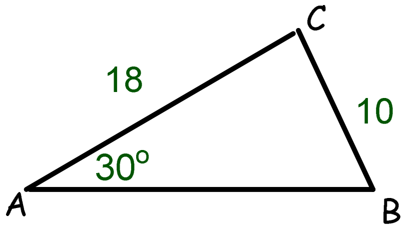
4.6A - Sine Law: AMBIGUOUS Case (Day 1)

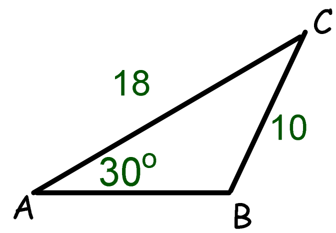
- When two sides and the non-included angle of a triangle are given, the triangle may not be unique. (SSA case)
- Each case must be examined to determine if there is no solution, one solution or two possible solutions.

Consider this question:

ΔABC, $\angle A = 30^\circ$, a = 10, b = 18. Solve for <B (nearest degree).



Which diagram is correct??
Are they both possibilities??



algebraically:

$$\frac{\sin B}{18} = \frac{\sin 30}{10}$$

$$\sin B = \frac{18 \sin 30}{10}$$

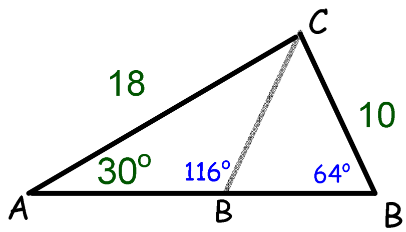
$$\sin B = 0.9$$

$$B = \sin^{-1}(0.9)$$

$$B = 64^\circ, 116^\circ$$

Sine is positive...
∴ one answer in Q1 (acute)
one answer in Q2 (obtuse)

second triangle — first triangle



What is the relationship between the 2 possible answers?

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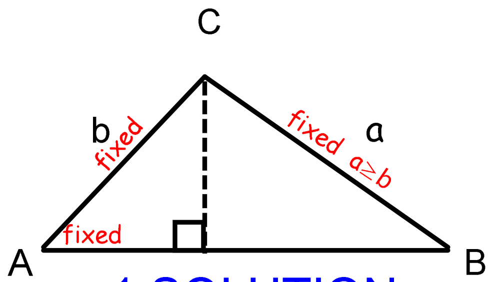
Let's explore all the cases (summary in handout package)

For each ΔABC, you are given a,b, and <A.... (SSA)

Hint:
Draw the Δ the same way every time!

Cases where <A is acute (A<90°)

Case 1: If a ≥ b



- if a≥b, a will be longer than the perpendicular from C
- AB will extend past the perpendicular from C

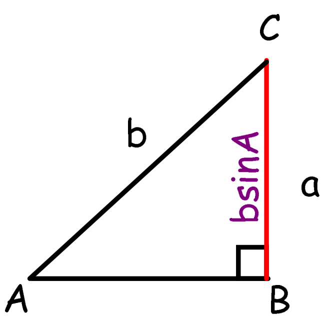
1 SOLUTION

(B is also acute)

Case 2: If a < b

If a<b, a could be:

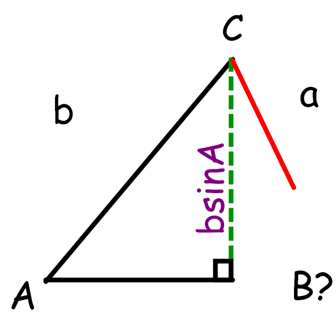
- the same length as the perpendicular from C
- shorter than the perpendicular from C
- longer than the perpendicular from C



1 SOLUTION

- there is 1 triangle
- a is the same length as the ⊥ from C

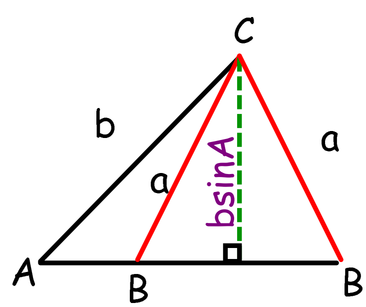
a = bsinA
B=90°



NO SOLUTION

- there is no triangle
- a is shorter than the ⊥ from C

a < bsinA
B...can't exist



2 SOLUTIONS

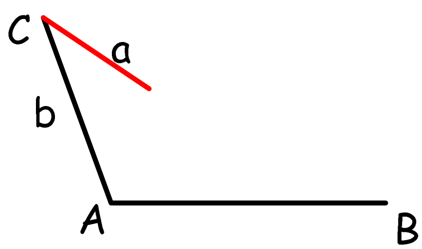
- there are two possible triangles
- a is longer than the ⊥ from C

a > bsinA
B is acute OR obtuse

In this case (A is acute and a < b)...the # of solutions depends on the value of bsinA.

Cases where <A is obtuse ($A \geq 90°$)

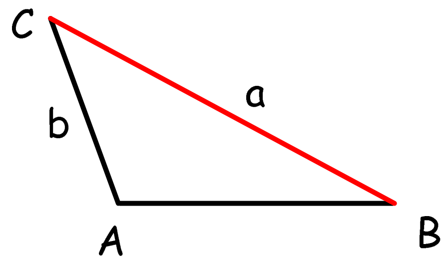
Case 1: **If $a \leq b$**



NO SOLUTION

- $a \leq b$
- it is not possible to draw a triangle

Case 2: **If $a > b$**



1 SOLUTION

- $a > b$
- there is only one triangle

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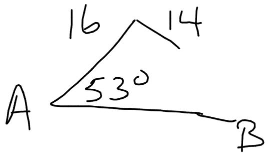
Ex. 1 For each given triangle, determine how many solutions there are. (don't solve...but include a diagram)

Hint

a) In ΔABC, a = 14, b = 16, and <A = 53°.

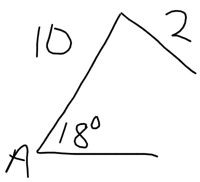


∠A acute
a < b

Use bsinA to compare
bsinA
≐ 16sin53°
≐ 12.8

a < b
a > bsinA
∴ 2 sol'ns

b) In ΔABC, <A=18°, a=2 cm, b=10 cm.

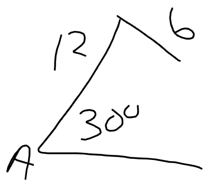


∠A acute
a < b
↑ Might be ambiguous!

bsinA
= 10 sin 18°
≐ 3.1

a < b
a < bsinA
∴ NO sol'ns

c) In ΔABC, <A=30°, a= 6m, b= 12m.



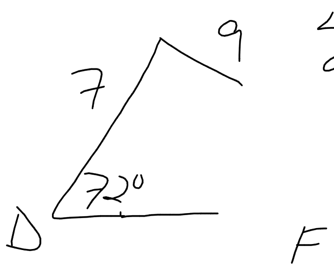
∠A acute
a < b
↑ might be ambiguous

bsinA
= 12 sin 30°
= 6

a = bsinA

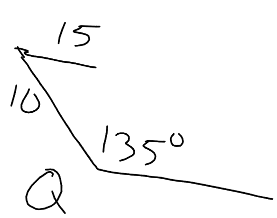
∴ ONE sol'n

d) In ΔDEF, <D= 72°, d= 9cm, f= 7cm.



∠D acute
d > f
∴ one solution

e) In ΔPQR, <Q= 135°, q= 15 cm, r= 10cm.



∠Q is obtuse
q > r
∴ one sol'n

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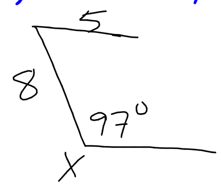
Ex. 2 Determine the measures of all angles in the given triangles.

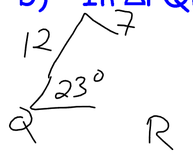
a) In ΔXYZ, ∠X = 97°, x = 5cm, y = 8cm.



∠X is obtuse

$x < y$ ∴ no sol^n

b) In ΔPQR, ∠Q = 23°, q = 7cm, r = 12cm.

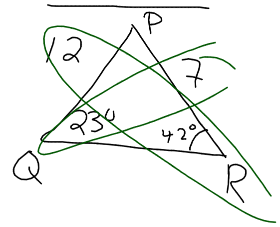


∠Q acute

$q < r$ ← might be ambiguous

$b \sin A$
$= 12 \sin 23°$
$\doteq 4.7$ ∴ 2 sol^ns

∠R acute



$$\frac{\sin R}{12} = \frac{\sin 23}{7}$$

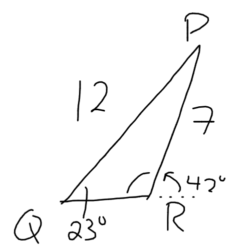
$$\sin R = \frac{12 \cdot \sin 23}{7}$$

$$R \doteq 42°$$

$$\angle P = 180° - 23° - 42° = 115°$$

∠Q = 23°
∠R = 42°
∠P = 115°

∠R obtuse

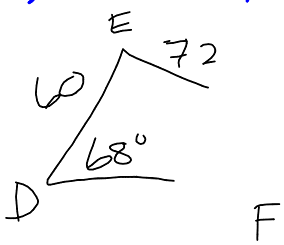


$$\angle R = 180° - 42° = 138°$$

$$\angle P = 180° - 138° - 23° = 19°$$

∠Q = 23°
∠R = 138°
∠P = 19°

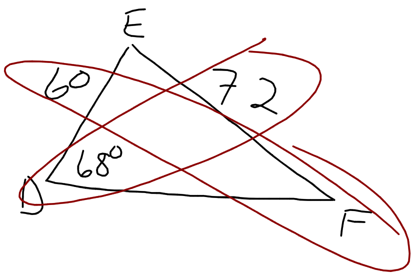
c) In ΔDEF, <D = 68°, d= 72 cm, f= 60cm.



$\angle D$ acute

$d > f$

$\therefore$ ONE SOL$^{\underline{N}}$



$$\frac{\sin F}{60} = \frac{\sin 68°}{72}$$

$$F = \sin^{-1}\left(60 \cdot \frac{\sin 68°}{72}\right)$$

$$\doteq 51°$$

$\angle E = 180° - 51° - 68°$

$= 61°$

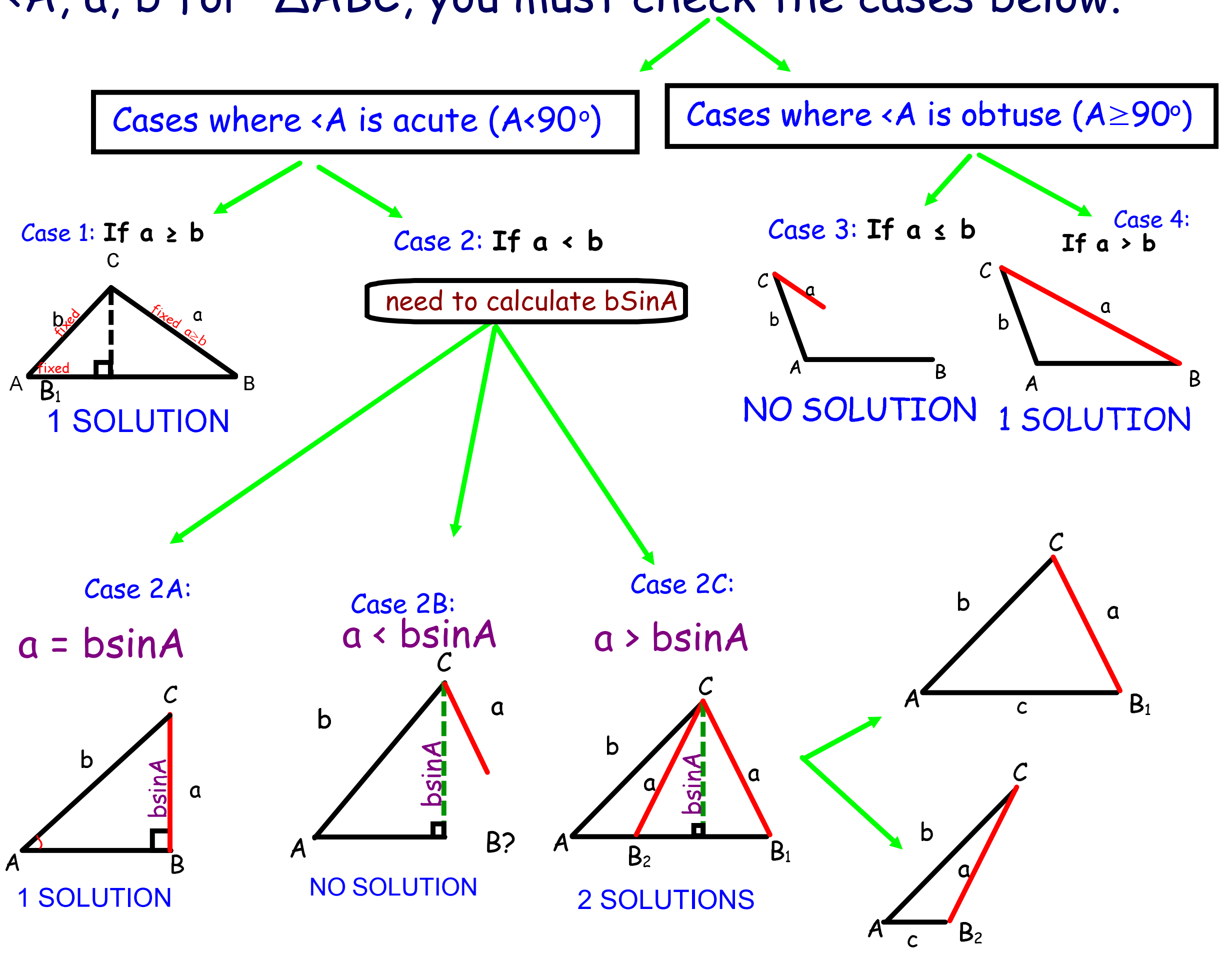
$\therefore \angle D = 68°$

$\angle E = 61°$

$\angle F = 51°$

Sine Law - Ambiguous Case Summary

If you are given 2 side lengths and an angle (not contained) $\angle A$, a, b for ΔABC, you must check the cases below.



Homework
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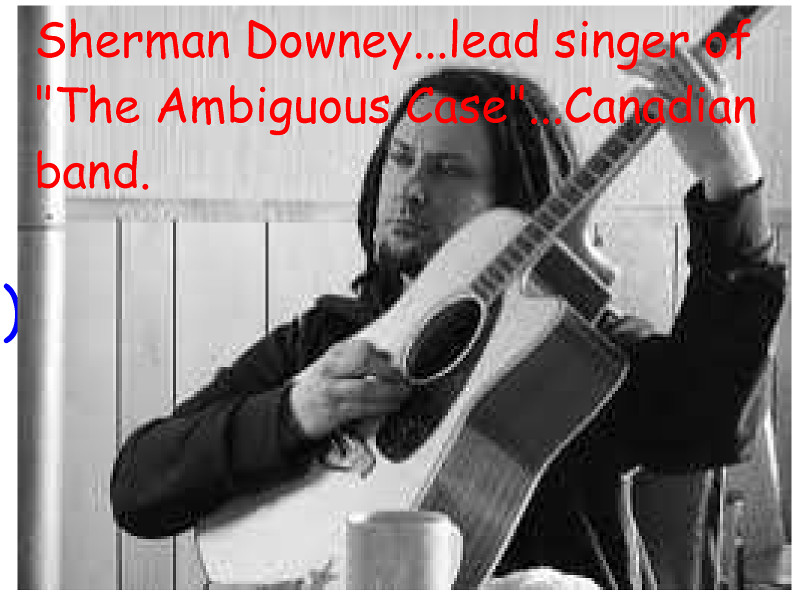

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