Name: Solutions.

Date: April 30, 2013

Content	Form	Total
52	5	%

MCR 3U - Unit 4 Test- Trigonometry

Unless directed otherwise, round lengths to one place after decimal and angles to nearest degree.

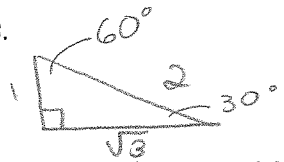
Part A: Fill in the blank with the simplified answer to each question. [19] [1 mark each]

1. State which trigonometric functions are positive in quadrant IV — cos x & sec x

2. State the reciprocal function of sine — csc x

3. Evaluate, to four decimal places. — cos 316° = 0.7193

4. Find the exact values of — tan 60° = $\sqrt{3}$



5. Evaluate angle θ to the nearest degree if $0° \le \theta \le 90°$, $\sin\theta = 0.7563$ — θ = 49°

6. Evaluate angle θ to the nearest degree if $180° \le \theta \le 270°$, $\tan\theta = \frac{1}{\sqrt{3}}$ — θ = 210°

7. State the following for angle 210° — a) the related acute angle: 30°

   b) the quadrant in which the terminal arm lies in: Q3

8. The point (-1,-1) lies on the terminal arm of θ. State the exact value of cos θ — $\frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

9. If $90° \le \theta \le 180°$ for which $\sin\theta = \frac{5}{13}$. Determine the exact value of $\tan\theta$ — $\frac{-5}{12}$

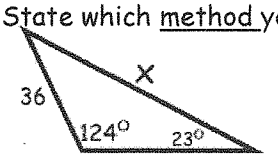
10. Determine the number of triangles that could be drawn with the given measures.

    a) ΔABC, where $\angle A = 30°$, a=18 m, b=24 m — a) 2

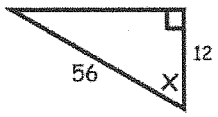
    b) ΔRST, where $\angle S = 52.5°$, s=30 cm, r=45 cm — b) 0

11. State which method you would use to solve for x

a)



b)



a) Sine law

b) SOH CAH TOA or $\cos\theta = \frac{adj}{hyp}$

12. Solve for x

a) $\tan x = \frac{13}{15}$ — a) ≐ 41°

b) $\sin 42° = \frac{13}{x}$ — b) ≐ 19.4

c) $x^2 = 3^2 + 5^2 - 2(3)(5)\cos 37°$ — c) ≐ 3.2

13. State an equivalent expression for each of the following:

a) $1-\sin^2\theta =$ $\cos^2\theta$

b) $\frac{\cos^2\theta}{\sin^2\theta} =$ $\cot^2\theta$

19

Part B: Write complete simplified solutions for each of the following questions. [33]

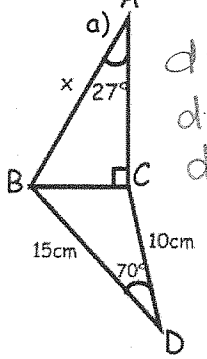
14. How do you decide when to use the sine law or the cosine law to solve a problem. [2]

Sine law → opposite pairs of angle/side length.

cosine law → 2 sides with contained angle → 3 sides.

15. Solve for the variable x [8]

a)



$d^2 = c^2 + b^2 - 2cd\cos D$

$d^2 = 15^2 + 10^2 - 2(15)(10)\cos 70°$

$d \doteq 14.9$

$< 27°$

$d \rightarrow$ opp

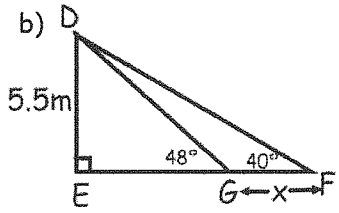
$x \rightarrow$ hyp

$\sin 27° = \frac{d}{x}$

$x = \frac{14.9}{\sin 27°}$

$\doteq 32.8\text{cm}$

b)



$< 48°$

$e \rightarrow$ hyp $5.5 \rightarrow$ opp

$\sin 48° = \frac{5.5}{e}$

$e = \frac{5.5}{\sin 48°}$

$\doteq 7.4$

$< DGF = 180° - 48°$ (SAT)

$= 132°$

$< GDF = 180° - 40° - 132°$ (CAST)

$= 8°$

$\frac{d}{\sin D} = \frac{f}{\sin F}$

$\frac{x}{\sin 8} = \frac{7.4}{\sin 40°}$

$x = \frac{\sin 8° \cdot 7.4}{\sin 40°}$

$\doteq 1.6\text{m}$

16. Find the exact simplified value of $\sin 225° \cos 315° + \tan 240°$ (show all work) [4]

$= \sin(180°+45°) \cdot \cos(360°-45°) + \tan(180°+60°)$

$= -\sin 45° \cdot \cos 45° + \tan 60°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + \sqrt{3}$

$= -\frac{2}{4} + \sqrt{3}$

$= \frac{-1+2\sqrt{3}}{2}$

17. If $\cos\theta = -\frac{6}{11}$ and $0° \le \theta \le 360°$, [4]

a) Determine θ.

$R\,\theta = \cos^{-1}\left(\frac{6}{11}\right)$

$\doteq 57°$

$Q2 \doteq 180 - 57$

$\doteq 123°$

$Q3 \doteq 180 + 57$

$\doteq 237°$

$\theta \doteq \{123°, 237°\}$

b) Determine the exact value of $\csc\theta$

$x = 6$, $r = 11$, $y = ?$

$x^2 + y^2 = r^2$ (P.T)

$(6)^2 + y^2 = (11)^2$

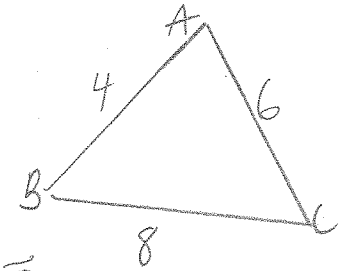
$y^2 = 121 - 36$

$y = \pm\sqrt{85}$

Q2 $\csc\theta = \frac{11}{\sqrt{85}} = \frac{11\sqrt{85}}{85}$

Q3 $\csc\theta = -\frac{11\sqrt{85}}{85}$

18. Find the measure of the smallest angle in a triangle with sides of 4 m, 6 m and 8 m. Draw the diagram.[3]



$\angle C = \cos^{-1}\left(\frac{6^2 + 8^2 - 4^2}{2(6)(8)}\right)$

$\doteq 29$

∴ the smallest angle is 29°

19. Prove the following identities [3,3]

a) $\tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$

$LS = \tan x + \frac{1}{\tan x}$

$= \frac{\sin x}{\cos x} + \frac{1}{\frac{\sin x}{\cos x}}$

$= \frac{\sin x}{\cos x} + 1 \cdot \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$= \frac{1}{\cos x \sin x}$

$\therefore LS = RS$

$\therefore$ QED

b) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \frac{\cot x - 1}{\cot x}$

$LS = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$

$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x (\cos x + \sin x)}$

$= \frac{\cos x - \sin x}{\cos x}$

$= \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}$

$= 1 - \tan x$

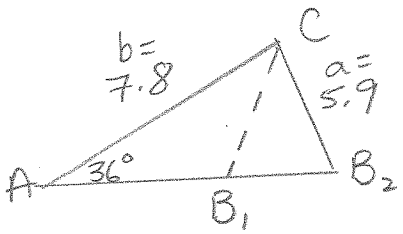
$= 1 - \frac{1}{\cot x}$

$= \frac{\cot x - 1}{\cot x}$

$\therefore LS = RS$

$\therefore$ QED

20. Aiden and Beth are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of the cloud. Aiden's rope to the cloud is 7.8 m long and makes an angle of 36° with the ground. Beth's rope to the same cloud is 5.9 m long . Determine distance(s) between Aiden and Beth to the nearest tenth of a meter. [6]



$\angle A$ is acute

$a < b$

$b \cdot \sin A = 7.8 \sin 36$

$\doteq 4.6$

$\therefore a < b \cdot \sin A$

$\therefore$ 2 solutions

$\angle A = 36°$ $\angle ACB_2 \doteq 93°$

$\angle B_2 \doteq 51°$ $\angle ACB_1 = 15°$

$\angle AB_1C \doteq 129°$

$b = 7.8$ m

$a = 5.9$ m

$AB_1 \doteq 2.6$ m

$AB_2 \doteq 10.0$ m

Big $\triangle AB_2C$

$\frac{\sin A}{a} = \frac{\sin B_2}{b}$

$\frac{\sin 36}{5.9} = \frac{\sin B_2}{7.8}$

$\angle B_2 \doteq \sin^{-1}\left(\frac{7.8 \cdot \sin 36}{5.9}\right)$

$\doteq 51°$

$\angle CB_1B_2 \doteq 51°$ (ITT)

$\angle B_1CB_2 = 180 - 51 - 36$ (ASTT)

$\doteq 93°$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$c \doteq \frac{\sin 93° \cdot 5.9}{\sin 36°}$

$\doteq 10.0$

Small $\triangle AB_1C$

$\angle AB_1C = 180 - 51$ (SA)

$\doteq 129°$

$\angle ACB_1 \doteq 180 - 36 - 129$ (ASTT)

$= 15°$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$c = \frac{\sin 15° \cdot 5.9}{\sin 36°}$

$\doteq 2.6$

$\therefore$ Aiden & Beth can be either 2.6 m or 10.0 m