{4x^2}{(2x-3)(2x+3)} \\ &= \frac{4x^2-9}{(2x-3)(2x+3)} + \frac{9}{(2x-3)(2x+3)} \\ &= \frac{(2x-3)(2x+3)}{(2x-3)(2x+3)} + \frac{9}{(2x-3)(2x+3)} \\ &= 1 + \frac{9}{(2x-3)(2x+3)} \\ &= 1 + \frac{3x+4.5-3x+4.5}{(2x-3)(2x+3)} \\ &= 1 + \frac{3x+4.5}{(2x-3)(2x+3)} - \frac{3x-4.5}{(2x-3)(2x+3)} \\ &= 1 + \frac{1.5(2x+3)}{(2x-3)(2x+3)} - \frac{1.5(2x-3)}{(2x-3)(2x+3)} \\ &= 1 + \frac{1.5}{2x-3} - \frac{1.5}{2x+3} \\ &= \frac{2x-3}{2x-3} + \frac{1.5}{2x-3} - \frac{1.5}{2x+3} \\ &= \frac{2x-1.5}{2x-3} - \frac{1.5}{2x+3}\end{aligned}$$

1.9 Solving First-Degree Inequalities

Practise

Section 1.9 Page 78 Question 1

a)

$$\begin{aligned} y + 9 &< 11 \\ y + 9 - 9 &< 11 - 9 \\ y &< 2 \end{aligned}$$

Check $y = 1$.

$$\begin{aligned} \textbf{L.S.} &= 1 + 9 \\ &= 10 \\ &< \textbf{R.S.} \end{aligned}$$

c)

$$\begin{aligned} 3x - 4 &\geq 5 \\ 3x - 4 + 4 &\geq 5 + 4 \\ 3x &\geq 9 \\ \frac{3x}{3} &\geq \frac{9}{3} \\ x &\geq 3 \end{aligned}$$

Check $x = 4$.

$$\begin{aligned} \textbf{L.S.} &= 3(4) - 4 \\ &= 8 \\ &\geq \textbf{R.S.} \end{aligned}$$

e)

$$\begin{aligned} -3x &< 6 \\ \frac{-3x}{-3} &> \frac{6}{-3} \\ x &> -2 \end{aligned}$$

Check $x = 0$.

$$\begin{aligned} \textbf{L.S.} &= -3(0) \\ &= 0 \\ &< \textbf{R.S.} \end{aligned}$$

g)

$$\begin{aligned} 2(m - 3) &\leq 0 \\ \frac{2(m-3)}{2} &\leq \frac{0}{2} \\ m - 3 &\leq 0 \\ m - 3 + 3 &\leq 0 + 3 \\ m &\leq 3 \end{aligned}$$

Check $m = 1$.

$$\begin{aligned} \textbf{L.S.} &= 2(1 - 3) \\ &= -4 \\ &\leq \textbf{R.S.} \end{aligned}$$

Section 1.9 Page 78 Question 2

a)

$$\begin{aligned} 2x + 1 &> 2 \\ 2x + 1 - 1 &> 2 - 1 \\ 2x &> 1 \\ \frac{2x}{2} &> \frac{1}{2} \\ x &> \frac{1}{2} \end{aligned}$$

Check $x = 1$.

$$\begin{aligned} \textbf{L.S.} &= 2(1) + 1 \\ &= 3 \\ &> \textbf{R.S.} \end{aligned}$$

c)

$$\begin{aligned} 6y + 4 &\leq 5y + 3 \\ 6y + 4 - 4 &\leq 5y + 3 - 4 \\ 6y &\leq 5y - 1 \\ 6y - 5y &\leq 5y - 1 - 5y \\ y &\leq -1 \end{aligned}$$

Check $y = -2$.

$$\begin{aligned} \textbf{L.S.} &= 6(-2) + 4 \\ &= -8 \\ \textbf{R.S.} &= 5(-2) + 3 \\ &= -7 \\ \textbf{L.S.} &\leq \textbf{R.S.} \end{aligned}$$

e)
$$\begin{aligned} 7 + 3x &< 2x + 9 \\ 7 + 3x - 7 &< 2x + 9 - 7 \\ 3x &< 2x + 2 \\ 3x - 2x &< 2x + 2 - 2x \\ x &< 2 \end{aligned}$$

Check $x = 0$.

$$\begin{aligned} \textbf{L.S.} &= 7 + 3(0) \\ &= 7 \\ \textbf{R.S.} &= 2(0) + 9 \\ &= 9 \\ \textbf{L.S.} &< \textbf{R.S.} \end{aligned}$$

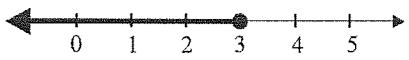
g)
$$\begin{aligned} 2(3x - 2) &\leq -4 \\ \frac{2(3x - 2)}{2} &\leq \frac{-4}{2} \\ 3x - 2 &\leq -2 \\ 3x - 2 + 2 &\leq -2 + 2 \\ 3x &\leq 0 \\ \frac{3x}{3} &\leq \frac{0}{3} \\ x &\leq 0 \end{aligned}$$

Check $x = -1$.

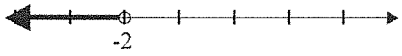
$$\begin{aligned} \textbf{L.S.} &= 2(3(-1) - 2) \\ &= -10 \\ &\leq \textbf{R.S.} \end{aligned}$$

Section 1.9 Page 78 Question 3

a)
$$\begin{aligned} 6x + 2 &\leq 4x + 8 \\ 6x + 2 - 2 &\leq 4x + 8 - 2 \\ 6x &\leq 4x + 6 \\ 6x - 4x &\leq 4x + 6 - 4x \\ 2x &\leq 6 \\ \frac{2x}{2} &\leq \frac{6}{2} \\ x &\leq 3 \end{aligned}$$

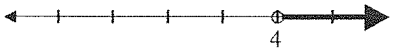


c)
$$\begin{aligned} 2(x + 3) &< x + 4 \\ 2x + 6 &< x + 4 \\ 2x + 6 - 6 &< x + 4 - 6 \\ 2x &< x - 2 \\ 2x - x &< x - 2 - x \\ x &< -2 \end{aligned}$$



e)
$$\begin{aligned} 3(y + 2) &\geq 2(y + 1) \\ 3y + 6 &\geq 2y + 2 \\ 3y + 6 - 6 &\geq 2y + 2 - 6 \\ 3y &\geq 2y - 4 \\ 3y - 2y &\geq 2y - 4 - 2y \\ y &\geq -4 \end{aligned}$$

g)
$$\begin{aligned} 6x - 3(x + 1) &> x + 5 \\ 6x - 3x - 3 &> x + 5 \\ 3x - 3 &> x + 5 \\ 3x - 3 + 3 &> x + 5 + 3 \\ 3x &> x + 8 \\ 3x - x &> x + 8 - x \\ 2x &> 8 \\ \frac{2x}{2} &> \frac{8}{2} \\ x &> 4 \end{aligned}$$



Section 1.9 Page 79 Question 4

a)
$$\begin{aligned} 6 - 2x &> 4 \\ 6 - 2x - 6 &> 4 - 6 \\ -2x &> -2 \\ \frac{-2x}{-2} &< \frac{-2}{-2} \\ x &< 1 \end{aligned}$$

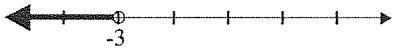
c)
$$\begin{aligned} 3y - 8 &\geq 7y + 8 \\ 3y - 8 + 8 &\geq 7y + 8 + 8 \\ 3y &\geq 7y + 16 \\ 3y - 7y &\geq 7y + 16 - 7y \\ -4y &\geq 16 \\ \frac{-4y}{-4} &\leq \frac{16}{-4} \\ y &\leq -4 \end{aligned}$$

e)
$$\begin{aligned} 4(1-x) &\geq 3(x-1) \\ 4-4x &\geq 3x-3 \\ 4-4x-4 &\geq 3x-3-4 \\ -4x &\geq 3x-7 \\ -4x-3x &\geq 3x-7-3x \\ -7x &\geq -7 \\ \frac{-7x}{-7} &\leq \frac{-7}{-7} \\ x &\leq 1 \end{aligned}$$

g)
$$\begin{aligned} 4x-3(2x+1) &\leq 4(x-3) \\ 4x-6x-3 &\leq 4x-12 \\ -2x-3 &\leq 4x-12 \\ -2x-3+3 &\leq 4x-12+3 \\ -2x &\leq 4x-9 \\ -2x-4x &\leq 4x-9-4x \\ -6x &\leq -9 \\ \frac{-6x}{-6} &\geq \frac{-9}{-6} \\ x &\geq \frac{3}{2} \end{aligned}$$

Section 1.9 Page 79 Question 5

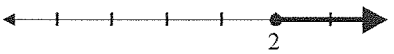
a)
$$\begin{aligned} \frac{y}{3}+2 &< 1 \\ \frac{y}{3}+2-2 &< 1-2 \\ \frac{y}{3} &< -1 \\ 3\times\frac{y}{3} &< -1\times 3 \\ y &< -3 \end{aligned}$$



c)
$$\begin{aligned} \frac{2x}{3}+1 &\geq 2 \\ \frac{2x}{3}+1-1 &\geq 2-1 \\ \frac{2x}{3} &\geq 1 \\ 3\times\frac{2x}{3} &\geq 1\times 3 \\ 2x &\geq 3 \\ \frac{2x}{2} &\geq \frac{3}{2} \\ x &\geq \frac{3}{2} \end{aligned}$$

e)
$$\begin{aligned} 1.2x-0.1 &> 3.5 \\ 1.2x-0.1+0.1 &> 3.5+0.1 \\ 1.2x &> 3.6 \\ \frac{1.2x}{1.2} &> \frac{3.6}{1.2} \\ x &> 3 \end{aligned}$$

g)
$$\begin{aligned} 1.9 &\geq 4.9-1.5q \\ 1.9-1.9 &\geq 4.9-1.5q-1.9 \\ 0 &\geq 3-1.5q \\ 0+1.5q &\geq 3-1.5q+1.5q \\ 1.5q &\geq 3 \\ \frac{1.5q}{1.5} &\geq \frac{3}{1.5} \\ q &\geq 2 \end{aligned}$$



Section 1.9 Page 79 Question 6

a)
$$\begin{aligned} 2(1.2a+2.5) &> 0.2 \\ 2.4a+5 &> 0.2 \\ 2.4a+5-5 &> 0.2-5 \\ 2.4a &> -4.8 \\ \frac{2.4a}{2.4} &> \frac{-4.8}{2.4} \\ a &> -2 \end{aligned}$$

c)
$$\begin{aligned} 0.75y-2.6 &< 0.25y-3.1 \\ 0.75y-2.6+2.6 &< 0.25y-3.1+2.6 \\ 0.75y &< 0.25y-0.5 \\ 0.75y-0.25y &< 0.25y-0.5-0.25y \\ 0.5y &< -0.5 \\ \frac{0.5y}{0.5} &< \frac{-0.5}{0.5} \\ y &< -1 \end{aligned}$$

e)
$$\begin{aligned}
1.5(x+2)+1 &> 2.5(1-x)-0.5\\
1.5x+3+1 &> 2.5-2.5x-0.5\\
1.5x+4 &> 2-2.5x\\
1.5x+4-4 &> 2-2.5x-4\\
1.5x &> -2.5x-2\\
1.5x+2.5x &> -2.5x-2+2.5x\\
4x &> -2\\
\frac{4x}{4} &> \frac{-2}{4}\\
x &> -\frac{1}{2}
\end{aligned}$$

Section 1.9 Page 79 Question 7

a)
$$\begin{aligned}
\frac{x+1}{2} &< \frac{x+2}{3}\\
6\times\frac{x+1}{2} &< \frac{x+2}{3}\times 6\\
3(x+1) &< 2(x+2)\\
3x+3 &< 2x+4\\
3x+3-3 &< 2x+4-3\\
3x &< 2x+1\\
3x-2x &< 2x+1-2x\\
x &< 1
\end{aligned}$$

c)
$$\begin{aligned}
\frac{z+2}{4} &> \frac{z-1}{5}+1\\
20\times\frac{z+2}{4} &> \left(\frac{z-1}{5}+1\right)\times 20\\
5(z+2) &> 4(z-1)+20\\
5z+10 &> 4z-4+20\\
5z+10-10 &> 4z-4+20-10\\
5z &> 4z+6\\
5z-4z &> 4z+6-4z\\
z &> 6
\end{aligned}$$

Apply, Solve, Communicate

Section 1.9 Page 79 Question 8

$$\begin{aligned}
4m+18 &\le 50\\
4m+18-18 &\le 50-18\\
4m &\le 32\\
\frac{4m}{4} &\le \frac{32}{4}\\
m &\le 8
\end{aligned}$$

Katrina can buy up to and including 8 markers.

Section 1.9 Page 79 Question 9

From $5x + 10 > 90$ and $x + 10 < 180$, x can be determined by composing and solving a three-term inequality.

$$\begin{aligned}
90 &< 5x+10 < 180\\
90-10 &< 5x+10-10 < 180-10\\
80 &< 5x < 170\\
\frac{80}{5} &< \frac{5x}{5} < \frac{170}{5}\\
16 &< x < 34
\end{aligned}$$

The expression $(5x + 10)°$ defines an obtuse angle if $16 < x < 34$.

Section 1.9 Page 79 Question 10

a) The total cost, C, of a pizza with n toppings can be defined as $C = 1.55n + 12.25$.

b)

$$\begin{aligned} C &\leq 20 \\ 1.55n + 12.25 &\leq 20 \\ 1.55n + 12.25 - 12.25 &\leq 20 - 12.25 \\ 1.55n &\leq 7.75 \\ \frac{1.55n}{1.55} &\leq \frac{7.75}{1.55} \\ n &\leq 5 \end{aligned}$$

You can afford up to and including 5 toppings.

Section 1.9 Page 79 Question 11

The total interior degree measure of a triangle is 180°.

$$\begin{aligned} 90 \leq 4x &< 180 \\ \frac{90}{4} \leq \frac{4x}{4} &< \frac{180}{4} \\ 22.5 \leq x &< 45 \end{aligned}$$

The values of x lie in the interval $22.5 \leq x < 45$.

Section 1.9 Page 80 Question 12

a) Mario's net earnings, N, at the end of the week after working t hours can be written as $N = 15t - 75$.

b)

$$\begin{aligned} N &\geq 450 \\ 15t - 75 &\geq 450 \\ 15t - 75 + 75 &\geq 450 + 75 \\ 15t &\geq 525 \\ \frac{15t}{15} &\geq \frac{525}{15} \\ t &\geq 35 \end{aligned}$$

Mario must work at least 35 h to have at least \$450 at the end of the week.

Section 1.9 Page 80 Question 13

Let n be the number of caps ordered. To raise money, the revenue from the sale of the caps must exceed the cost of the caps.

$$\begin{aligned} \text{Revenue} &> \text{Cost} \\ 15n &> 7n + 500 \\ 15n - 7n &> 7n + 500 - 7n \\ 8n &> 500 \\ \frac{8n}{8} &> \frac{500}{8} \\ n &> 62.5 \end{aligned}$$

The college baseball team must order and sell a minimum of 63 caps to raise money.

Section 1.9 Page 80 Question 14

The population of Paris increased by an average of 80 per year, so its population can be expressed as $P = 80t + 8600$. The population of Aylmer increased by an average of 160 per year, so its population is $A = 160t + 6200$, where t is measured in years. The number of years can be determined by solving an inequality.

$$\begin{aligned} A &> P \\ 160t + 6200 &> 80t + 8600 \\ 80t &> 2400 \\ t &> 30 \end{aligned}$$

The population of Aylmer will exceed the population of Paris in the year 2021 (1991 + 30).

Section 1.9 Page 80 Question 15

a)

$$\begin{aligned} 2(3x-7)+2(2) &> 32 \\ \frac{2(3x-7)+2(2)}{2} &> \frac{32}{2} \\ 3x-7+2 &> 16 \\ 3x-5 &> 16 \\ 3x-5+5 &> 16+5 \\ 3x &> 21 \\ \frac{3x}{3} &> \frac{21}{3} \\ x &> 7 \end{aligned}$$

The value of x must be greater than 7.

b)

$$\begin{aligned} 2(3x-7) &< 40 \\ \frac{2(3x-7)}{2} &< \frac{40}{2} \\ 3x-7 &< 20 \\ 3x-7+7 &< 20+7 \\ 3x &< 27 \\ x &< 9 \end{aligned}$$

The rectangle will have an area less than 40 cm^2 if x is less than 9.

c) Each side must be of positive length.

$$\begin{aligned} 3x-7 &> 0 \\ 3x-7+7 &> 0+7 \\ 3x &> 7 \\ \frac{3x}{3} &> \frac{7}{3} \\ x &> \frac{7}{3} \end{aligned}$$

The value of x must exceed $\frac{7}{3}$ cm.

Section 1.9 Page 80 Question 16

Let P be the perimeter of the triangle. A three-term inequality can be composed and solved.

$$\begin{aligned} 12 \le P &\le 15 \\ 12 \le 2(x-1)+3x-1+x &\le 15 \\ 12 \le 2x-2+3x-1+x &\le 15 \\ 12 \le 6x-3 &\le 15 \\ 12+3 \le 6x-3+3 &\le 15+3 \\ 15 \le 6x &\le 18 \\ \frac{15}{6} \le \frac{6x}{6} &\le \frac{18}{6} \\ \frac{5}{2} \le x &\le 3 \end{aligned}$$

The value of x must lie in the interval $\frac{5}{2} \le x \le 3$.

Section 1.9 Page 80 Question 17

$$\begin{aligned} 3(x+2)-5 &\neq 2(1-x)+4 \\ 3x+6-5 &\neq 2-2x+4 \\ 5x &\neq 5 \\ x &\neq 1 \end{aligned}$$

Section 1.9 Page 80 Question 18

a) Let p be the ticket price.

$$1500p = 90\,000 + 4500$$
$$1500p = 94\,500$$
$$p = 63$$

The ticket price should be set to \$63.

b) If the charity sold 2000 tickets, the revenue would be 2000(63) or \$126 000. The profit would range from \$90 000 to \$126 000 − \$4500 or \$121 500.

Section 1.9 Page 81 Question 19

Answers may vary.

Section 1.9 Page 81 Question 20

Let t be the number of hours after 10:00. Jason's distance from Hamilton can be expressed as $J = 80t$. Hakim's distance from Hamilton can be expressed as $H = 100(t - 1)$.

$$H > J$$
$$100(t - 1) > 80t$$
$$100t - 100 > 80t$$
$$100t - 100 + 100 > 80t + 100$$
$$100t > 80t + 100$$
$$100t - 80t > 80t + 100 - 80t$$
$$20t > 100$$
$$t > 5$$

Hakim is farther from Hamilton after 15:00. But Hakim arrives in Montreal at 17:12 and Jason arrives at 17:45. So Hakim is farther away from Hamilton between 15:00 and 17:45.

Section 1.9 Page 81 Question 21

a)
$$4x + 2(x + 1) = 6x - 2$$
$$4x + 2x + 2 = 6x - 2$$
$$6x + 2 = 6x - 2$$
$$6x + 2 - 2 = 6x - 2 - 2$$
$$6x = 6x - 4$$
$$6x - 6x = 6x - 4 - 6x$$
$$0 = -4$$

c)
$$4x + 2(x + 1) > 6x - 2$$
$$4x + 2x + 2 > 6x - 2$$
$$6x + 2 > 6x - 2$$
$$6x + 2 - 2 > 6x - 2 - 2$$
$$6x > 6x - 4$$
$$6x - 6x > 6x - 4 - 6x$$
$$0 > -4$$

b) There are no real solutions to the given equation.

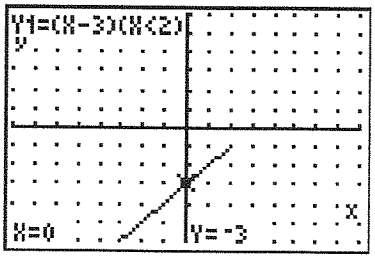
d) Since the final inequality is always true, the original inequality is true for all real values of x.

Section 1.9 Page 81 Question 22

a) The definition

$$y = (x - 3)(x < 2)$$

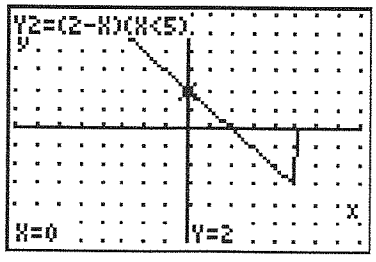
restricts the graph of $y = x - 3$ to the left of $x = 2$.



b) The definition

$$y = (2 - x)(x < 5)$$

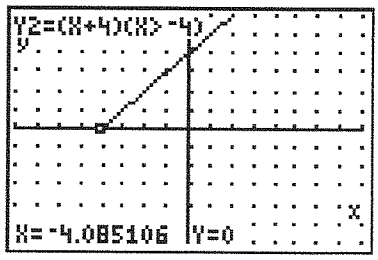
restricts the graph of $y = 2 - x$ to the left of $x = 5$.



c) The definition

$$y = (x + 4)(x > -4)$$

restricts the graph of $y = x + 4$ to the right of $x = -4$.



Review of Key Concepts

Section Review Page 85 Question 1

a) $5^{-2} = \frac{1}{5^2}$
$= \frac{1}{25}$

b) $6^0 = 1$

c) $3^{-3} = \frac{1}{3^3}$
$= \frac{1}{27}$

d) $(-3)^{-4} = \frac{1}{(-3)^4}$
$= \frac{1}{81}$

e) $(5^{-1})^2 = \left(\frac{1}{5}\right)^2$
$= \frac{1}{25}$

f) $\frac{1}{(-3)^{-1}} = (-3)^1$
$= -3$

g) $\frac{2^3}{2^0 - 2^{-1}} = \frac{8}{1 - \frac{1}{2}}$
$= \frac{8}{\frac{1}{2}}$
$= 16$

h) $\frac{4^{-1} + 2^{-2}}{2^{-3}} = \frac{\frac{1}{4} + \frac{1}{4}}{\frac{1}{8}}$
$= \frac{\frac{1}{2}}{\frac{1}{8}}$
$= 4$

i) $\frac{a^0 + 3^2}{2^4 - b^0} = \frac{1 + 9}{16 - 1}$
$= \frac{10}{15}$
$= \frac{2}{3}$

Section Review Page 85 Question 2

a) $m^2 \times m^5 = m^{2+5}$
$= m^7$

b) $y^{-3} \times y^{-2} = y^{-3+(-2)}$
$= y^{-5}$
$= \frac{1}{y^5}$

c) $t^7 \div t^4 = t^{7-4}$
$= t^3$

d) $m^{-7} \div m^{-2} = m^{-7-(-2)}$
$= m^{-7+2}$
$= m^{-5}$
$= \frac{1}{m^5}$

e) $(x^2y^3)^4 = (x^2)^4(y^3)^4$
$= x^8y^{12}$

f) $(y^3)^0 = y^0$
$= 1$

g) $(x^{-2}y^3)^{-2} = (x^{-2})^{-2}(y^3)^{-2}$
$= x^4y^{-6}$
$= \frac{x^4}{y^6}$

h) $\left(\frac{m^3}{n^2}\right)^4 = \frac{(m^3)^4}{(n^2)^4}$
$= \frac{m^{12}}{n^8}$

i) $\left(\frac{x^{-3}}{y^{-2}}\right)^{-2} = \frac{(x^{-3})^{-2}}{(y^{-2})^{-2}}$
$= \frac{x^6}{y^4}$

Section Review Page 85 Question 3

a) $(-2x^2y^3)(-5x^3y^4) = 10x^{2+3}y^{3+4}$
$= 10x^5y^7$

b) $(-18a^3b^2) \div (-2a^2b) = \frac{-18a^3b^2}{-2a^2b}$
$= 9a^{3-2}b^{2-1}$
$= 9ab$

c) $3m^{-2} \times 4m^6 = 12m^{-2+6}$
$= 12m^4$

d) $10x^{-2} \div (-2x^{-3}) = \frac{10x^{-2}}{-2x^{-3}}$
$= -5x^{-2-(-3)}$
$= -5x^1$
$= -5x$

e) $(-2a^5b^3)^2 = (-2)^2(a^5)^2(b^3)^2$
$= 4a^{10}b^6$

f) $(-3m^{-3}n^{-1})^{-3} = (-3)^{-3}(m^{-3})^{-3}(n^{-1})^{-3}$
$= \frac{1}{(-3)^3}m^9n^3$
$= \frac{1}{-27}m^9n^3$
$= -\frac{m^9n^3}{27}$

g) $\left(\frac{3m^2}{2n^3}\right)^3 = \frac{(3m^2)^3}{(2n^3)^3}$
$= \frac{(3)^3(m^2)^3}{(2)^3(n^3)^3}$
$= \frac{27m^6}{8n^9}$

h) $\left(\frac{-2x^{-3}}{3y^{-4}}\right)^{-2} = \frac{(-2x^{-3})^{-2}}{(3y^{-4})^{-2}}$
$= \frac{(-2)^{-2}(x^{-3})^{-2}}{(3)^{-2}(y^{-4})^{-2}}$
$= \frac{\frac{1}{(-2)^2}x^6}{\frac{1}{3^2}y^8}$
$= \frac{\frac{1}{4}x^6}{\frac{1}{9}y^8}$
$= \frac{9x^6}{4y^8}$

i) $\frac{(3x^3y)(6xy^4)}{-9xy^2} = \frac{18x^{3+1}y^{1+4}}{-9xy^2}$
$= \frac{18x^4y^5}{-9xy^2}$
$= -2x^{4-1}y^{5-2}$
$= -2x^3y^3$

j) $\frac{3ab^4}{2a^3b^2} \times \frac{12a^5b}{15a^4b} = \frac{3ab^4 \times 12a^5b}{2a^3b^2 \times 15a^4b}$
$= \frac{36a^{1+5}b^{4+1}}{30a^{3+4}b^{2+1}}$
$= \frac{6a^6b^5}{5a^7b^3}$
$= \frac{6b^{5-3}}{5a^{7-6}}$
$= \frac{6b^2}{5a}$

k) $\frac{(-2s^{-2}t)(5s^{-3}t^2)}{4s^2t^{-3}} = \frac{-10s^{-2+(-3)}t^{1+2}}{4s^2t^{-3}}$
$= \frac{-10s^{-5}t^3}{4s^2t^{-3}}$
$= \frac{-5t^{3-(-3)}}{2s^{2-(-5)}}$
$= -\frac{5t^6}{2s^7}$

l) $\left(\frac{6a^{-2}b^{-3}}{2a^2b^{-1}}\right)^{-2} = \frac{(6)^{-2}(a^{-2})^{-2}(b^{-3})^{-2}}{(2)^{-2}(a^2)^{-2}(b^{-1})^{-2}}$
$= \frac{\frac{1}{6^2}a^4b^6}{\frac{1}{2^2}a^{-4}b^2}$
$= \frac{\frac{1}{36}a^4b^6}{\frac{1}{4}a^{-4}b^2}$
$= \frac{4a^{4-(-4)}b^{6-2}}{36}$
$= \frac{a^8b^4}{9}$

Section Review Page 85 Question 4

a) $6^{\frac{1}{2}} = \sqrt{6}$

b) $5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}}$
$= \frac{1}{\sqrt{5}}$

c) $7^{\frac{3}{5}} = \left(7^{\frac{1}{5}}\right)^3$
$= \left(\sqrt[5]{7}\right)^3$

d) $10^{-\frac{4}{3}} = \left(\sqrt[3]{10}\right)^{-4}$
$= \frac{1}{\left(\sqrt[3]{10}\right)^4}$

Section Review Page 85 Question 5

a) $\sqrt[3]{-8} = (-8)^{\frac{1}{3}}$

b) $\left(\sqrt[3]{m}\right)^5 = \left(m^{\frac{1}{3}}\right)^5$
$= m^{\frac{5}{3}}$

c) $\sqrt[3]{x^2} = \left(x^2\right)^{\frac{1}{3}}$
$= x^{\frac{2}{3}}$

d) $\sqrt{\sqrt[5]{4a^4}} = \left(\left(4a^4\right)^{\frac{1}{5}}\right)^{\frac{1}{2}}$
$= \left(\left(4a^4\right)^{\frac{1}{2}}\right)^{\frac{1}{5}}$
$= \left(2a^2\right)^{\frac{1}{5}}$
$= 2^{\frac{1}{5}}a^{\frac{2}{5}}$

Section Review Page 85 Question 6

a) $25^{\frac{1}{2}} = \sqrt{25}$
$= 5$

b) $\left(\frac{1}{27}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{1}{27}}$
$= \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$
$= \frac{1}{3}$

c) $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}}$
$= \frac{1}{\sqrt{49}}$
$= \frac{1}{7}$

d) $1^{-\frac{1}{4}} = \frac{1}{1^{\frac{1}{4}}}$
$= \frac{1}{\sqrt[4]{1}}$
$= \frac{1}{1}$
$= 1$

e) $0.09^{0.5} = (0.09)^{\frac{1}{2}}$
$= \sqrt{0.09}$
$= \sqrt{\frac{9}{100}}$
$= \frac{3}{10}$

f) $(-8)^{-\frac{1}{3}} = \frac{1}{(-8)^{\frac{1}{3}}}$
$= \frac{1}{\sqrt[3]{-8}}$
$= \frac{1}{-2}$
$= -\frac{1}{2}$

g) $(0.008)^{-\frac{1}{3}} = \frac{1}{(0.008)^{\frac{1}{3}}}$
$= \frac{1}{\sqrt[3]{0.008}}$
$= \frac{1}{0.2}$
$= \frac{1}{\frac{1}{5}}$
$= 5$

h) $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$
$= 3^2$
$= 9$

i) $-16^{-\frac{3}{4}} = -\frac{1}{16^{\frac{3}{4}}}$
$= -\frac{1}{\left(\sqrt[4]{16}\right)^3}$
$= -\frac{1}{2^3}$
$= -\frac{1}{8}$

j) $\left(\frac{81}{16}\right)^{\frac{5}{4}} = \left(\frac{\sqrt[4]{81}}{\sqrt[4]{16}}\right)^5$
$= \left(\frac{3}{2}\right)^5$
$= \frac{3^5}{2^5}$
$= \frac{243}{32}$

k) $\left(\frac{1}{9}\right)^{2.5} = \left(\frac{1}{9}\right)^{\frac{5}{2}}$
$= \left(\sqrt{\frac{1}{9}}\right)^5$
$= \left(\frac{1}{3}\right)^5$
$= \frac{1}{243}$

l) $\left(\frac{27}{125}\right)^{-\frac{2}{3}} = \left(\sqrt[3]{\frac{27}{125}}\right)^{-2}$
$= \left(\frac{3}{5}\right)^{-2}$
$= \frac{1}{\left(\frac{3}{5}\right)^2}$
$= \frac{1}{\frac{9}{25}}$
$= \frac{25}{9}$

m) $(-32)^{\frac{4}{5}} = \left(\sqrt[5]{-32}\right)^4$
$= (-2)^4$
$= 16$

n) $(-8^{-1})^{-\frac{1}{3}} = (-1 \times 8^{-1})^{-\frac{1}{3}}$
$= (-1)^{-\frac{1}{3}} \times (8^{-1})^{-\frac{1}{3}}$
$= \frac{1}{(-1)^{\frac{1}{3}}} \times 8^{\frac{1}{3}}$
$= \frac{1}{\sqrt[3]{-1}} \times \sqrt[3]{8}$
$= \frac{1}{-1} \times 2$
$= -2$

o) $\sqrt{\sqrt{16}} = \sqrt{4}$
$= 2$

Section Review Page 86 Question 7

a) $\sqrt{\sqrt[3]{y^4}} = \left(\left(y^4\right)^{\frac{1}{3}}\right)^{\frac{1}{2}}$
$= y^{\frac{4}{6}}$
$= y^{\frac{2}{3}}$

b) $\sqrt{\sqrt{81m^8}} = \sqrt{9m^4}$
$= 3m^2$

c) $\sqrt[3]{-8x} = (-8x)^{\frac{1}{3}}$
$= (-8)^{\frac{1}{3}} x^{\frac{1}{3}}$
$= -2x^{\frac{1}{3}}$

d) $(\sqrt{x^3})(\sqrt{x}) = \sqrt{x^3 \times x}$
$= \sqrt{x^{3+1}}$
$= \sqrt{x^4}$
$= x^2$

e) $(\sqrt[3]{-64})x = -4x$

f) $\sqrt[3]{-64x} = (-64x)^{\frac{1}{3}}$
$= (-64)^{\frac{1}{3}} \times x^{\frac{1}{3}}$
$= -4x^{\frac{1}{3}}$

Section Review Page 86 Question 8

Let V be the volume of the cube.

$$V = \left(\left(\frac{5}{2}\right)^{-\frac{2}{3}}\right)^3$$
$$= \left(\frac{5}{2}\right)^{-2}$$
$$= \frac{4}{25}$$

The volume of the cube is $\frac{4}{25}$ cubic units.

Section Review Page 86 Question 9

a) $2^x = 64$
$2^x = 2^6$
$x = 6$
Check $x = 6$.
L.S. $= 2^6$
$= 64$
$=$ **R.S.**

b) $(-5)^x = -125$
$(-5)^x = (-5)^3$
$x = 3$
Check $x = 3$.
L.S. $= (-5)^3$
$= -125$
$=$ **R.S.**

c) $2^{x+3} = 128$
$2^{x+3} = 2^7$
$x + 3 = 7$
$x = 4$
Check $x = 4$.
L.S. $= 2^{4+3}$
$= 2^7$
$= 128$
$=$ **R.S.**

d)

$$\frac{5^{x-1}}{25} = 1$$
$$\frac{5^{x-1}}{5^2} = 5^0$$
$$5^{x-1-2} = 5^0$$
$$5^{x-3} = 5^0$$
$$x - 3 = 0$$
$$x = 3$$

Check $x = 3$.

$$\text{L.S.} = \frac{5^{3-1}}{25}$$
$$= \frac{5^2}{25}$$
$$= 1$$
$$= \text{R.S.}$$

e)

$$5^{y+2} = 1$$
$$5^{y+2} = 5^0$$
$$y + 2 = 0$$
$$y = -2$$

Check $y = -2$.

$$\text{L.S.} = 5^{-2+2}$$
$$= 5^0$$
$$= 1$$
$$= \text{R.S.}$$

f)

$$4^{2x+1} = 8$$
$$\left(2^2\right)^{2x+1} = 2^3$$
$$2^{4x+2} = 2^3$$
$$4x + 2 = 3$$
$$4x = 1$$
$$x = \frac{1}{4}$$

Check $x = \frac{1}{4}$.

$$\text{L.S.} = 4^{2\left(\frac{1}{4}\right)+1}$$
$$= 4^{\frac{3}{2}}$$
$$= 2^3$$
$$= 8$$
$$= \text{R.S.}$$

g)

$$2\left(3^{n+2}\right) = 18$$
$$3^{n+2} = 9$$
$$3^{n+2} = 3^2$$
$$n + 2 = 2$$
$$n = 0$$

Check $n = 0$.

$$\text{L.S.} = 2\left(3^{0+2}\right)$$
$$= 2(3^2)$$
$$= 2(9)$$
$$= 18$$
$$= \text{R.S.}$$

h)

$$4^{x-2} + 1 = 5$$
$$4^{x-2} = 4^1$$
$$x - 2 = 1$$
$$x = 3$$

Check $x = 3$.

$$\text{L.S.} = 4^{3-2} + 1$$
$$= 4^1 + 1$$
$$= 5$$
$$= \text{R.S.}$$

Section Review Page 86 Question 10

a)

$$2^{x+5} = 2^{2x-1}$$
$$x + 5 = 2x - 1$$
$$x = 6$$

b)

$$27^{x-2} = 3^{x+6}$$
$$\left(3^3\right)^{x-2} = 3^{x+6}$$
$$3^{3x-6} = 3^{x+6}$$
$$3x - 6 = x + 6$$
$$2x = 12$$
$$x = 6$$

c)

$$8^{2m+2} = 16^{m-2}$$
$$\left(2^3\right)^{2m+2} = \left(2^4\right)^{m-2}$$
$$2^{6m+6} = 2^{4m-8}$$
$$6m + 6 = 4m - 8$$
$$2m = -14$$
$$m = -7$$

d)

$$5^{y-1} = 25^{2y-1}$$
$$5^{y-1} = \left(5^2\right)^{2y-1}$$
$$5^{y-1} = 5^{4y-2}$$
$$y - 1 = 4y - 2$$
$$3y = 1$$
$$y = \frac{1}{3}$$

e)

$$4^{2t+1} = 8^{2t-1}$$
$$\left(2^2\right)^{2t+1} = \left(2^3\right)^{2t-1}$$
$$2^{4t+2} = 2^{6t-3}$$
$$4t + 2 = 6t - 3$$
$$2t = 5$$
$$t = \frac{5}{2}$$

f)

$$6^{3x+5} = 36^{3x+6}$$
$$6^{3x+5} = \left(6^2\right)^{3x+6}$$
$$6^{3x+5} = 6^{6x+12}$$
$$3x + 5 = 6x + 12$$
$$3x = -7$$
$$x = -\frac{7}{3}$$

Section Review Page 86 Question 11

a)
$$\begin{aligned} 2^{x+3} + 2^x &= 288 \\ 2^x\left(2^3 + 1\right) &= 288 \\ 2^x(9) &= 288 \\ 2^x &= 32 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

Check $x = 5$.

$$\begin{aligned} \textbf{L.S.} &= 2^{5+3} + 2^5 \\ &= 256 + 32 \\ &= 288 \\ &= \textbf{R.S.} \end{aligned}$$

b)
$$\begin{aligned} 3^{g+3} - 3^{g+2} &= 1458 \\ 3^g\left(3^3 - 3^2\right) &= 1458 \\ 3^g(18) &= 1458 \\ 3^g &= 81 \\ 3^g &= 3^4 \\ g &= 4 \end{aligned}$$

Check $g = 4$.

$$\begin{aligned} \textbf{L.S.} &= 3^{4+3} - 3^{4+2} \\ &= 3^7 - 3^6 \\ &= 2187 - 729 \\ &= 1458 \\ &= \textbf{R.S.} \end{aligned}$$

c)
$$\begin{aligned} -500 &= 5^{y+1} - 5^{y+2} \\ -500 &= 5^y\left(5^1 - 5^2\right) \\ -500 &= 5^y(-20) \\ 25 &= 5^y \\ 5^2 &= 5^y \\ y &= 2 \end{aligned}$$

Check $y = 2$.

$$\begin{aligned} \textbf{R.S.} &= 5^{2+1} - 5^{2+2} \\ &= 5^3 - 5^4 \\ &= 125 - 625 \\ &= -500 \\ &= \textbf{L.S.} \end{aligned}$$

Section Review Page 86 Question 12

Let t be the time taken to double.

$$\begin{aligned} 30\,000\left(2^{\frac{t}{3.75}}\right) &= 7\,680\,000 \\ 2^{\frac{t}{3.75}} &= 256 \\ 2^{\frac{t}{3.75}} &= 2^8 \\ \frac{t}{3.75} &= 8 \\ t &= 30 \end{aligned}$$

It will take 30 h for the number of bacteria to double from 30 000 to 7 680 000.

Section Review Page 86 Question 13

a)
$$\begin{aligned} (5x^2 - 4x - 2) + (8x^2 + 3x - 3) &= 5x^2 - 4x - 2 + 8x^2 + 3x - 3 \\ &= 13x^2 - x - 5 \end{aligned}$$

b)
$$\begin{aligned} (2x^2 - 6xy + 7y^2) + (4x^2 + 3xy - 11y^2) &= 2x^2 - 6xy + 7y^2 + 4x^2 + 3xy - 11y^2 \\ &= 6x^2 - 3xy - 4y^2 \end{aligned}$$

Section Review Page 86 Question 14

a)
$$\begin{aligned} (7y^2 + 4y - 7) - (9y^2 + 3y - 3) &= 7y^2 + 4y - 7 - 9y^2 - 3y + 3 \\ &= -2y^2 + y - 4 \end{aligned}$$

b)
$$\begin{aligned} (3m^2 + mn - 7n^2) - (5m^2 + 3mn - 8n^2) &= 3m^2 + mn - 7n^2 - 5m^2 - 3mn + 8n^2 \\ &= -2m^2 - 2mn + n^2 \end{aligned}$$

Section Review Page 86 Question 15

a)
$$\begin{aligned} 4(x + 5) + 3(x - 7) &= 4x + 20 + 3x - 21 \\ &= 7x - 1 \end{aligned}$$

b)
$$\begin{aligned} 6(3s - 4t) - (7s - t) + 5 &= 18s - 24t - 7s + t + 5 \\ &= 11s - 23t + 5 \end{aligned}$$

c) $$\begin{aligned} 2x(x+3) - x(3x+8) &= 2x^2 + 6x - 3x^2 - 8x \\ &= -x^2 - 2x \end{aligned}$$

d) $$\begin{aligned} 3y(y-2) + 2y(3y+4) - 4y(2y-3) &= 3y^2 - 6y + 6y^2 + 8y - 8y^2 + 12y \\ &= y^2 + 14y \end{aligned}$$

Section Review Page 87 Question 16

a) $$\begin{aligned} 3[4 - 2(y-3)] + 4[3(2-y) - 5] &= 3(4 - 2y + 6) + 4(6 - 3y - 5) \\ &= 3(-2y + 10) + 4(-3y + 1) \\ &= -6y + 30 - 12y + 4 \\ &= -18y + 34 \end{aligned}$$

b) $$\begin{aligned} 2x[2 - x(x-1)] - [3 - x(x+20)] &= 2x(2 - x^2 + x) - (3 - x^2 - 20x) \\ &= 4x - 2x^3 + 2x^2 - 3 + x^2 + 20x \\ &= -2x^3 + 3x^2 + 24x - 3 \end{aligned}$$

Section Review Page 87 Question 17

a) $$\begin{aligned} (y-8)(y-9) &= y^2 - 9y - 8y + 72 \\ &= y^2 - 17y + 72 \end{aligned}$$

b) $$\begin{aligned} 2(7-3x)(4+x) &= 2(28 + 7x - 12x - 3x^2) \\ &= 2(28 - 5x - 3x^2) \\ &= 56 - 10x - 6x^2 \end{aligned}$$

c) $$\begin{aligned} 3(3x-1)^2 &= 3(9x^2 - 6x + 1) \\ &= 27x^2 - 18x + 3 \end{aligned}$$

d) $$\begin{aligned} (4x+3y)(2x-5y) &= 8x^2 - 20xy + 6xy - 15y^2 \\ &= 8x^2 - 14xy - 15y^2 \end{aligned}$$

Section Review Page 87 Question 18

a) $$\begin{aligned} (m-4)(m+4) + (m-3)^2 &= m^2 - 16 + m^2 - 6m + 9 \\ &= 2m^2 - 6m - 7 \end{aligned}$$

b) $$\begin{aligned} (x+6)^2 - (x+4)(x-7) &= x^2 + 12x + 36 - (x^2 - 7x + 4x - 28) \\ &= x^2 + 12x + 36 - (x^2 - 3x - 28) \\ &= x^2 + 12x + 36 - x^2 + 3x + 28 \\ &= 15x + 64 \end{aligned}$$

c) $$\begin{aligned} 3(4y+1)^2 + 2(3y-4)(2y-3) &= 3(16y^2 + 8y + 1) + 2(6y^2 - 9y - 8y + 12) \\ &= 48y^2 + 24y + 3 + 2(6y^2 - 17y + 12) \\ &= 48y^2 + 24y + 3 + 12y^2 - 34y + 24 \\ &= 60y^2 - 10y + 27 \end{aligned}$$

d) $$\begin{aligned} 2(3x-2y)(x+3y) - 2(2x-y)^2 &= 2(3x^2 + 9xy - 2xy - 6y^2) - 2(4x^2 - 4xy + y^2) \\ &= 2(3x^2 + 7xy - 6y^2) - 8x^2 + 8xy - 2y^2 \\ &= 6x^2 + 14xy - 12y^2 - 8x^2 + 8xy - 2y^2 \\ &= -2x^2 + 22xy - 14y^2 \end{aligned}$$

Section Review Page 87 Question 19

a) $$\begin{aligned}(x-3)(x^2-3x+2) &= x^3-3x^2+2x-3x^2+9x-6\\ &= x^3-6x^2+11x-6\end{aligned}$$

b) $$\begin{aligned}(2t+1)(3t^2-t-1) &= 6t^3-2t^2-2t+3t^2-t-1\\ &= 6t^3+t^2-3t-1\end{aligned}$$

c) $$\begin{aligned}(x^2+2x+3)(x^2-x-1) &= x^4-x^3-x^2+2x^3-2x^2-2x+3x^2-3x-3\\ &= x^4+x^3-5x-3\end{aligned}$$

d) $$\begin{aligned}(3z^2-2z+1)(2z^2+2z-3) &= 6z^4+6z^3-9z^2-4z^3-4z^2+6z+2z^2+2z-3\\ &= 6z^4+2z^3-11z^2+8z-3\end{aligned}$$

Section Review Page 87 Question 20

Let ΔA be the increase in area.

$$\begin{aligned}\Delta A &= A_{\text{enlarged}} - A_{\text{original}}\\ &= (2x+1+2)(x-1+2)-(2x+1)(x-1)\\ &= (2x+3)(x+1)-(2x+1)(x-1)\\ &= 2x^2+5x+3-(2x^2-x-1)\\ &= 2x^2+5x+3-2x^2+x+1\\ &= 6x+4\end{aligned}$$

The area is increased by $6x+4$ square units.

Section Review Page 87 Question 21

a) $$\begin{aligned}\frac{3x}{3x+9} &= \frac{3x}{3(x+3)}\\ &= \frac{x}{x+3},\ x\neq -3\end{aligned}$$

b) $$\begin{aligned}\frac{8y^2-10xy}{4y} &= \frac{2y(4y-5x)}{4y}\\ &= \frac{4y-5x}{2},\ y\neq 0\end{aligned}$$

c) $$\begin{aligned}\frac{5x-5y}{7x-7y} &= \frac{5(x-y)}{7(x-y)}\\ &= \frac{5}{7},\ x\neq y\end{aligned}$$

d) $$\begin{aligned}\frac{6x-10}{5-3x} &= \frac{2(3x-5)}{5-3x}\\ &= -2,\ x\neq \frac{5}{3}\end{aligned}$$

e) $$\begin{aligned}\frac{3w}{3w^2-12w} &= \frac{3w}{3w(w-4)}\\ &= \frac{1}{w-4},\ w\neq 0,4\end{aligned}$$

f) $$\begin{aligned}\frac{3m^2-3m}{4m^2-4m} &= \frac{3m(m-1)}{4m(m-1)}\\ &= \frac{3}{4},\ m\neq 0,1\end{aligned}$$

g) $$\begin{aligned}\frac{t-2}{t^2-3t+2} &= \frac{t-2}{(t-2)(t-1)}\\ &= \frac{1}{t-1},\ t\neq 1,2\end{aligned}$$

h) $$\begin{aligned}\frac{2a^2-7a-15}{a-5} &= \frac{(a-5)(2a+3)}{a-5}\\ &= 2a+3,\ a\neq 5\end{aligned}$$

i) $$\begin{aligned}\frac{y^2-9}{y^2+y-12} &= \frac{(y-3)(y+3)}{(y-3)(y+4)}\\ &= \frac{y+3}{y+4},\ y\neq -4,3\end{aligned}$$

j) $$\begin{aligned}\frac{6n^2-7n-3}{12n^2+7n+1} &= \frac{(2n-3)(3n+1)}{(4n+1)(3n+1)}\\ &= \frac{2n-3}{4n+1},\ n\neq -\frac{1}{4},-\frac{1}{3}\end{aligned}$$

Section Review Page 87 Question 22

a) Rearrange the formula for area to isolate length.

$$\text{area} = \text{length} \times \text{width}$$

$$\text{length} = \frac{\text{area}}{\text{width}}$$

$$= \frac{2x^2 + 4x + 2}{x + 1}$$

$$= \frac{2(x^2 + 2x + 1)}{x + 1}$$

$$= \frac{2(x + 1)^2}{x + 1}$$

$$= 2(x + 1),\ x \neq -1$$

b)

$$\text{length} : \text{width} = 2(x + 1) : x + 1$$

$$= 2 : 1$$

For an Alberta flag, length : width = 2 : 1.

Section Review Page 88 Question 23

a)

$$\frac{5x^3}{2y} \times \frac{8y}{15x^2} = \frac{40x^3y}{30x^2y}$$

$$= \frac{4x}{3},\ x, y \neq 0$$

b)

$$\frac{-4a^3}{3b} \div \frac{2a}{3b^2} = \frac{-4a^3}{3b} \times \frac{3b^2}{2a}$$

$$= \frac{-12a^3b^2}{6ab}$$

$$= -2a^2b,\ a, b \neq 0$$

c)

$$\frac{3a^2b}{-4xy} \times \frac{-5x^2y}{6ab^2} = \frac{-15a^2bx^2y}{-24ab^2xy}$$

$$= \frac{5ax}{8b},\ a, b, x, y \neq 0$$

d)

$$\frac{b^2}{8x^3y} \div \frac{3b}{4xy} = \frac{b^2}{8x^3y} \times \frac{4xy}{3b}$$

$$= \frac{4b^2xy}{24bx^3y}$$

$$= \frac{b}{6x^2},\ b, x, y \neq 0$$

e)

$$\frac{3x - 3}{2x + 2} \times \frac{5x + 5}{6x - 6} = \frac{3(x - 1)}{2(x + 1)} \times \frac{5(x + 1)}{6(x - 1)}$$

$$= \frac{15(x - 1)(x + 1)}{12(x + 1)(x - 1)}$$

$$= \frac{5}{4},\ x \neq \pm 1$$

f)

$$\frac{4m + 8}{3n - 3} \div \frac{2m + 6}{7n - 7} = \frac{4m + 8}{3n - 3} \times \frac{7n - 7}{2m + 6}$$

$$= \frac{4(m + 2)}{3(n - 1)} \times \frac{7(n - 1)}{2(m + 3)}$$

$$= \frac{28(m + 2)(n - 1)}{6(n - 1)(m + 3)}$$

$$= \frac{14(m + 2)}{3(m + 3)},\ m \neq -3,\ n \neq 1$$

g)

$$\frac{t^2 + 4t + 4}{t - 2} \div \frac{3t + 6}{t^2 - 5t + 6} = \frac{t^2 + 4t + 4}{t - 2} \times \frac{t^2 - 5t + 6}{3t + 6}$$

$$= \frac{(t + 2)^2}{t - 2} \times \frac{(t - 2)(t - 3)}{3(t + 2)}$$

$$= \frac{(t + 2)^2(t - 2)(t - 3)}{3(t - 2)(t + 2)}$$

$$= \frac{(t + 2)(t - 3)}{3},\ t \neq \pm 2, 3$$

h)

$$\frac{2x^2 - 5x - 3}{2x^2 - 5x + 2} \times \frac{2x^2 + 3x - 2}{x^2 - 4x + 3} = \frac{(2x + 1)(x - 3)}{(2x - 1)(x - 2)} \times \frac{(2x - 1)(x + 2)}{(x - 1)(x - 3)}$$

$$= \frac{(2x + 1)(x - 3)(2x - 1)(x + 2)}{(x - 1)(x - 3)(2x - 1)(x - 2)}$$

$$= \frac{(2x + 1)(x + 2)}{(x - 1)(x - 2)},\ x \neq \frac{1}{2}, 1, 2, 3$$

i)
$$\begin{aligned}\frac{6y^2-5y+1}{12y^2-5y-2} \div \frac{3y^2-4y+1}{4y^2+3y-1} &= \frac{6y^2-5y+1}{12y^2-5y-2} \times \frac{4y^2+3y-1}{3y^2-4y+1} \\ &= \frac{(2y-1)(3y-1)}{(4y+1)(3y-2)} \times \frac{(y+1)(4y-1)}{(3y-1)(y-1)} \\ &= \frac{(2y-1)(3y-1)(y+1)(4y-1)}{(4y+1)(3y-2)(3y-1)(y-1)} \\ &= \frac{(2y-1)(y+1)(4y-1)}{(3y-2)(4y+1)(y-1)},\ y \neq \frac{2}{3}, \pm\frac{1}{4}, \frac{1}{3}, \pm 1\end{aligned}$$

Section Review Page 88 Question 24

a) $w_B = \dfrac{2t^2-3t+1}{2t-1}$

b) $w_C = \dfrac{3t^2-2t-1}{3t+1}$

c)
$$\begin{aligned}w_B \times w_C &= \frac{2t^2-3t+1}{2t-1} \times \frac{3t^2-2t-1}{3t+1} \\ &= \frac{(2t-1)(t-1)}{2t-1} \times \frac{(3t+1)(t-1)}{3t+1} \\ &= (t-1)(t-1) \\ &= (t-1)^2,\ t \neq -\frac{1}{3}, \frac{1}{2}\end{aligned}$$

d) Rectangle A is a square because its length and width are equal.

Section Review Page 88 Question 25

a)
$$\begin{aligned}\frac{5}{x}+\frac{1}{x}-\frac{8}{x} &= \frac{5+1-8}{x} \\ &= -\frac{2}{x},\ x \neq 0\end{aligned}$$

b)
$$\begin{aligned}\frac{2m+1}{m-2}+\frac{3m-5}{m-2} &= \frac{2m+1+3m-5}{m-2} \\ &= \frac{5m-4}{m-2},\ m \neq 2\end{aligned}$$

c)
$$\begin{aligned}\frac{4z-3}{z^2}-\frac{3z-1}{z^2} &= \frac{4z-3-(3z-1)}{z^2} \\ &= \frac{4z-3-3z+1}{z^2} \\ &= \frac{z-2}{z^2},\ z \neq 0\end{aligned}$$

d)
$$\begin{aligned}\frac{2t}{3}-\frac{3t}{4}+\frac{t}{6} &= \frac{4}{4} \times \frac{2t}{3}-\frac{3}{3} \times \frac{3t}{4}+\frac{2}{2} \times \frac{t}{6} \\ &= \frac{8t}{12}-\frac{9t}{12}+\frac{2t}{12} \\ &= \frac{8t-9t+2t}{12} \\ &= \frac{t}{12}\end{aligned}$$

e)
$$\begin{aligned}\frac{4x+1}{5}+\frac{2x-1}{4} &= \frac{4}{4} \times \frac{4x+1}{5}+\frac{5}{5} \times \frac{2x-1}{4} \\ &= \frac{16x+4}{20}+\frac{10x-5}{20} \\ &= \frac{16x+4+10x-5}{20} \\ &= \frac{26x-1}{20}\end{aligned}$$

f)
$$\begin{aligned}\frac{2a-3b}{6}-\frac{3a-2b}{4} &= \frac{2}{2} \times \frac{2a-3b}{6}-\frac{3}{3} \times \frac{3a-2b}{4} \\ &= \frac{4a-6b-(9a-6b)}{12} \\ &= \frac{4a-6b-9a+6b}{12} \\ &= -\frac{5a}{12}\end{aligned}$$

g) $$\frac{4}{2y-5}+\frac{2}{5-2y}=\frac{4}{2y-5}+\frac{-2}{2y-5}$$
$$=\frac{4-2}{2y-5}$$
$$=\frac{2}{2y-5},\ y\neq\frac{5}{2}$$

h) $$\frac{x^2+5}{x^2-4}-\frac{x^2-2}{4-x^2}=\frac{x^2+5}{x^2-4}-\frac{2-x^2}{x^2-4}$$
$$=\frac{x^2+5-2+x^2}{x^2-4}$$
$$=\frac{2x^2+3}{x^2-4},\ x\neq\pm2$$

i) $$\frac{2x+5}{x^2+3x+2}-\frac{x+4}{x^2+3x+2}=\frac{2x+5-x-4}{x^2+3x+2}$$
$$=\frac{x+1}{(x+1)(x+2)}$$
$$=\frac{1}{x+2},\ x\neq-1,-2$$

Section Review Page 88 Question 26

Let l represent the length of the unknown side.

$$l=\frac{9x+1}{4}-\frac{x+1}{2}-\frac{2x-1}{2}$$
$$=\frac{9x+1}{4}-\frac{2}{2}\times\frac{x+1}{2}-\frac{2}{2}\times\frac{2x-1}{2}$$
$$=\frac{9x+1}{4}-\frac{2x+2}{4}-\frac{4x-2}{4}$$
$$=\frac{9x+1-2x-2-4x+2}{4}$$
$$=\frac{3x+1}{4}$$

The length of the third side is $\frac{3x+1}{4}$ units.

Section Review Page 89 Question 27

a) $$\frac{2}{y}+\frac{4}{y^2}-\frac{1}{y}=\frac{y}{y}\times\frac{2}{y}+\frac{4}{y^2}-\frac{y}{y}\times\frac{1}{y}$$
$$=\frac{2y}{y^2}+\frac{4}{y^2}-\frac{y}{y^2}$$
$$=\frac{2y+4-y}{y^2}$$
$$=\frac{y+4}{y^2},\ y\neq0$$

b) $$\frac{4}{x^2}-\frac{5}{xy}+\frac{2}{y^2}=\frac{y^2}{y^2}\times\frac{4}{x^2}-\frac{xy}{xy}\times\frac{5}{xy}+\frac{x^2}{x^2}\times\frac{2}{y^2}$$
$$=\frac{4y^2}{x^2y^2}-\frac{5xy}{x^2y^2}+\frac{2x^2}{x^2y^2}$$
$$=\frac{4y^2-5xy+2x^2}{x^2y^2},\ x,y\neq0$$

c) $$\frac{a}{2a-2}+\frac{2}{3a-3}=\frac{a}{2(a-1)}+\frac{2}{3(a-1)}$$
$$=\frac{3}{3}\times\frac{a}{2(a-1)}+\frac{2}{2}\times\frac{2}{3(a-1)}$$
$$=\frac{3a}{6(a-1)}+\frac{4}{6(a-1)}$$
$$=\frac{3a+4}{6(a-1)},\ a\neq1$$

d) $$\frac{2}{x+3}-\frac{4}{x+1}=\frac{x+1}{x+1}\times\frac{2}{x+3}-\frac{x+3}{x+3}\times\frac{4}{x+1}$$
$$=\frac{2(x+1)}{(x+1)(x+3)}-\frac{4(x+3)}{(x+1)(x+3)}$$
$$=\frac{2(x+1)-4(x+3)}{(x+1)(x+3)}$$
$$=\frac{2x+2-4x-12}{(x+3)(x+1)}$$
$$=\frac{-2x-10}{(x+3)(x+1)}$$
$$=\frac{-2(x+5)}{(x+3)(x+1)},\ x\neq-3,-1$$

e)
$$\begin{aligned}\frac{2}{t^2+3t+2}-\frac{1}{t^2+t-2} &= \frac{2}{(t+1)(t+2)}-\frac{1}{(t-1)(t+2)}\\ &= \frac{t-1}{t-1}\times\frac{2}{(t+1)(t+2)}-\frac{t+1}{t+1}\times\frac{1}{(t-1)(t+2)}\\ &= \frac{2(t-1)}{(t-1)(t+1)(t+2)}-\frac{t+1}{(t+1)(t-1)(t+2)}\\ &= \frac{2(t-1)-(t+1)}{(t+1)(t-1)(t+2)}\\ &= \frac{2t-2-t-1}{(t+1)(t-1)(t+2)}\\ &= \frac{t-3}{(t+1)(t-1)(t+2)},\ t\neq -2,\pm 1\end{aligned}$$

f)
$$\begin{aligned}\frac{x+1}{3x^2+4x+1}+\frac{2x-1}{3x^2-5x-2} &= \frac{x+1}{(3x+1)(x+1)}+\frac{2x-1}{(3x+1)(x-2)}\\ &= \frac{1}{3x+1}+\frac{2x-1}{(3x+1)(x-2)}\\ &= \frac{x-2}{x-2}\times\frac{1}{3x+1}+\frac{2x-1}{(3x+1)(x-2)}\\ &= \frac{x-2}{(3x+1)(x-2)}+\frac{2x-1}{(3x+1)(x-2)}\\ &= \frac{x-2+2x-1}{(3x+1)(x-2)}\\ &= \frac{3x-3}{(3x+1)(x-2)}\\ &= \frac{3(x-1)}{(3x+1)(x-2)},\ x\neq -1,-\frac{1}{3},2\end{aligned}$$

Section Review Page 89 Question 28

a)
$$\begin{aligned}y+3 &< 9\\ y &< 9-3\\ y &< 6\end{aligned}$$
Check. Try $y=0$.
$$\begin{aligned}\textbf{L.S.} &= 0+3\\ &= 3\\ &< \textbf{R.S.}\end{aligned}$$

b)
$$\begin{aligned}3w+4 &> 10\\ 3w &> 6\\ w &> 2\end{aligned}$$
Check. Try $w=3$.
$$\begin{aligned}\textbf{L.S.} &= 3(3)+4\\ &= 13\\ &> \textbf{R.S.}\end{aligned}$$

c)
$$\begin{aligned}2x-5 &\geq -7\\ 2x &\geq -2\\ x &\geq -1\end{aligned}$$
Check. Try $x=0$.
$$\begin{aligned}\textbf{L.S.} &= 2(0)-5\\ &= -5\\ &\geq \textbf{R.S.}\end{aligned}$$

d)
$$\begin{aligned}4z-5 &\leq 3\\ 4z &\leq 8\\ z &\leq 2\end{aligned}$$
Check. Try $z=0$.
$$\begin{aligned}\textbf{L.S.} &= 4(0)-5\\ &= -5\\ &\leq \textbf{R.S.}\end{aligned}$$

e)
$$\begin{aligned}-5k &< 10\\ k &> -2\end{aligned}$$
Check. Try $k=0$.
$$\begin{aligned}\textbf{L.S.} &= -5(0)\\ &= 0\\ &< \textbf{R.S.}\end{aligned}$$

f)
$$\begin{aligned}2t &> t-8\\ t &> -8\end{aligned}$$
Check. Try $t=0$.
$$\begin{aligned}\textbf{L.S.} &= 2(0)\\ &= 0\\ \textbf{R.S.} &= 0-8\\ &= -8\\ &< \textbf{L.S.}\end{aligned}$$

g)
$$3(m-2) \le 6$$
$$m-2 \le 2$$
$$m \le 4$$

Check. Try $m = 0$.

L.S. $= 3(0-2)$
$= -6$
$\le$ **R.S.**

h)
$$2(n+4) \ge 0$$
$$n+4 \ge 0$$
$$n \ge -4$$

Check. Try $n = 0$.

L.S. $= 2(0+4)$
$= 8$
$\ge$ **R.S.**

Section Review Page 89 Question 29

a)
$$3x+2 > -10$$
$$3x > -12$$
$$x > -4$$

b)
$$5y+1 < 1$$
$$5y < 0$$
$$y < 0$$

c)
$$7m+3 \le 6m+2$$
$$m+3 \le 2$$
$$m \le -1$$

d)
$$3z-8 > 2z+3$$
$$z-8 > 3$$
$$z > 11$$

e)
$$9+5b < 6b+1$$
$$9-b < 1$$
$$-b < -8$$
$$b > 8$$

f)
$$4(2q-1) > 4$$
$$2q-1 > 1$$
$$2q > 2$$
$$q > 1$$

g)
$$3(2b-2) \le 6$$
$$2b-2 \le 2$$
$$2b \le 4$$
$$b \le 2$$

h)
$$2(4-n) \ge 0$$
$$4-n \ge 0$$
$$-n \ge -4$$
$$n \le 4$$

Section Review Page 89 Question 30

a)
$$5m+4 \le 3m+10$$
$$2m+4 \le 10$$
$$2m \le 6$$
$$m \le 3$$

b)
$$w+2 > 6w-8$$
$$-5w > -10$$
$$w < 2$$

c)
$$2(x-7) < x+3$$
$$2x-14 < x+3$$
$$x-14 < 3$$
$$x < 17$$

d)
$$4(3z-1) \le 2(5-z)$$
$$12z-4 \le 10-2z$$
$$14z-4 \le 10$$
$$14z \le 14$$
$$z \le 1$$

e)
$$\begin{aligned} 2(y-3)+1 &\geq -4(2-y)+7 \\ 2y-6+1 &\geq -8+4y+7 \\ 2y-5 &\geq 4y-1 \\ -2y-5 &\geq -1 \\ -2y &\geq 4 \\ y &\leq -2 \end{aligned}$$

f)
$$\begin{aligned} 5n-2(n+3)-1 &< 2(n-5)+6 \\ 5n-2n-6-1 &< 2n-10+6 \\ 3n-7 &< 2n-4 \\ n-7 &< -4 \\ n &< 3 \end{aligned}$$

Section Review Page 89 Question 31

a)
$$\begin{aligned} \frac{x}{4}-3 &> -1 \\ x-12 &> -4 \\ x &> 8 \end{aligned}$$

b)
$$\begin{aligned} \frac{w}{5}+5 &\leq 2 \\ w+25 &\leq 10 \\ w &\leq -15 \end{aligned}$$

c)
$$\begin{aligned} 1.9m+2.4 &< 6.2 \\ 1.9m &< 3.8 \\ m &< 2 \end{aligned}$$

d)
$$\begin{aligned} 3.3-2.6p &\geq 8.5 \\ -2.6p &\geq 5.2 \\ p &\leq -2 \end{aligned}$$

e)
$$\begin{aligned} \frac{x+1}{2} &> \frac{5-x}{2}-2 \\ x+1 &> 5-x-4 \\ 2x+1 &> 1 \\ 2x &> 0 \\ x &> 0 \end{aligned}$$

f)
$$\begin{aligned} \frac{5-w}{4} &\geq \frac{5-w}{5} \\ 5(5-w) &\geq 4(5-w) \\ 25-5w &\geq 20-4w \\ 25-w &\geq 20 \\ -w &\geq -5 \\ w &\leq 5 \end{aligned}$$

g)
$$\begin{aligned} 1.4(y+3)+6.1 &> 2.5(1-y) \\ 1.4y+4.2+6.1 &> 2.5-2.5y \\ 1.4y+10.3 &> 2.5-2.5y \\ 3.9y+10.3 &> 2.5 \\ 3.9y &> -7.8 \\ y &> -2 \end{aligned}$$

h)
$$\begin{aligned} 3(1.2k+2)-12.6 &\leq 5(0.3k+0.8)-2.2 \\ 3.6k+6-12.6 &\leq 1.5k+4.0-2.2 \\ 3.6k-6.6 &\leq 1.5k+1.8 \\ 2.1k-6.6 &\leq 1.8 \\ 2.1k &\leq 8.4 \\ k &\leq 4 \end{aligned}$$

Section Review Page 89 Question 32

Let p represent the perimeter of the triangle.

$$\begin{aligned} p &= (2x+1)+(2x+3)+(2x-2) \\ &= 6x+2 \end{aligned}$$

a)
$$\begin{aligned} p &\geq 44 \\ 6x+2 &\geq 44 \\ 6x &\geq 42 \\ x &\geq 7 \end{aligned}$$

Values of x greater than or equal to 7 give a perimeter of 44 or more.

b)
$$\begin{aligned} p &< 56 \\ 6x+2 &< 56 \\ 6x &< 54 \\ x &< 9 \end{aligned}$$

Values of x less than 9 give a perimeter less than 56.

Section Review Page 89 Question 33

To find the amount raised, P, subtract the cost from the revenue. Let n represent the number of people.

$$\begin{aligned} P &> 10\,000 \\ 100n - (200 + 60n) &> 10\,000 \\ 100n - 200 - 60n &> 10\,000 \\ 40n - 200 &> 10\,000 \\ 40n &> 10\,200 \\ n &> 255 \end{aligned}$$

A total of 255 or more tickets must be sold to raise more the \$10 000.

Chapter Test

Section Chapter Test Page 90 Question 1

a) $\dfrac{1}{(-5)^2} = \dfrac{1}{25}$

b) $\dfrac{5^2 - 5^1}{5^{-1}} = \dfrac{25 - 5}{\frac{1}{5}}$

$= \dfrac{20}{\frac{1}{5}}$

$= 100$

c) $\dfrac{3^{-1}}{3^{-2} + 3^0} = \dfrac{\frac{1}{3}}{\frac{1}{9} + 1}$

$= \dfrac{\frac{1}{3}}{\frac{10}{9}}$

$= \dfrac{9}{30}$

$= \dfrac{3}{10}$

Section Chapter Test Page 90 Question 2

a) $\left(\dfrac{s^{-2}}{t^3}\right)^{-3} = \dfrac{s^6}{t^{-9}}$

$= s^6 t^9$

b) $30a^4b^2 \div (-5ab) = \dfrac{30a^4b^2}{-5ab}$

$= -6a^3b$

c) $(-3a^2b^5)^2 = 9a^4b^{10}$

d) $\dfrac{10m^2n^{-2} \times 2m^{-1}n^4}{-4mn^{-3}} = \dfrac{20m^{2+(-1)}n^{-2+4}}{-4mn^{-3}}$

$= \dfrac{20mn^2}{-4mn^{-3}}$

$= -5n^5$

e) $\dfrac{(-4s^{-2}t^{-3})^{-2}}{-s^2t^{-1}} = \dfrac{(-4)^{-2} \times (s^{-2})^{-2} \times (t^{-3})^{-2}}{-s^2t^{-1}}$

$= \dfrac{1}{(-4)^2} \times \dfrac{s^4t^6}{-s^2t^{-1}}$

$= -\dfrac{s^4t^6}{16s^2t^{-1}}$

$= -\dfrac{s^2t^7}{16}$

Section Chapter Test Page 90 Question 3

a)
$$\begin{aligned} -100^{-\frac{3}{2}} &= -\frac{1}{100^{\frac{3}{2}}} \\ &= -\frac{1}{\left(\sqrt{100}\right)^3} \\ &= -\frac{1}{10^3} \\ &= -\frac{1}{1000} \end{aligned}$$

b)
$$\begin{aligned} 81^{-\frac{3}{4}} &= \frac{1}{81^{\frac{3}{4}}} \\ &= \frac{1}{\left(\sqrt[4]{81}\right)^3} \\ &= \frac{1}{3^3} \\ &= \frac{1}{27} \end{aligned}$$

c)
$$\begin{aligned} \left(\frac{8}{-27}\right)^{-\frac{2}{3}} &= \left(\left(\frac{8}{-27}\right)^{\frac{1}{3}}\right)^{-2} \\ &= \left(\frac{\sqrt[3]{8}}{\sqrt[3]{-27}}\right)^{-2} \\ &= \left(\frac{2}{-3}\right)^{-2} \\ &= \frac{1}{\left(\frac{2}{-3}\right)^2} \\ &= \frac{1}{\frac{4}{9}} \\ &= \frac{9}{4} \end{aligned}$$

Section Chapter Test Page 90 Question 4

Given $(w, l) = (3 + \sqrt{2}, 3 - \sqrt{2})$, notice that the measurements are conjugate binomials.

$$\begin{aligned} A &= w \times l \\ &= (3 + \sqrt{2})(3 - \sqrt{2}) \\ &= 3^2 - (\sqrt{2})^2 \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

The area of the rectangle is 7 square units.

Section Chapter Test Page 90 Question 5

a)
$$\begin{aligned} (-3)^x &= 81 \\ (-3)^x &= (-3)^4 \\ x &= 4 \end{aligned}$$

Check $x = 4$.

$$\begin{aligned} \textbf{L.S.} &= (-3)^4 \\ &= 81 \\ &= \textbf{R.S.} \end{aligned}$$

b)
$$\begin{aligned} 2^{x-3} &= 64 \\ 2^{x-3} &= 2^6 \\ x - 3 &= 6 \\ x &= 9 \end{aligned}$$

Check $x = 9$.

$$\begin{aligned} \textbf{L.S.} &= 2^{9-3} \\ &= 2^6 \\ &= 64 \\ &= \textbf{R.S.} \end{aligned}$$

c)
$$\begin{aligned} -5^{x+2} &= -1 \\ 5^{x+2} &= 5^0 \\ x + 2 &= 0 \\ x &= -2 \end{aligned}$$

Check $x = -2$.

$$\begin{aligned} \textbf{L.S.} &= -5^{-2+2} \\ &= -5^0 \\ &= -1 \\ &= \textbf{R.S.} \end{aligned}$$

d)

$$3^{2y-3} = 9$$
$$3^{2y-3} = 3^2$$
$$2y - 3 = 2$$
$$2y = 5$$
$$y = \frac{5}{2}$$

Check $y = \frac{5}{2}$.

$$\text{L.S.} = 3^{2(\frac{5}{2})-3}$$
$$= 3^{5-3}$$
$$= 3^2$$
$$= 9$$
$$= \text{R.S.}$$

e)

$$2^{3x+2} = \frac{1}{16}$$
$$2^{3x+2} = 2^{-4}$$
$$3x + 2 = -4$$
$$3x = -6$$
$$x = -2$$

Check $x = -2$.

$$\text{L.S.} = 2^{3(-2)+2}$$
$$= 2^{-4}$$
$$= \frac{1}{16}$$
$$= \text{R.S.}$$

f)

$$4\left(6^{g+2}\right) = 144$$
$$6^{g+2} = 36$$
$$6^{g+2} = 6^2$$
$$g + 2 = 2$$
$$g = 0$$

Check $g = 0$.

$$\text{L.S.} = 4\left(6^{0+2}\right)$$
$$= 4(36)$$
$$= 144$$
$$= \text{R.S.}$$

Section Chapter Test Page 90 Question 6

a)

$$3^{x-2} = 3^{2x+1}$$
$$x - 2 = 2x + 1$$
$$x = -3$$

Check $x = -3$.

$$\text{L.S.} = 3^{-3-2}$$
$$= 3^{-5}$$
$$\text{R.S.} = 3^{2(-3)+1}$$
$$= 3^{-5}$$
$$= \text{L.S.}$$

b)

$$2^{x+2} = 4^{x+3}$$
$$2^{x+2} = 2^{2x+6}$$
$$x + 2 = 2x + 6$$
$$x = -4$$

Check $x = -4$.

$$\text{L.S.} = 2^{-4+2}$$
$$= 2^{-2}$$
$$\text{R.S.} = 4^{-4+3}$$
$$= 4^{-1}$$
$$= 2^{-2}$$
$$= \text{L.S.}$$

c)

$$5^{4x+2} = 25^{x-1}$$
$$5^{4x+2} = 5^{2x-2}$$
$$4x + 2 = 2x - 2$$
$$2x = -4$$
$$x = -2$$

Check $x = -2$.

$$\text{L.S.} = 5^{4(-2)+2}$$
$$= 5^{-6}$$
$$\text{R.S.} = 25^{-2-1}$$
$$= 25^{-3}$$
$$= 5^{-6}$$
$$= \text{L.S.}$$

d)

$$6^{x+2} + 6^x = 222$$
$$6^x(6^2 + 1) = 222$$
$$6^x(37) = 222$$
$$6^x = 6^1$$
$$x = 1$$

Check $x = 1$.

$$\text{L.S.} = 6^{1+2} + 6^1$$
$$= 216 + 6$$
$$= 222$$
$$= \text{R.S.}$$

e)

$$2^{x+2} - 2^{x+3} = -64$$
$$2^x(2^2 - 2^3) = -64$$
$$2^x(-4) = -64$$
$$2^x = 16$$
$$x = 4$$

Check $x = 4$.

$$\text{L.S.} = 2^{4+2} - 2^{4+3}$$
$$= 2^6 - 2^7$$
$$= 64 - 128$$
$$= -64$$
$$= \text{R.S.}$$

Section Chapter Test Page 90 Question 7

a)

$$(2x^2 + 3x - 7) + (7x^2 - 6x - 11) = 2x^2 + 3x - 7 + 7x^2 - 6x - 11$$
$$= 9x^2 - 3x - 18$$

b)

$$(4y^2 - 7y - 7) - (8y^2 + 5y - 9) = 4y^2 - 7y - 7 - 8y^2 - 5y + 9$$
$$= -4y^2 - 12y + 2$$

Section Chapter Test Page 90 Question 8

a) $$\begin{aligned} 3t(t-7)-2t(4t+5) &= 3t^2-21t-8t^2-10t \\ &= -5t^2-31t \end{aligned}$$

b) $$\begin{aligned} 4w(2w-3)-2w(w+5)-3w(2w-1) &= 8w^2-12w-2w^2-10w-6w^2+3w \\ &= -19w \end{aligned}$$

c) $$\begin{aligned} (x-5)(x+11) &= x^2+11x-5x-55 \\ &= x^2+6x-55 \end{aligned}$$

d) $$\begin{aligned} 3(2x-y)(x-3y) &= 3(2x^2-7xy+3y^2) \\ &= 6x^2-21xy+9y^2 \end{aligned}$$

e) $$\begin{aligned} -2(2s+3t)^2 &= -2(4s^2+12st+9t^2) \\ &= -8s^2-24st-18t^2 \end{aligned}$$

f) $$\begin{aligned} 2(x-3)^2-(2x+1)(3x+2) &= 2(x^2-6x+9)-(6x^2+7x+2) \\ &= 2x^2-12x+18-6x^2-7x-2 \\ &= -4x^2-19x+16 \end{aligned}$$

g) $$\begin{aligned} 3(2x-3y)(2x+3y)-(x-y)(3x+y) &= 3(4x^2-9y^2)-(3x^2-2xy-y^2) \\ &= 12x^2-27y^2-3x^2+2xy+y^2 \\ &= 9x^2+2xy-26y^2 \end{aligned}$$

Section Chapter Test Page 90 Question 9

a) $$\begin{aligned} \frac{3x-3y}{5x-5y} &= \frac{3(x-y)}{5(x-y)} \\ &= \frac{3}{5},\ x \neq y \end{aligned}$$

b) $$\begin{aligned} \frac{2y^2+4y}{3y^2+6y} &= \frac{2y(y+2)}{3y(y+2)} \\ &= \frac{2}{3},\ y \neq -2, 0 \end{aligned}$$

c) $$\begin{aligned} \frac{t^2-16}{t^2-t-12} &= \frac{(t-4)(t+4)}{(t-4)(t+3)} \\ &= \frac{t+4}{t+3},\ t \neq -3, 4 \end{aligned}$$

d) $$\begin{aligned} \frac{2m^2+m-3}{3m^2+2m-5} &= \frac{(2m+3)(m-1)}{(3m+5)(m-1)} \\ &= \frac{2m+3}{3m+5},\ m \neq -\frac{5}{3}, 1 \end{aligned}$$

Section Chapter Test Page 91 Question 10

a) $$\begin{aligned} \frac{x^2+2x-3}{x^2+6x+8} \times \frac{x^2+2x-8}{x^2+x-6} &= \frac{(x+3)(x-1)}{(x+2)(x+4)} \times \frac{(x+4)(x-2)}{(x+3)(x-2)} \\ &= \frac{(x+3)(x-1)(x+4)(x-2)}{(x+2)(x+4)(x+3)(x-2)} \\ &= \frac{x-1}{x+2},\ x \neq -4, -3, \pm 2 \end{aligned}$$

b) $$\begin{aligned} \frac{2a^2-a-1}{3a^2+a-2} \div \frac{2a^2-3a-2}{3a^2-11a+6} &= \frac{2a^2-a-1}{3a^2+a-2} \times \frac{3a^2-11a+6}{2a^2-3a-2} \\ &= \frac{(2a+1)(a-1)}{(3a-2)(a+1)} \times \frac{(3a-2)(a-3)}{(2a+1)(a-2)} \\ &= \frac{(2a+1)(a-1)(3a-2)(a-3)}{(2a+1)(a-2)(3a-2)(a+1)} \\ &= \frac{(a-1)(a-3)}{(a-2)(a+1)},\ a \neq -1, -\frac{1}{2}, \frac{2}{3}, 2, 3 \end{aligned}$$

c)
$$\begin{aligned}\frac{n+2}{3}+\frac{2n-1}{4}-\frac{3n+1}{2} &= \frac{4}{4}\times\frac{n+2}{3}+\frac{3}{3}\times\frac{2n-1}{4}-\frac{6}{6}\times\frac{3n+1}{2}\\ &= \frac{4n+8}{12}+\frac{6n-3}{12}-\frac{18n+6}{12}\\ &= \frac{4n+8+6n-3-18n-6}{12}\\ &= \frac{-8n-1}{12}\end{aligned}$$

d)
$$\begin{aligned}\frac{4}{2x-3}-\frac{1}{3-2x} &= \frac{4}{2x-3}-\frac{-1}{2x-3}\\ &= \frac{5}{2x-3},\ x\neq\frac{3}{2}\end{aligned}$$

e)
$$\begin{aligned}\frac{2}{x^2+5x+4}-\frac{3}{x^2-3x-4} &= \frac{2}{(x+4)(x+1)}-\frac{3}{(x-4)(x+1)}\\ &= \frac{x-4}{x-4}\times\frac{2}{(x+4)(x+1)}-\frac{x+4}{x+4}\times\frac{3}{(x-4)(x+1)}\\ &= \frac{2(x-4)}{(x+4)(x-4)(x+1)}-\frac{3(x+4)}{(x+4)(x-4)(x+1)}\\ &= \frac{2(x-4)-3(x+4)}{(x+4)(x-4)(x+1)}\\ &= \frac{2x-8-3x-12}{(x+4)(x-4)(x+1)}\\ &= \frac{-x-20}{(x+4)(x-4)(x+1)},\ x\neq-1,\pm4\end{aligned}$$

Section Chapter Test Page 91 Question 11

a)
$$\begin{aligned}2z+5 &\geq z-3\\ z+5 &\geq -3\\ z &\geq -8\end{aligned}$$

b)
$$\begin{aligned}3(x+2) &> -1(x-2)\\ 3x+6 &> -x+2\\ 4x+6 &> 2\\ 4x &> -4\\ x &> -1\end{aligned}$$

c)
$$\begin{aligned}3(3z+1) &\leq -2(9-z)\\ 9z+3 &\leq -18+2z\\ 7z+3 &\leq -18\\ 7z &\leq -21\\ z &\leq -3\end{aligned}$$

d)
$$\begin{aligned}3(y-1)+10 &\geq -5(2-y)-7\\ 3y-3+10 &\geq -10+5y-7\\ 3y+7 &\geq 5y-17\\ -2y+7 &\geq -17\\ -2y &\geq -24\\ y &\leq 12\end{aligned}$$

e)
$$\frac{h-5}{3}+4>\frac{h}{2}+1$$
$$2(h-5)+6(4)>3h+6(1)$$
$$2h-10+24>3h+6$$
$$2h+14>3h+6$$
$$-h>-8$$
$$h<8$$

f)
$$2.7(y-2)<3(0.2y+2.1)-1.2$$
$$2.7y-5.4<0.6y+6.3-1.2$$
$$2.7y-5.4<0.6y+5.1$$
$$2.1y-5.4<5.1$$
$$2.1y<10.5$$
$$y<5$$

Section Chapter Test Page 91 Question 12

a) The decay of Cobalt-60 is represented by the equation, $A_L = A_0\left(\frac{1}{2}\right)^{\frac{t}{5.3}}$. After 10.6 years,

$$A_L = A_0\left(\frac{1}{2}\right)^{\frac{10.6}{5.3}}$$
$$= A_0\left(\frac{1}{2}\right)^{2}$$
$$= \frac{1}{4}A_0$$

There will be $\frac{1}{4}$ of the original sample remaining after 10.6 years.

b) Let t be the time taken for the sample to decay to 12.5%.

$$\left(\frac{1}{2}\right)^{\frac{t}{5.3}} = 0.125$$
$$\left(\frac{1}{2}\right)^{\frac{t}{5.3}} = \frac{1}{8}$$
$$\left(\frac{1}{2}\right)^{\frac{t}{5.3}} = \left(\frac{1}{2}\right)^{3}$$
$$\frac{t}{5.3} = 3$$
$$t = 15.9$$

It will take 15.9 years until there is only 12.5% of the original sample remaining.

Section Chapter Test Page 91 Question 13

a) Let P represent the perimeter.

$$P = 2\left(\frac{3x+1}{2}+\frac{2x-1}{3}\right)$$
$$= 2\left(\frac{9x+3}{6}+\frac{4x-2}{6}\right)$$
$$= \frac{13x+1}{3} \quad (1)$$

The perimeter is $P = \frac{13x+1}{3}$ units.

b) The three smallest values for a are 2, 5, and 8. The substitutions define the smallest values of the numerator of (1) that are multiples of 3.

Challenge Problems

Section Challenge Problems Page 92 Question 1

$$\begin{aligned}\sqrt{\sqrt{\sqrt{x}}} &= \left(\left((x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} \\ &= x^{\frac{1}{8}} \\ &= \sqrt[8]{x}\end{aligned}$$

The correct answer is **c)**.

Section Challenge Problems Page 92 Question 2

$$\begin{aligned}n^{200} &< 5^{300} \\ \left(n^{200}\right)^{\frac{1}{200}} &= \left(5^{300}\right)^{\frac{1}{200}} \\ n &< 5^{\frac{3}{2}} \qquad (1)\end{aligned}$$

Since $5^{\frac{3}{2}} \doteq 11.18$, the largest integer satisfying (1) is 11. The correct answer is **d)**.

Section Challenge Problems Page 92 Question 3

$$\begin{aligned}a^b x^b &= (3a)^{3b} \\ (ax)^b &= \left((3a)^3\right)^b \\ ax &= (3a)^3 \\ ax &= 27a^3 \\ x &= 27a^2\end{aligned}$$

The correct answer is **c)**.

Section Challenge Problems Page 92 Question 4

To avoid division by zero, $x \neq 2$.

Consider real values for $2 - x < 0$ or $x > 2$.

$$\begin{aligned}\frac{3}{2-x} &\leq 1 \\ 3 &\geq 2 - x \\ x &\geq -1\end{aligned}$$

Discard the values in the interval $-1 \leq x \leq 2$ because of the initial assumption $x > 2$. Hence $x > 2$.

Consider real values for $2 - x > 0$ or $x < 2$.

$$\begin{aligned}\frac{3}{2-x} &\leq 1 \\ 3 &\leq 2 - x \\ x &\leq -1\end{aligned}$$

Hence $x \leq -1$.

The correct answer is **d)**.

Section Challenge Problems Page 92 Question 5

$$\begin{aligned}60^a &= 3 \qquad (1) \\ 60^b &= 5 \qquad (2) \\ (12 \cdot 5)^b &= 5 \\ 12^b 5^b &= 5 \\ 12^b &= 5^{1-b} \\ 12 &= 5^{\frac{1-b}{b}} \qquad (3)\end{aligned}$$

Prepare the given expression for the above substitutions.

$$\begin{aligned}12^{\frac{1-a-b}{2(1-b)}} &= \left(12^{\frac{1-b-a}{1-b}}\right)^{\frac{1}{2}} \\ &= \left(12^{1-\frac{a}{1-b}}\right)^{\frac{1}{2}} \\ &= \left(\frac{12}{12^{\frac{a}{1-b}}}\right)^{\frac{1}{2}} \qquad (4)\end{aligned}$$

Substitute (3) into (4).

$$\begin{aligned}&= \left(\frac{12}{\left(5^{\frac{1-b}{b}}\right)^{\frac{a}{1-b}}}\right)^{\frac{1}{2}} \\ &= \left(\frac{12}{5^{\frac{a}{b}}}\right)^{\frac{1}{2}} \qquad (5)\end{aligned}$$

Substitute (2) into (5).

$$\begin{aligned}&= \left(\frac{12}{\left(60^b\right)^{\frac{a}{b}}}\right)^{\frac{1}{2}} \qquad (5) \\ &= \left(\frac{12}{60^a}\right)^{\frac{1}{2}} \qquad (6)\end{aligned}$$

Substitute (1) into (6).

$$\begin{aligned}&= \left(\frac{12}{3}\right)^{\frac{1}{2}} \\ &= 4^{\frac{1}{2}} \\ &= 2\end{aligned}$$

The correct answer is **b)**.

Section Challenge Problems Page 92 Question 6

Let t represent the number of 94% tests.

$$\begin{aligned}\frac{94t + 4(84)}{t + 4} &= 90\\ 94t + 336 &= 90(t + 4)\\ 94t + 336 &= 90t + 360\\ 4t &= 24\\ t &= 6\end{aligned}$$

You need 6 more tests at 94% to bring your average up to 90%.

Section Challenge Problems Page 92 Question 7

$$\begin{aligned}\frac{1}{\frac{1}{x} - 1} &= -2\\ \frac{1}{x} - 1 &= -\frac{1}{2}\\ \frac{1}{x} &= \frac{1}{2}\\ x &= 2\end{aligned}$$

Section Challenge Problems Page 92 Question 8

$$\begin{aligned}\frac{1}{x} + \frac{x}{y} + \frac{1}{xy} &= 1\\ \frac{y}{xy} + \frac{x^2}{xy} + \frac{1}{xy} &= 1\\ \frac{x^2 + y + 1}{xy} &= 1\\ x^2 + y + 1 &= xy\\ xy - y &= x^2 + 1\\ y(x - 1) &= x^2 + 1\\ y &= \frac{x^2 + 1}{x^2 - 1}\\ &= x + 1 + \frac{2}{x - 1}\end{aligned}$$

Because of the third term, every x-value greater than 3 yields a fractional y-value. Thus, the only solutions are $(2, 5)$ and $(3, 5)$.

Section Challenge Problems Page 92 Question 9

$$\begin{aligned}\left(x^2yz^3\right)\left(xy^2\right) &= \left(7^3\right)\left(7^9\right)\\ x^3y^3z^3 &= 7^{12}\\ (xyz)^3 &= \left(7^4\right)^3\\ xyz &= 7^4\\ &= 2401\end{aligned}$$

Using the Strategies

Section Problem Solving Page 95 Question 1

January 1	Number of Mondays	January 31
Sunday	5	Tuesday
Monday	5	Wednesday
Tuesday	4	Thursday
Wednesday	4	Friday
Thursday	4	Saturday
Friday	4	Sunday
Saturday	5	Monday

A review of the above table shows that January 31 does not fall on Monday, Tuesday, or Wednesday if the month has 4 Mondays.

Section Problem Solving Page 95 Question 2

Let n be the number of rows of the square. Thus, there are n^2 members of the band. If the number of rows is increased by 5, the number of columns, c, can be expressed as

$$c = \frac{n^2}{n+5} \tag{1}$$

Sampling natural numbers for n until a natural number for c is found yields $n = 20$. There are 20×20 or 400 members in the band.

Section Problem Solving Page 95 Question 3

Each term in the sequence is found by squaring the units digit of the previous number and adding its tens digit. The sequence can be continued.

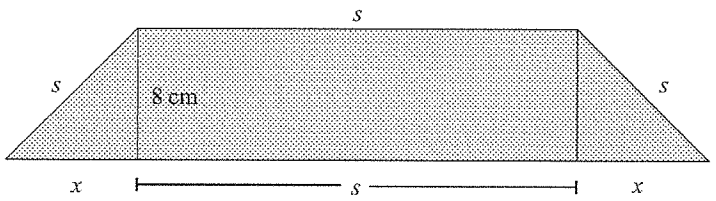
$$15, 26, 38, 67, 55, 30, 3, 9, 81, 9, 81 \ldots$$

The terms 9 and 81 repeat.

Section Problem Solving Page 95 Question 4

Let s be the length of the equal sides. A partition of the trapezoid consisting of a rectangle and two right triangles yields a system of equations.

$$x^2 + 8^2 = s^2 \tag{1}$$
$$2x + s = 3s - 2 \tag{2}$$



Solve for x in (2).

$$\begin{aligned} 2x + s &= 3s - 2 \\ 2x &= 2s - 2 \\ x &= s - 1 \end{aligned} \tag{3}$$

Substitute (3) into (1) to determine s.

$$\begin{aligned} (s-1)^2 + 8^2 &= s^2 \\ s^2 - 2s + 1 + 64 &= s^2 \\ 2s &= 65 \\ s &= \frac{65}{2} \end{aligned} \tag{4}$$

The formula for the area, A, of a trapezoid with sides s_1 and s_2 and height h is given by

$$A = \frac{h}{2}(s_1 + s_2) \tag{5}$$

From the given information, (5) can be simplified.

$$\begin{aligned} A &= \frac{8}{2}(s + 3s - 2) \\ &= 4(4s - 2) \\ &= 16s - 8 \end{aligned} \qquad (6)$$

Substitute (4) into (6).

$$\begin{aligned} A &= 16\left(\frac{65}{2}\right) - 8 \\ &= 8(65) - 8 \\ &= 512 \end{aligned}$$

The area of the trapezoid is 512 cm^2.

Section Problem Solving Page 95 Question 5

An examination of the powers of 6317 reveals a pattern in the units digits.

$$6317^1 = 6317$$
$$6317^2 = 39\,904\,489$$

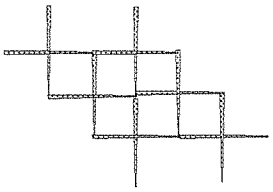
Continuing to multiply the units digit by 7 yields the units sequence 7, 9, 3, 1, 7, 9, ... This is referred to as a modulo 4 sequence. The units digit, d, of 6317^n can be expressed as

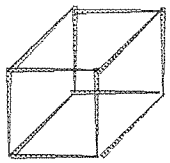
$$d = \begin{cases} 7 & \text{if } n \bmod 4 \equiv 1 \\ 9 & \text{if } n \bmod 4 \equiv 2 \\ 3 & \text{if } n \bmod 4 \equiv 3 \\ 1 & \text{if } n \bmod 4 \equiv 0 \end{cases}$$

where n mod 4 is the remainder when n is divided by 4. Since 458 mod 4 $\equiv$ 2, the units digit of 6317^{458} is 9.

Section Problem Solving Page 95 Question 6

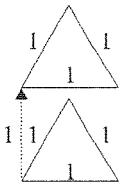
Two possible diagrams are presented below.





Section Problem Solving Page 95 Question 7

The points are the vertices of two equilateral triangles of side length 1 unit. The second triangle is a translation of 1 unit upward of the first triangle.



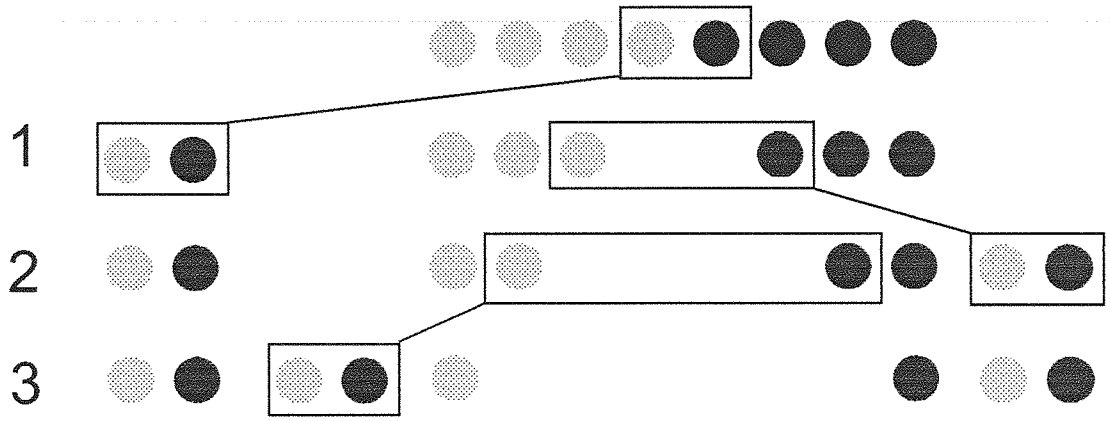
Section Problem Solving Page 95 Question 8

If digits D and F are 1 and 9 respectively, E can be any other single digit and the sum is true.

$$\begin{array}{r} 1 \\ E \\ +\ 9 \\ \hline 1E \end{array}$$

Section Problem Solving Page 95 Question 9

Answers may vary. One solution involving 3 moves is depicted below.



Section Problem Solving Page 95 Question 10

Answers may vary. Assumptions:

- There are 2 000 000 cars in Ontario.
- Each car travels an average of 50 km per week.
- The average car gets 20 km/L of gasoline.

The number of litres, l, of gasoline consumed in a week in Ontario is determined as follows.

$$l = \frac{2\,000\,000 \times 50}{20}$$
$$= 5\,000\,000$$

Approximately 5 000 000 L of gasoline are consumed in Ontario per week.

Section Problem Solving Page 95 Question 11

Number the given system as on the right. First, determine a system of equations involving E and F.

$$A + B = C \quad (1)$$
$$C + D = E \quad (2)$$
$$A + E = F \quad (3)$$
$$B + D + F = 20 \quad (4)$$
$$A = 4 \quad (5)$$

Substitute (5) into (3).

$$4 + E = F \quad (6)$$

Substitute (1) into (2) and solve for B+D.

$$A + B + D = E$$
$$B + D = E - A \quad (7)$$

Substitute (7) and (5) into (4).

$$E - A + F = 20$$
$$E + F = 24 \quad (8)$$

Finally, solving the system involving (6) and (8) yields E = 10 and F = 14.

Section Problem Solving Page 95 Question 12

The distances covered by each cyclist in a given time interval maintain the ratio

$$S : B : J = 36 : 24 : 18$$

where S, B, and J are Suzanne's, Beth's, and Jasmin's distances respectively. Set $B = 12$ to determine the distance Jasmin will cover until Beth crosses the finish line.

$$\begin{aligned} S : B : J &= 36 : 24 : 18 \\ S : 12 : J &= 36 : 24 : 18 \\ \frac{12}{24} &= \frac{J}{18} \\ J &= \frac{18 \times 12}{24} \\ &= 9 \end{aligned}$$

Jasmin will cover 9 km before Beth crosses the finish line, leaving her $18 - 9$ or 9 km from the finish line.