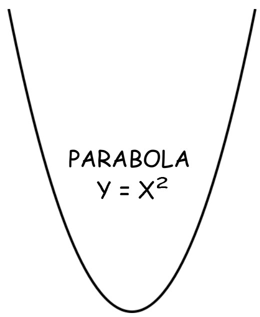
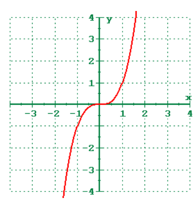
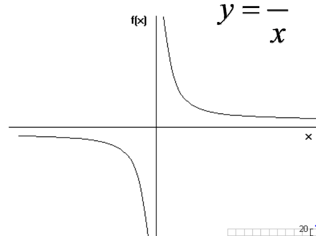
# Unit 3: Transformations of Functions



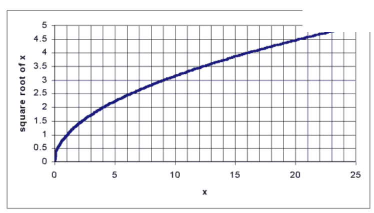

CUBIC
$Y = X^3$



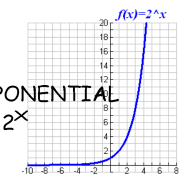
$y = \frac{1}{x}$

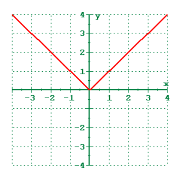


$y = \sqrt{x}$



EXPONENTIAL
$Y = 2^x$



$y = |x|$

**In This Unit**

- What functions are
- The "Base" Functions
- Translations
- Reflections
- Stretches
- Combinations of Transformations
- The Exponential Function

# 3.1 - Functions

## A. Relation vs. Function

Relation -> any set of ordered pairs (one # related to another)

ex. { (-3,4), (-2,1), (-2, 7), (5,-3) }

Function -> special type of relation
-> for every "x" value, there is only one y-value

*Parentheses { } are used to represent the "set" of something. A set is a collection of things.*

Ex. Which of the following relations are functions?

a)

| x | y |
|---|---|
| -3 | 1 |
| -2 | 4 |
| -1 | 5 |

FN

NO REPEAT Xs

b)

| x | y |
|---|---|
| -2 | 4 |
| -3 | 5 |
| -2 | 7 |

NOT FN

c) A = {(3,4),(2,-1),(5,-1),(6,4)}

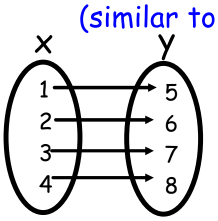
FN

d) B = {(2,2),(3,-4),(2,3),(4,-1)}

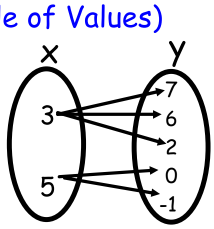
NOT FN

### Mapping Diagrams

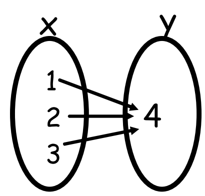
A different way of expressing a relation
(similar to a Table of Values)




one to one
is a function




one to many
not a function

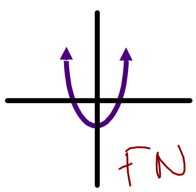



many to one
is a function

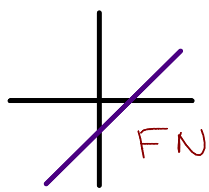
*How can we test if a relation*
*is a function when we are given a graph???*

The **VERTICAL LINE TEST** - if a relation is graphed, it is a function if a vertical line touches in no more the one place everywhere on the graph

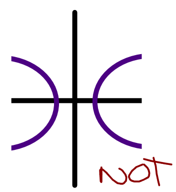
e)



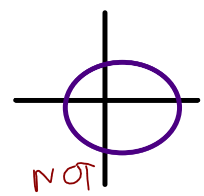

f)



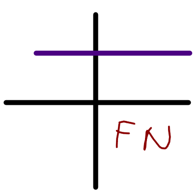

g)



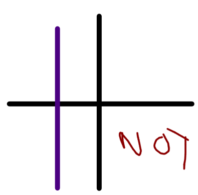

h)




i)




j)

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## *B. Finding the Domain and Range*

**Domain** - set of all "input" values (usually "x")
- values of x that can be used/make sense

**Range** - set of all "output" values (usually "y")
- values of y that are possible

We use **set notation** to describe the domain and range.

D = { } R = { }

Ex. Find the domain and range.

a) {(0,-3),(1,-4),(2,-3),(5,-1),(7,-4)}

$D = \{0, 1, 2, 5, 7\}$

$R = \{-4, -3, -1\}$

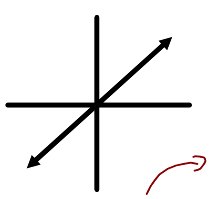
b)

| x | y |
|---|---|
| -3 | 0 |
| -2 | 1 |
| -1 | 0 |
| 0 | 1 |

$D = \{-3, -2, -1, 0\}$

$R = \{0, 1\}$

c)

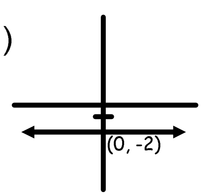


Element of Real

$D = \{x \in \mathbb{R}\}$

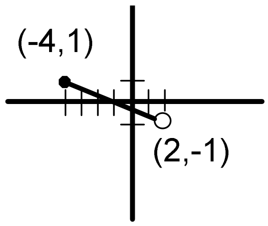
$R = \{y \in \mathbb{R}\}$

d)




$D = \{x \in \mathbb{R}\}$

$R = \{-2\}$

e) 


Dots: closed-value exists at that point
open: values exist up to BUT NOT including that point

## Reading math:

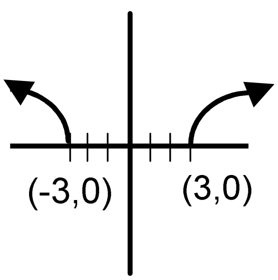
The Domain is equal to $D = \{\, x \in \mathbb{R} \;/\; -4 \leq x < 2 \,\}$

the set of

all x as an element of the Reals

such that

x is greater than or equal to negative four and less than 2

f) 


Now you try:

$R = \{\, y \in \mathbb{R} \;/\; y \geq 0 \,\}$

$D = \{\, x \in \mathbb{R} \;/\; x \leq -3 \text{ OR } x \geq 3 \,\}$

Union
"$\cup$"

$D = \{\, x \in \mathbb{R} \;/\; x \leq -3 \cup x \geq 3 \,\}$

Try picturing these first, then state the Domain and Range...

g) $y = 5x - 2$ line

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R}\}$

h) $y = -4(x-3)^2 + 2$

parabola
opens down
max of 2

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} / y \leq 2\}$

i) $x^2 + y^2 = 49$

- circle
- radius 7

$D = \{x \in \mathbb{R} / -7 \leq x \leq 7\}$

$R = \{y \in \mathbb{R} / -7 \leq y \leq 7\}$

Oct 12-9:52 AM

## C. Notation...

| Standard Notation | vs. | Function Notation |
|---|---|---|
| $y = x + 3$ | | $f(x) = x + 3$ |
| Solve for y when x = 1 | | Find $f(1)$ |
| $y = 1 + 3$<br>$y = 4$ | | $f(1) = 1 + 3$<br>$= 4$ |

## D. Function Values

1. If $f(x) = 3x^2 - 2x + 1$ , find $f(-1)$

$f(-1) = 3(-1)^2 - 2(-1) + 1$
$= 3 + 2 + 1$
$= 6$

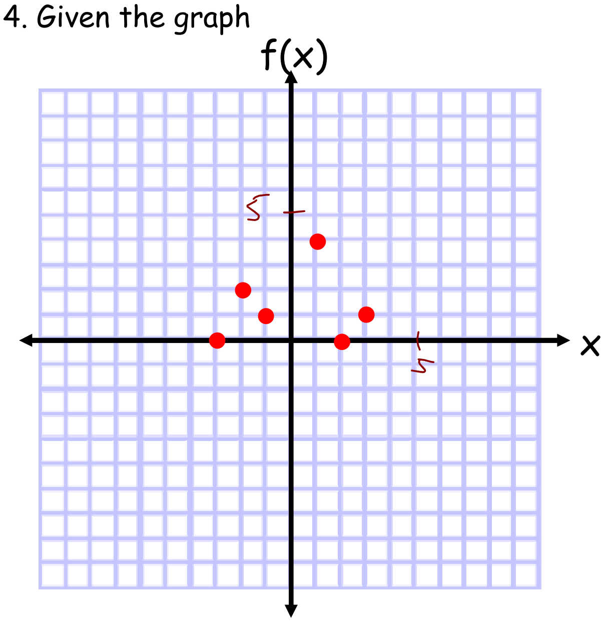
2. If $f(x) = -3x+2$ , find $x$ if $f(x) = 0$

$f(x) = -3x+2$
$0 = -3x+2$
$\frac{2}{3} = x$

3. If $f(x) = x^2 - 6x$ , find $x$ if $f(x) = 16$

$16 = x^2 - 6x$
$0 = x^2 - 6x - 16$ $\quad \therefore x = 8, -2$
$= (x-8)(x+2)$

4. Given the graph




Find:

f(1) = 4

f(-3) = 0

f(7) = DNE

5. If $f(x) = 2x^2-3x$

a) find $3f(-1)$

TWO STEPS

$f(-1) = 2(-1)^2-3(-1)$
$= 5$

$3f(-1) = 3(5)$
$= 15$

One Step

$3f(-1) = 3\left[2(-1)^2-3(-1)\right]$
$= 3(2+3)$
$= 3(5)$
$= 15$

b) $f(m+1)$

$f(m+1) = 2(m+1)^2 - 3(m+1)$
$= 2(m^2+2m+1) - 3m - 3$
$= 2m^2+4m+2-3m-3$
$= 2m^2+m-1$

c) $f(f(x))$

$f(f(x)) = 2(2x^2-3x)^2 - 3(2x^2-3x)$

$= 8x^4 - 24x^3 + 12x^2 + 9x$

Oct 18-2:18 PM

Practice

p. 178 #1-12 eoo, 15, 17, 18, 26