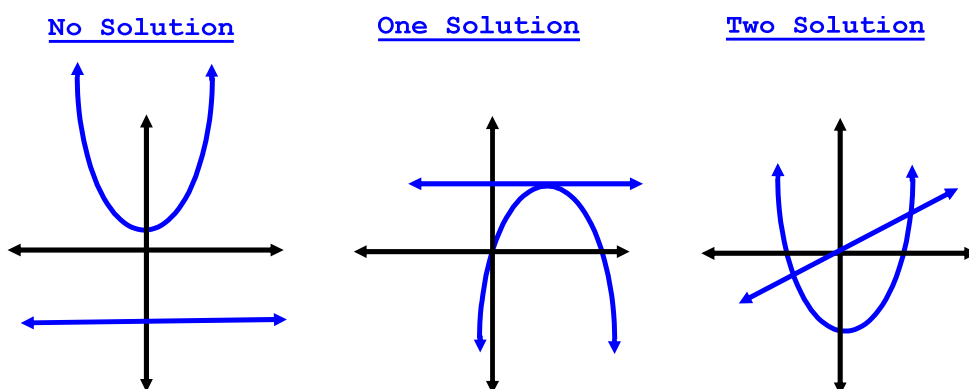


2.7 - Intersection of Lines and Quadratics

A **system of equations** consists of 2 or more equations. If the graphs are a straight line and quadratic (parabola), the system could have **no solution**, **one solution**, or **two solutions**.



Method for solving algebraically:

1. Isolate one variable from the linear equation.
2. Sub into the quadratic
3. Solve for the remaining variable.
4. Sub answer(s) back into the linear equation to find the coordinate(s) of intersection, if they exist.

Ex 1 - Solve the system.

① $y = x^2 - 3$

② $2x + y = -3$

③ $y = -3 - 2x$

Sub into ①

$$-3 - 2x = x^2 - 3$$

$$0 = x^2 + 2x$$

$$= x(x + 2)$$

$$\swarrow$$

$$x = 0$$

$$\searrow$$

$$x = -2$$

POIs

@ $x = 0$

$$y = -3 - 2(0)$$

$$y = -3$$

$$(0, -3)$$

@ $x = -2$

$$y = -3 - 2(-2)$$

$$= 1$$

$$(-2, 1)$$

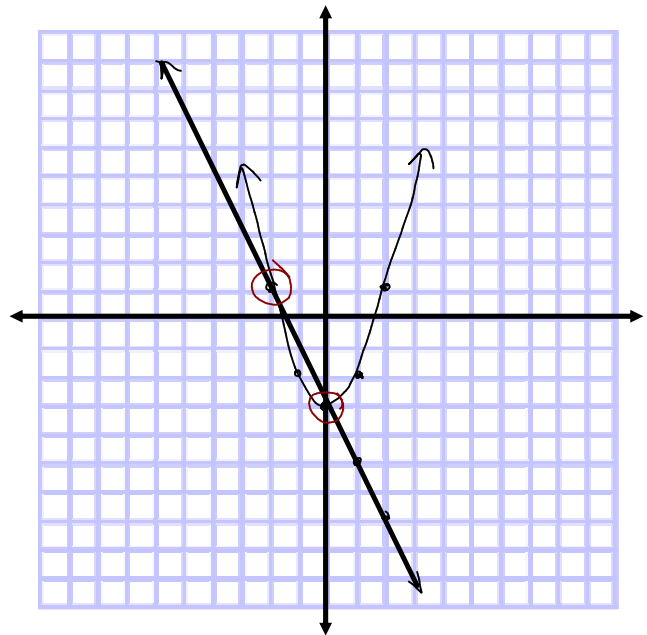
∴ The solutions are $(0, -3)$

$$\text{and } (-2, 1)$$

1. Isolate one variable from the linear equation.
2. Sub into the quadratic
3. Solve for the remaining variable.
4. Sub answer(s) back into the linear equation to find the coordinate(s) of intersection, if they exist.



Steps for solving algebraically



Ex 2 - Solve the system.

$$\textcircled{1} \quad x^2 - 64y^2 = 1$$

$$\textcircled{2} \quad x + 8y = 0$$

$$\textcircled{2} \quad x = -8y$$

Sub into $\textcircled{1}$

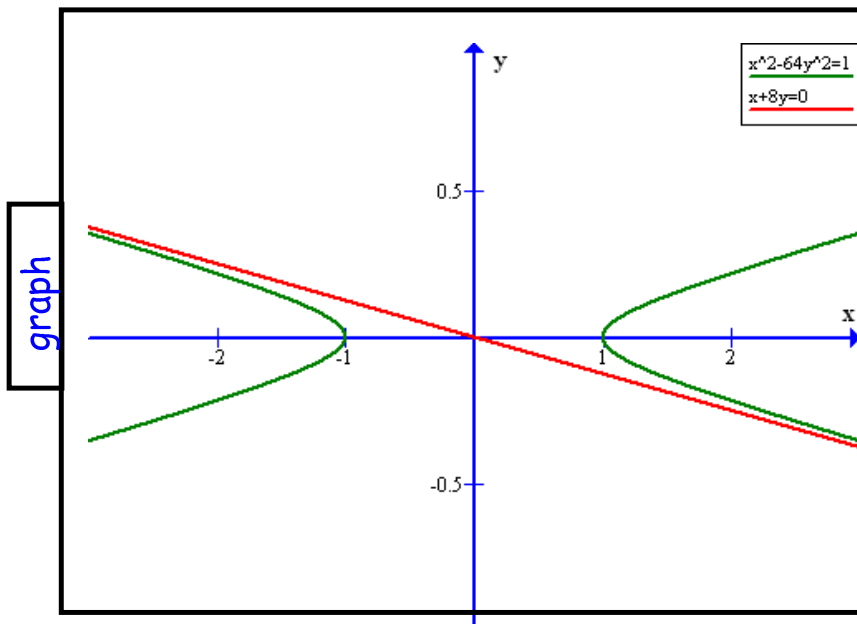
$$(-8y)^2 - 64y^2 = 1$$

$$64y^2 - 64y^2 = 1$$

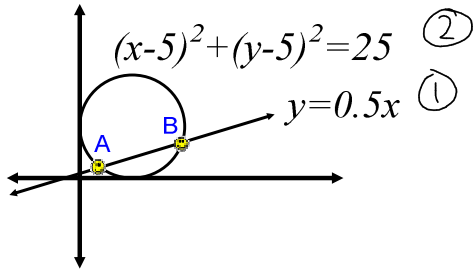
$$0 = 1$$

NOT POSSIBLE!!!

\therefore No SOLⁿ



Ex 3 - Find the length of the chord AB on the circle given the following information.
(round to two decimal places)



Sub (1) into (2)

$$\begin{aligned}(x-5)^2 + (0.5x-5)^2 &= 25 \\ x^2 - 10x + 25 + 0.25x^2 - 5x + 25 &= 25 \\ 1.25x^2 - 15x + 25 &= 0 \\ 1.25(x^2 - 12x + 20) &= 0 \\ 1.25(x-10)(x-2) &= 0 \\ \downarrow & \quad \quad \downarrow \\ x=10 & \quad \quad x=2\end{aligned}$$

POIs

Use (1) $y = 0.5x$

@ $x = 2$
 $y = 1$ (2, 1)

@ $x = 10$
 $y = 5$ (10, 5)

Distance of a line segment formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Length of chord

$$\begin{aligned}d &= \sqrt{(10-2)^2 + (5-1)^2} \\ &= \sqrt{64+16} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \\ &\approx 8.94\end{aligned}$$

\therefore The length of the chord is $4\sqrt{5}$

Practice - p. 684 #1a, 3fi, 7, 9, 13

