2.4 B Partial Factoring and the Discriminant

The Discriminant

Ex: 1 Solve each of the following equations using the Quadratic formula:

a) $x^2 + 2x + 1 = 0$

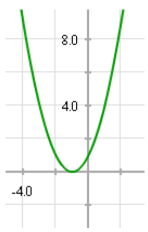
$x = \frac{-2 \pm \sqrt{4 - 4(1)(1)}}{2(1)}$

$= \frac{-2 \pm \sqrt{0}}{2}$ $= 0$

$= -1$

$\therefore$ One Sol^{n}

Picture it:



b) $2x^2 - 4x + 1 = 0$

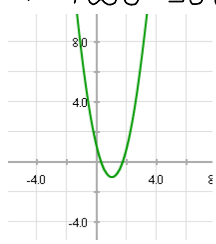
$x = \frac{4 \pm \sqrt{16 - 4(2)(1)}}{2(2)}$

$= \frac{4 \pm \sqrt{8}}{4}$

$= \frac{4 \pm 2\sqrt{2}}{4}$ > 0

$= \frac{2 \pm \sqrt{2}}{2}$

$\therefore$ Two Sol^{ns}

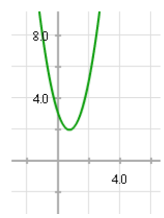


c) $2x^2 - 3x + 3 = 0$

$x = \frac{3 \pm \sqrt{9 - 4(2)(3)}}{2(2)}$

$= \frac{3 \pm \sqrt{-15}}{4}$ < 0

$\therefore$ NO SOL^{NS}



What do you notice about the number of roots?

We can determine the number of roots by looking under the radical sign

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is know as the Discriminant $b^2 - 4ac$

If $b^2 - 4ac > 0$ then there is two real roots

If $b^2 - 4ac = 0$ then there is one real root

If $b^2 - 4ac < 0$ then there is no real roots

Oct 7-9:37 AM

Ex 2: Determine the number of real solutions:

a) $2x^2 - x + 5 = 0$

$D = b^2 - 4ac$
$= (-1)^2 - 4(2)(5)$
$= -39$

$D < 0$ ∴ NO REAL ROOTS

b) $4(x + 1)^2 - 7 = 0$

Vertex Form

Opens up

Vertex below x-axis

∴ 2 real roots

c) $(x - 6)^2 = 0$

Vertex/Factored form

- perfect square
- vertex is on axis

∴ One sol$^{\underline{n}}$

d) $(x-3)(x+2) = 0$

Factored form

∴ 2 real roots

Ex 3:

For what values of k does $y = x^2 + kx + 9$ have 2 distinct real solutions?

$D > 0$

$k^2 - 4(1)(9) > 0$

$k^2 > 36$

$k > 6$ $k < -6$

Finding the Vertex by Partial Factoring

To find the vertex we could:
Complete the square (time consuming)
OR
Factor and we know the vertex falls halfway between the zeros
(sometimes we cannot factor a quadratic easily)

Let's look at another way to find the vertex:

PARTIAL factoring:

$$y = ax(x-s)+t$$

When you have these 2 points at the same height, you can use symmetry to find the vertex.

- one of your points is the y intercept!
- determines 2 points that are the same height (0,t) and (s,t)
- these points can then be used to determine the VERTEX
- Identify the axis of symmetry (halfway between 0 and s)
- Substitute the value of the axis of symmetry for x in the initial equation to find the y coordinate of the vertex.

These points are NOT the Zeros!!!

Ex:4 Use partial factoring to determine the vertex.

a) $y = 2x^2 + 10x + 1$

$= 2x(x+5) + 1$

$x = 0$ $x = -5$

values for which $y = 1$

We now have two symmetrical points.

Axis of symmetry

$x = \frac{0+(-5)}{2}$

$= \frac{-5}{2}$

Sub in to find vertex

$y = 2\left(-\frac{5}{2}\right)^2 + 10\left(-\frac{5}{2}\right) + 1$

$= \frac{25}{2} - 25 + 1$

$= \frac{25}{2} - \frac{50}{2} + \frac{2}{2}$

$= -\frac{23}{2}$

$\left(-\frac{5}{2}, -\frac{23}{2}\right)$

b) $y = -x^2 + 5x - 3$

$y = -x(x-5) - 3$

$x = 0$ $x = 5$

axis of symmetry

$x = \frac{0+5}{2}$

$= \frac{5}{2}$

find vertex

$y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) - 3$

$= -\frac{25}{4} + \frac{25}{2} - 3$

$= -\frac{25}{4} + \frac{50}{4} - \frac{12}{4}$

$= \frac{13}{4}$

$\therefore V\left(\frac{5}{2}, \frac{13}{4}\right)$

More Mind Map...

What can you tell from
standard form?
$y = ax^2 + bx + c$

- direction of opening and stretch a
- the y intercept c

Expand

Factor out ax from first two terms

What can you tell from
Partial Factored form?
$y = ax(x - s) + t$

- direction of opening and stretch a
- two points (0,t) and (s,t)

Can find:

- vertex (half way between s and 0)
- sub axis of symmetry to find y value of vertex.

$\boxed{\text{Hmwk: 2.4 B Handout}}$

Mar 4-8:16 AM