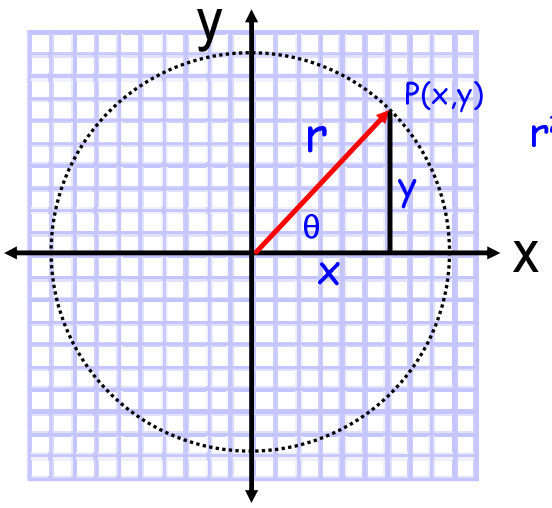
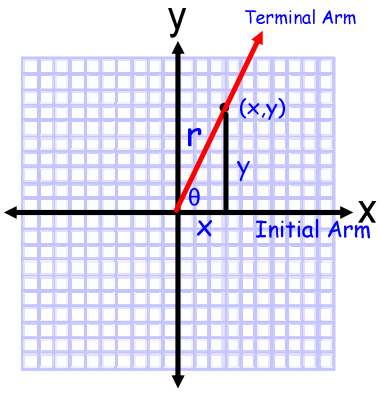
4.2 CAST Rule and Angles Greater Than 90°

Lets explore the primary trig ratios on a coordinate grid



$r^2=x^2+y^2$

I) If $\theta < 90^o$ Terminal arm is in Quadrant 1

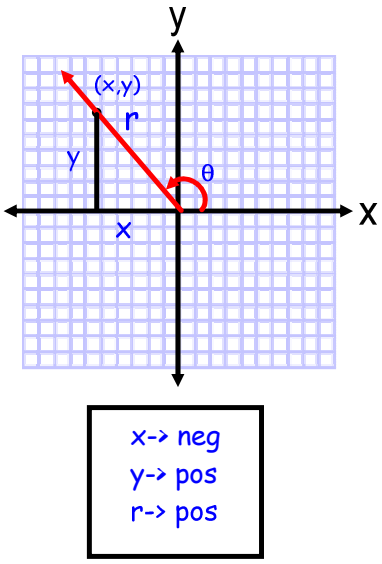


$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

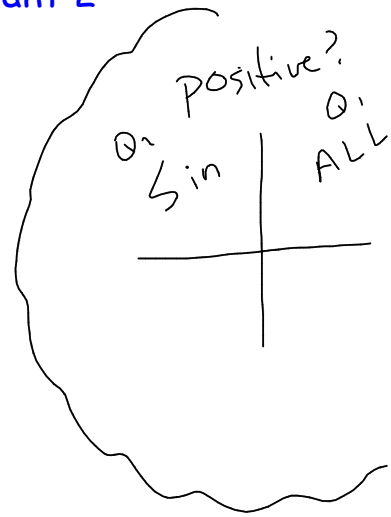
II) If $90^0<\theta<180^0$ Terminal arm is in Quadrant 2



$$\sin\theta = \frac{y}{r} \quad +$$

$$\cos\theta = \frac{x}{r} \quad -$$

$$\tan\theta = \frac{y}{x} \quad -$$



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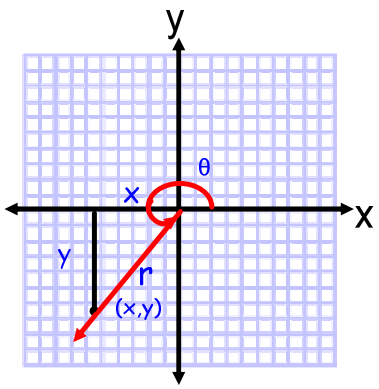
III) If $180^o<\theta<270^o$

Terminal arm is in Quadrant 3



x-> neg
y-> neg
r-> pos

$\sin\theta = \frac{y}{r}$ —

$\cos\theta = \frac{x}{r}$ —

$\tan\theta = \frac{y}{x}$ +

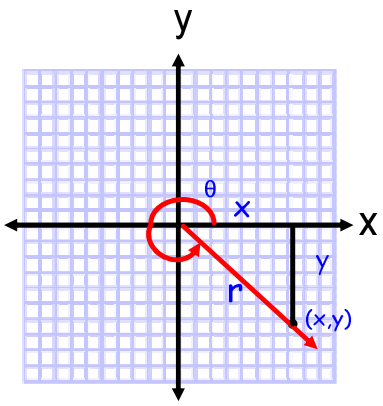
positive

SIN | ALL

TAN

IV) If $270^o<\theta<360^o$

Terminal arm is in Quadrant 4



x-> pos
y-> neg
r-> pos

$\sin\theta = \frac{y}{r}$ —

$\cos\theta = \frac{x}{r}$ +

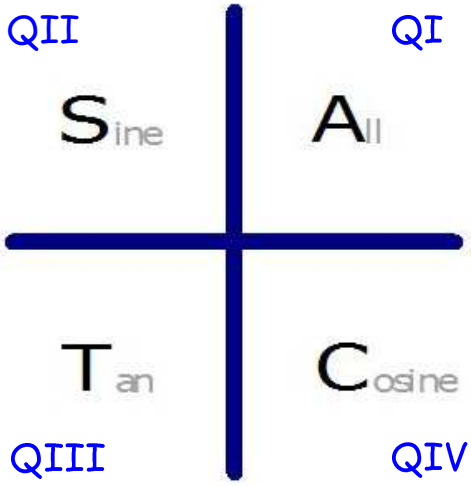
$\tan\theta = \frac{y}{x}$ —

positive?

SIN | ALL

TAN | COS

The CAST RULE tells us which trig ratios are positive and which are negative for a given quadrant



Principal Angle-> The angle measured between the initial arm and the terminal arm

Reference Angle or Related Acute Angle->The angle between the terminal arm and the x-axis ($\theta < 90$)

Note:
The Principal angle and Reference angle have the same trig ratios but the sign's may be different.

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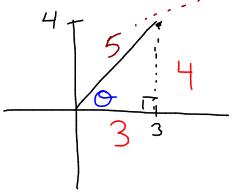
Ex.1 Given the point A, on the terminal arm, determine the primary trig ratios for θ. Include a diagram.

a) (3,4)



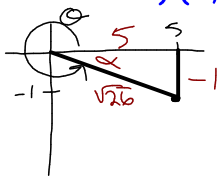
$5^2 = 3^2 + 4^2$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

b) (5,-1)



$r^2 = 5^2 + (-1)^2$

$r^2 = 26$

$r = \sqrt{26}$

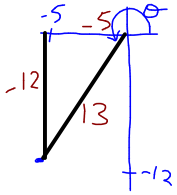
α = related acute angle

$\sin\theta = -\frac{1}{\sqrt{26}}$

$\cos\theta = \frac{5}{\sqrt{26}}$

$\tan\theta = -\frac{1}{5}$

c) (-5,-12)



$\sin\theta = -\frac{12}{13}$

$\cos\theta = \frac{-5}{13}$

$\tan\theta = \frac{12}{5}$

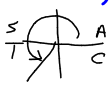
Ex.2

i) Evaluate, to four decimal places.

a) cos 154°

$= -0.8988$

b) tan 230°



$= 1.1918$

ii) Now undo the function and what angle do you get? Explain

2ND cos⁻¹ −0.8988

$\doteq 154°$

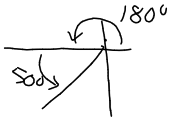
2ND TAN⁻¹ 1.1918

$\doteq 50°$

???

Same as tan 50°!!!

Since we know we are looking for angle in Q_3



$= 180° + 50°$

$= 230°$

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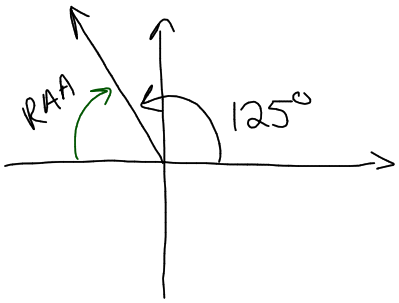
Ex.3 Draw the angle 125^0 in standard position. What is the principal angle? What is the related acute angle?

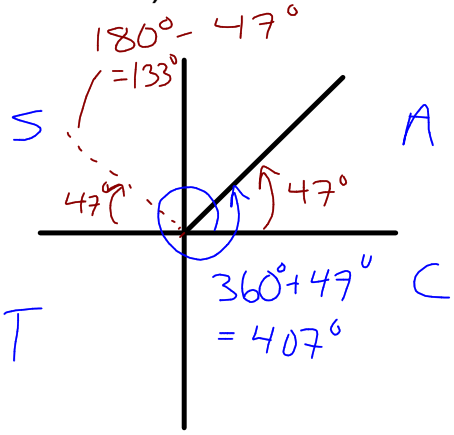


Principal angle $= 125^\circ$

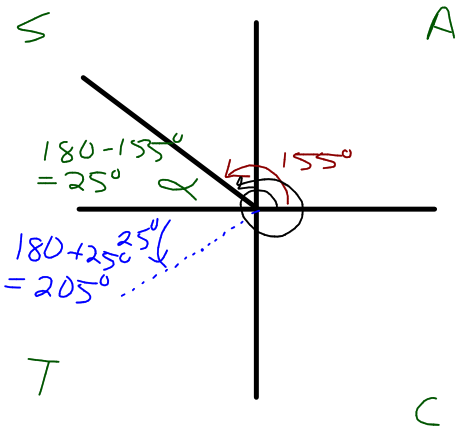
Related Acute angle $= 180^\circ - 125^\circ$
$= 55^\circ$

Ex.4 Determine two other angles that have the same trig ratio as the following.

a) $\sin 47^0$



b) $\cos 155^0$



$360^\circ + 155^\circ$
$= 515^\circ$

$\sin 47^\circ$
$= \sin 133^\circ$
$= \sin 407^\circ$

$\cos 155^\circ$
$= \cos 205^\circ$
$= \cos 515^\circ$

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Ex.5 Determine all angles between 0^0 and 360^0 that have the following trig ratios:

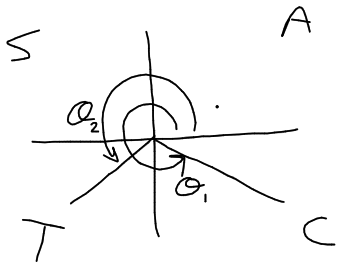
a) $\sin\theta=-0.26$

$\theta = \sin^{-1}(-0.26)$

$= -15^\circ$

$RAA = |-15^\circ|$

$= 15^\circ$



$\theta_1 = 360^\circ - 15^\circ$

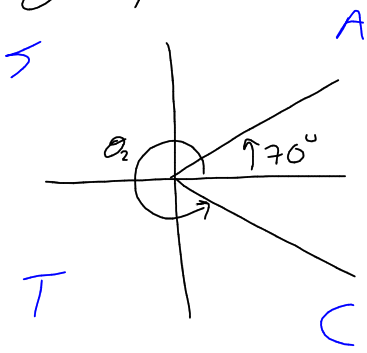
$= 345^\circ$

$\theta_2 = 180^\circ + 15^\circ$

$= 195^\circ$

b) $\cos\theta=0.34$

$\theta \doteq 70^\circ$



$\theta_1 = 70^\circ$

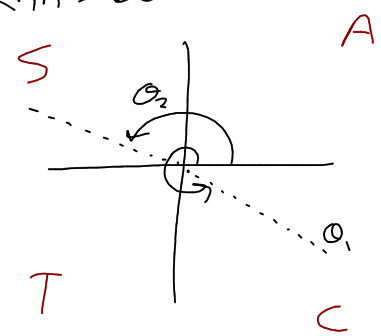
$\theta_2 = 360^\circ - 70^\circ$

$= 290^\circ$

c) $\tan\theta=-2.14$

$\theta = -65^\circ$

$RAA = 65^\circ$



$\theta_1 = 360^\circ - 65^\circ$

$= 295^\circ$

$\theta_2 = 180^\circ - 65^\circ$

$= 115^\circ$

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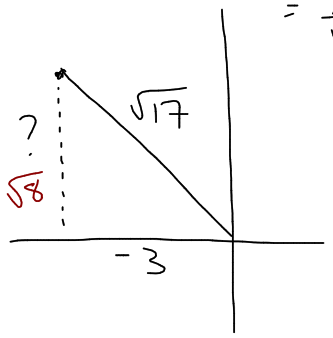
Ex.6 If $\cos\theta = -\frac{3}{\sqrt{17}}$ where $90^0 < \theta < 180^0$, determine sinθ and tanθ.

$= \frac{x}{r}$

Q_2



$y^2 = (\sqrt{17})^2 - (-3)^2$

$y = \sqrt{8}$

$\sin\theta = \frac{\sqrt{8}}{\sqrt{17}}$

$= \frac{2\sqrt{2}}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}}$

$= \frac{2\sqrt{34}}{17}$

$\tan\theta = -\frac{\sqrt{8}}{3}$

$= -\frac{2\sqrt{2}}{3}$

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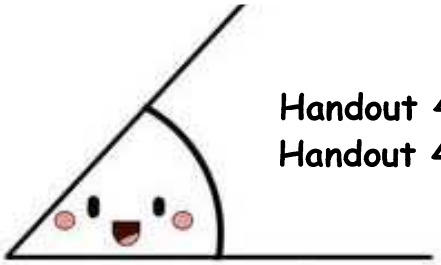
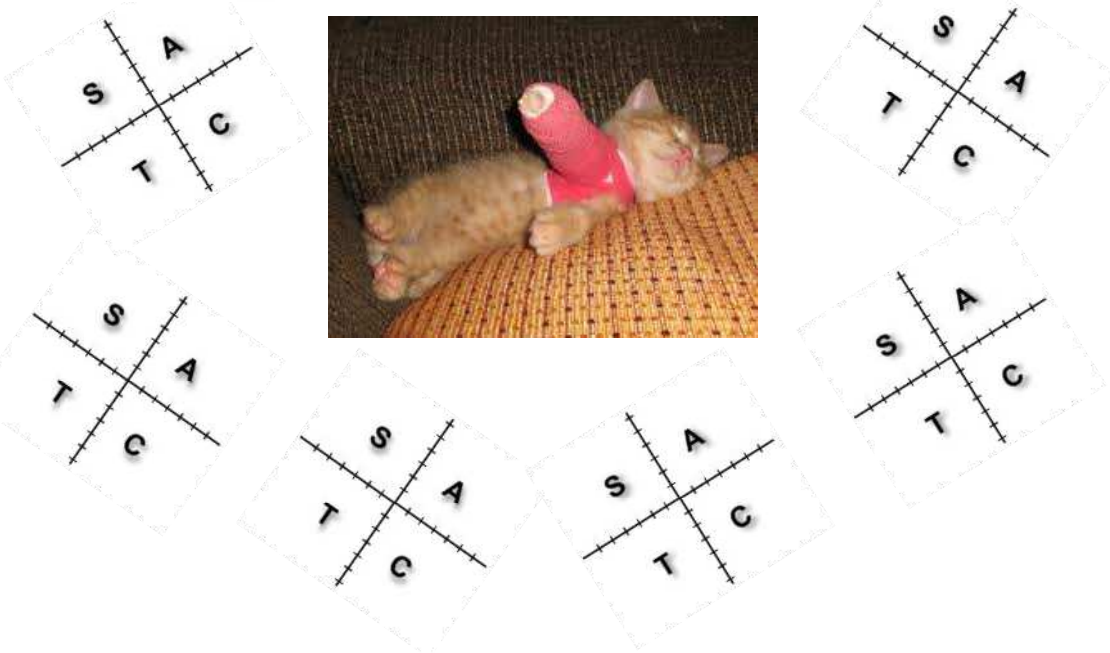


Handout 4.2A #1-3,5-7,8b,9,10
Handout 4.2B #2,4,6,9



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