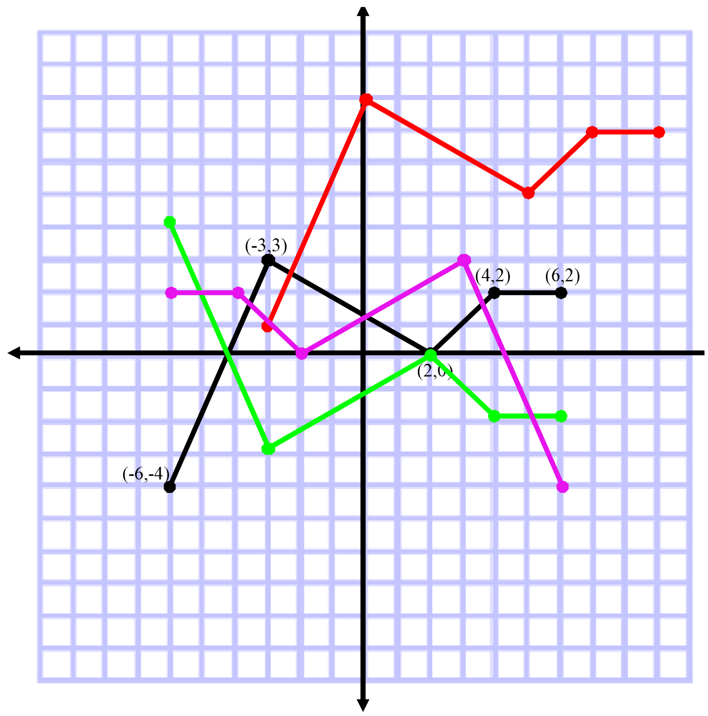
Warm-Up!

#1 Given y=f(x), graph

a) $y = f(x-3)+5$

b) $y = -f(x)$

c) $y = f(-x)$



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3.5 Inverse Functions

The INVERSE of a relation is the reverse of the original function.

NOTATION: If a function is called $f(x)$, then its' inverse function is called $f^{-1}(x)$.

Note: $f^{-1}(x)$ this is not an exponent!

Therefore $f^{-1}(x) \neq 1/f(x)$

There are three ways to find the inverse of a function.

1) given the coordinates, interchange x and y $(a,b) \Rightarrow (b,a)$
2) given the equation, interchange x and y and solve for y.
3) given the graph, reflect over y=x.

NOTE: Not all inverse relations are functions.
The domain of a function is the range of its inverse.
The range of a function is the domain of its inverse.

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Example 1:

Determine the inverse of the following function. State the domain and range for both the function and its inverse.

$f(x) = \{(1,2),(2,3),(3,6)\}$

D: $\{1, 2, 3\}$
R: $\{2, 3, 6\}$

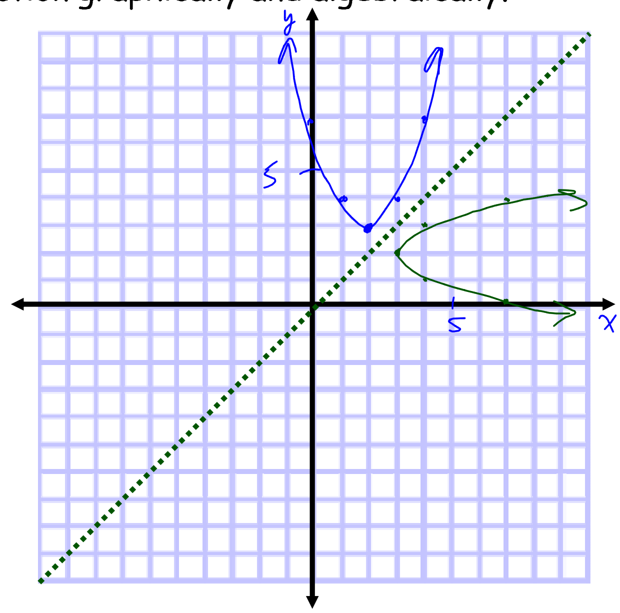
$f^{-1}(x) = \{(2,1),(3,2),(6,3)\}$

D: $\{2, 3, 6\}$
R: $\{1, 2, 3\}$

Example 2:

Determine the inverse of the following function graphically and algebraically.

$f(x) = (x-2)^2 + 3$



Alg

$y = (x-2)^2 + 3$

swap $y = x$, $x = y$

$x = (y-2)^2 + 3$

$\pm\sqrt{x-3} = y - 2$

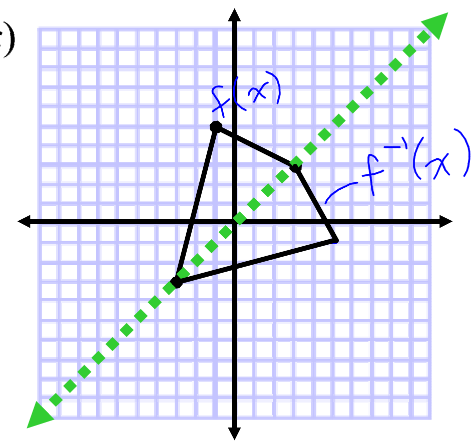
$y = 2 \pm \sqrt{x-3}$

$f^{-1}(x) = \pm\sqrt{x-3} + 2$

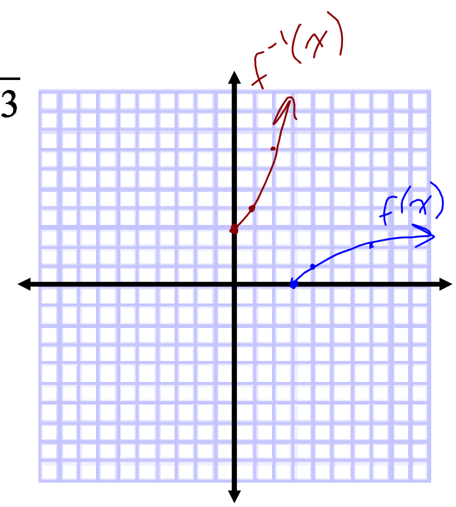
Example 3:

Graph $f^{-1}(x)$ given $f(x)$.

a) $f(x)$



b) $f(x)=\sqrt{x-3}$



points on the line
y=x are invariant!

Example 4:

Determine the equation of the inverse for each of the following functions.

a) $f(x) = \dfrac{1}{x+5}$

$y = \dfrac{1}{x+5}$

Swap

$x = \dfrac{1}{y+5}$

$x(y+5) = 1$

$y + 5 = \dfrac{1}{x}$

$y = \dfrac{1}{x} - 5$

$\therefore f^{-1}(x) = \dfrac{1}{x} - 5$

b) $f(x) = -3(x-4)^2 + 2$

$y = -3(x-4)^2 + 2$

INV

$x = -3(y-4)^2 + 2$

$\dfrac{x-2}{-3} = (y-4)^2$

$\dfrac{-(x-2)}{3} = (y-4)^2$

$\pm\sqrt{\dfrac{-(x-2)}{3}} = y - 4$

$y = \pm\sqrt{\dfrac{-(x-2)}{3}} + 4$

$\therefore f^{-1}(x) = \pm\sqrt{\dfrac{-(x-2)}{3}} + 4$

c) $f(x) = \sqrt{x-2}$

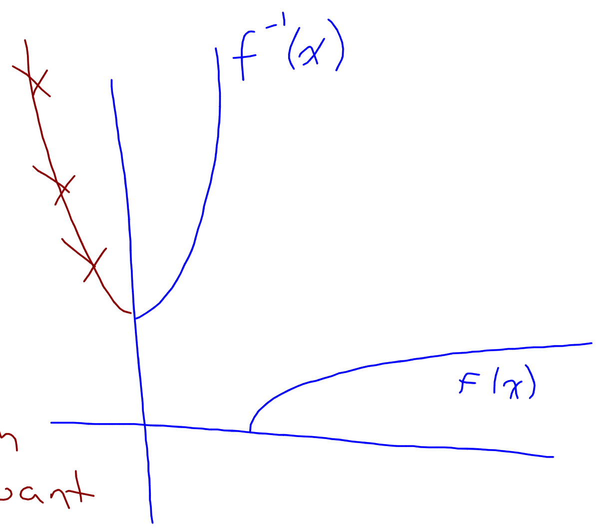
$y = \sqrt{x-2}$

Inv

$x = \sqrt{y-2}$

$x^2 = y - 2$

$y = x^2 + 2$



★ Restrict the domain to get the part you want

$f^{-1}(x) = x^2 + 2$,

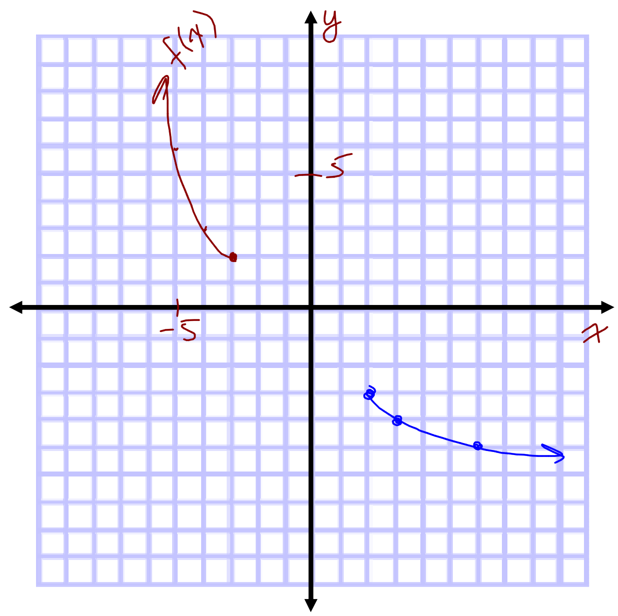
Restricting the Domain of $f(x)$:

Given $f(x) = (x+3)^2 + 2$,

a) restrict the domain of $f(x)$ so that $f^{-1}(x)$ is also a function.

b) graph $f(x)$ and $f^{-1}(x)$

When restricting the domain identify the vertex.



a) $f(x) = (x+3)^2 + 2 \, , \; x \le -3$

OR

$f(x) = (x+3)^2 + 2 \, , \; x \ge -3$

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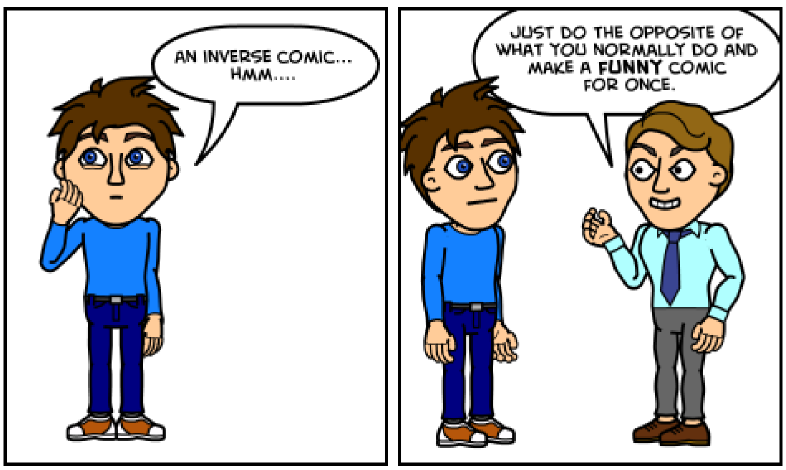
Homework:
p. 215 #2,3,5,10,13-16(eoo)



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