

4.5 Sine Law and Cosine Law

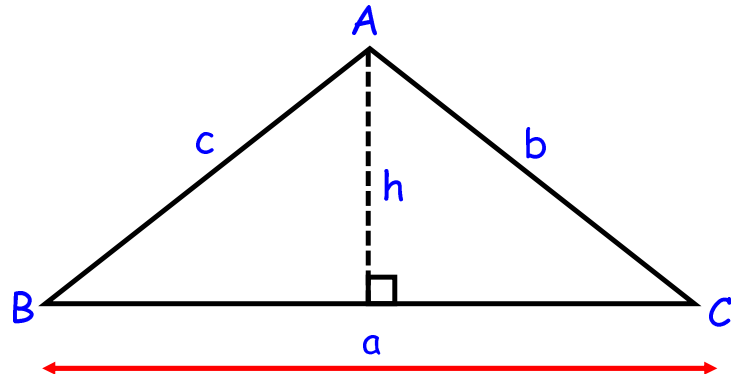
Lets develop the Sine Law from primary ratios:

$$\sin C = \frac{h}{b}$$

$$h = b \sin C$$

$$\sin B = \frac{h}{c}$$

$$h = c \sin B$$

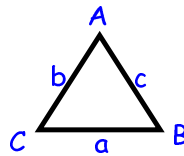


$$b \sin C = c \sin B$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

The Sine Law

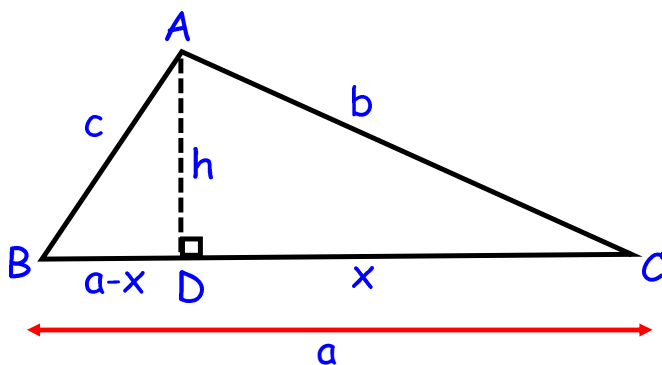
In $\triangle ABC$,



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Sine Law

Lets develop the Cosine Law



In $\triangle ADC$

$$\cos C = \frac{x}{b}$$

$$x = b \cos C$$

$$x^2 + h^2 = b^2$$

In $\triangle ABD$

$$c^2 = h^2 + (a-x)^2$$

$$c^2 = \underline{h^2} + a^2 - 2ax + \underline{x^2}$$

$$c^2 = \underbrace{h^2 + x^2}_{b^2} + a^2 - 2ax$$

$$c^2 = b^2 + a^2 - 2ax$$

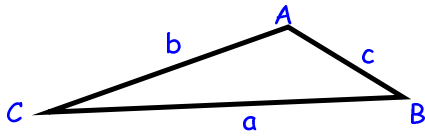
$$c^2 = b^2 + a^2 - 2ab \cos C$$

The Cosine Law

In $\triangle ABC$,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

(finding a side)



you can rearrange

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

(finding an angle)

So how do you know when to use what???



1

CHECK

Right angle
 $\Delta \rightarrow$ Use SOH
 CAH TOA or
 Pythagorean
 Theorem



2

matching
 angle/side pair use
SINE LAW



3

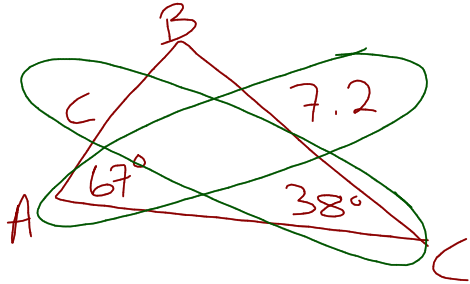
Otherwise use the **COSINE LAW**



Take note this only occurs:

- 3 sides
- 2 sides and the contained angle

Ex. 1 Solve for c , if $\angle A = 67^\circ$, $\angle C = 38^\circ$, $a = 7.2$ cm



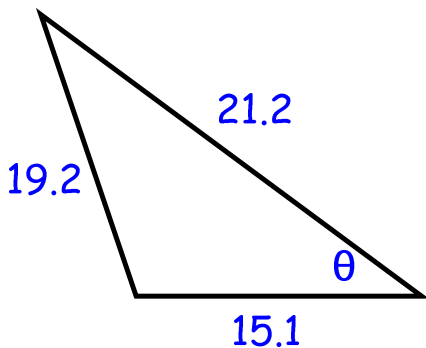
$$\frac{c}{\sin 38^\circ} = \frac{7.2}{\sin 67^\circ}$$

$$c = \sin 38^\circ \cdot \frac{7.2}{\sin 67^\circ}$$

$$\approx 4.8$$

$$\therefore c = 4.8 \text{ cm}$$

Ex. 2 Solve for the unknown

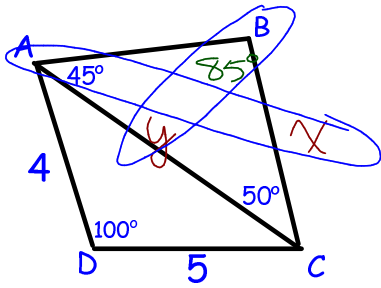


$$\cos \theta = \frac{21.2^2 + 15.1^2 - 19.2^2}{2(21.2)(15.1)}$$

$$\theta = \cos^{-1} \left(\frac{(21.2^2 + 15.1^2 - 19.2^2)}{(2 * 21.2 * 15.1)} \right)$$

$$\approx 61^\circ$$

Ex.3 Find the length of BC



$$y^2 = 4^2 + 5^2 - 2(4)(5) \cos 100^\circ$$

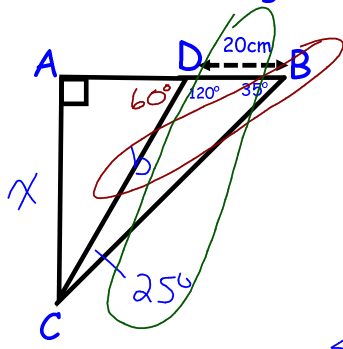
$$y = 6.9$$

$$\frac{x}{\sin 45^\circ} = \frac{6.9}{\sin 85^\circ}$$

$$x = 4.9$$

∴ BC is 4.9 units

Ex.4 Find the length of AC to the nearest tenth.



$$\frac{b}{\sin 35^\circ} = \frac{20}{\sin 25^\circ}$$

$$b = 27.1$$

$$\sin 60^\circ = \frac{x}{27.1}$$

$$x = 23.5$$

∴ Side AC is 23.5 cm

Homework - p. 290
#1-4 (a & d), 8, 9,
10, 15

