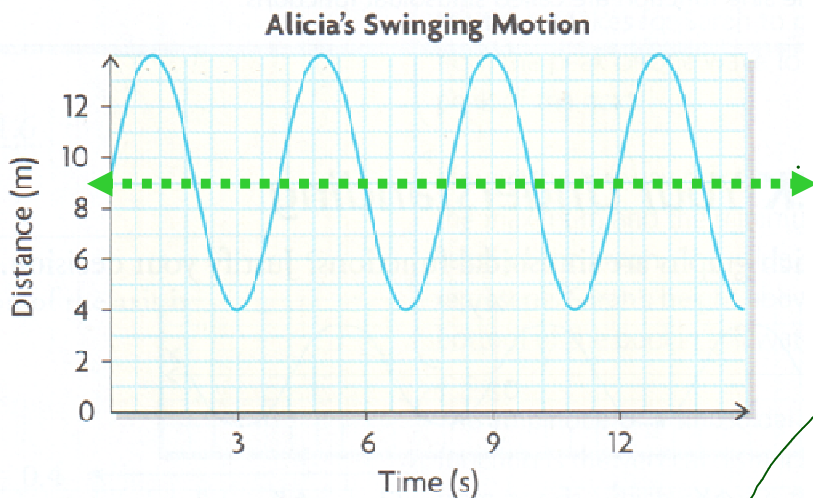


5.6 - Applications of Trig Functions

Ex 1 Alicia was swinging back and forth in front of a motion detector. Her distance from the detector was modeled by the following graph:



$$\frac{\text{Max} + \text{min}}{2} = \frac{14 + 4}{2}$$

• What is the equation of the axis? $d=9$ Counting
Algebraically:

• What is the amplitude? 5 Counting
Algebraically: $\frac{14-4}{2}$

• What is the period of the function approx 4

• How close did Alicia get to the motion detector? 4m

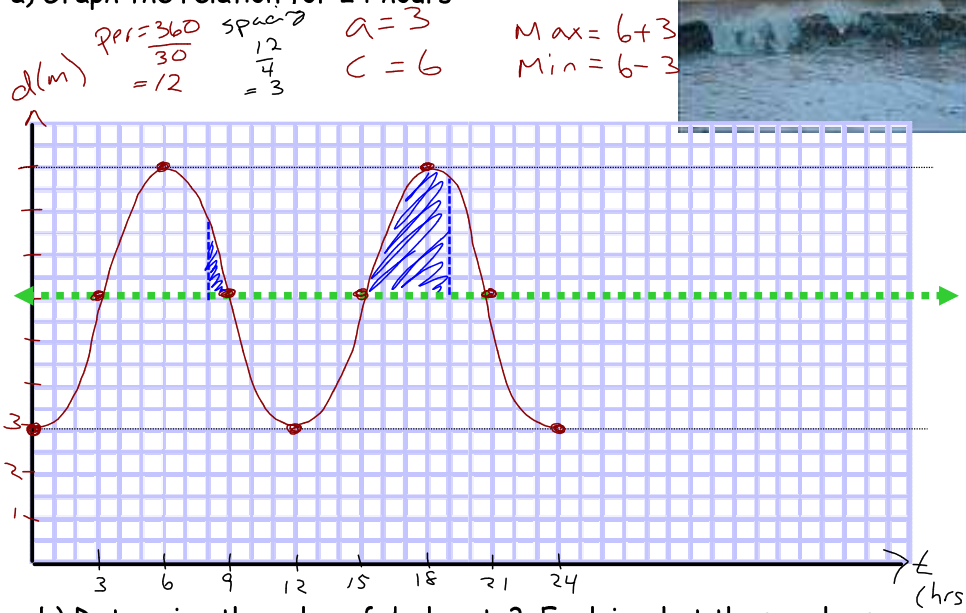
• At $t=8$ sec would it be safe to run between Alicia and the motion detector?

-Yes
-She's approx. 9m away
-She's moving away

Note: She's never closer than 4m
∴ Always safe

Ex2. The depth of the water in a harbour fluctuates because of the tide and is modeled by the equation $d = -3\cos(30t) + 6$, where d represents the depth of the water in metres, and t represents the number of hours after midnight. (ie. $t=0$ means midnight, $t=3$ means 3 A.M. etc)

a) Graph the relation for 24 hours



b) Determine the value of d when $t=3$. Explain what these values represent.

With graph: $d=6$

With equation:

$$d = -3\cos(30 \cdot 3) + 6 = 6$$

c) Determine the maximum depth of the water

$\text{Max} = c + |a|$ $\text{Min} = c - |a|$ 🖐

$\text{Max} = 6 + 3 = 9$ $\therefore \text{Max depth is } 9\text{m}$

d) Surfing is allowed between 8 A.M. and 7 P.M., but only when the depth of the water is 6 metres or more. For how many hours each day is surfing allowed? Explain.

Between 8-9am \rightarrow 1hr
 3-7pm \rightarrow 4hr

 5hr

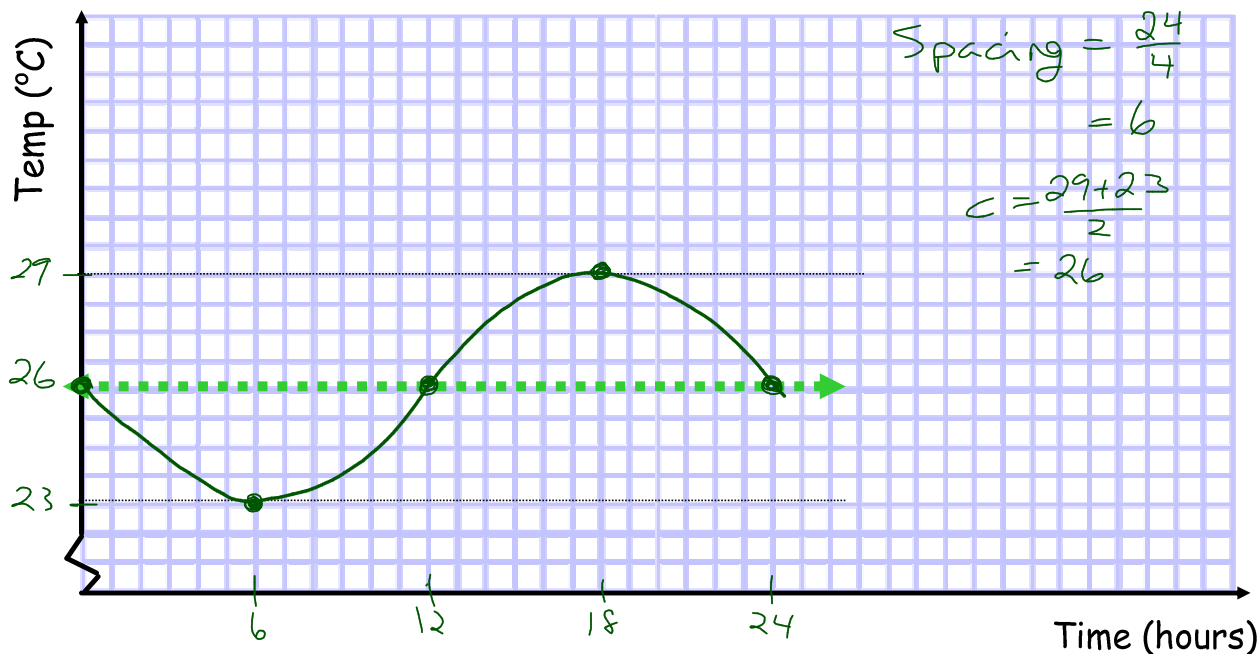
\therefore Surfing is allowed for 5 hrs.

Ex3. The temperature of a solar-heated pool changes throughout a sunny day and is modeled by a trigonometric relation. The temperature ranges from 23°C at 6 A.M. to 29°C at 6 P.M.

$$a = \frac{29 - 23}{2} = 3$$

lowest - highest \Rightarrow 12 hrs
 \therefore period = 24hr

a) Graph the relation for a 24 hour period starting at midnight (t=0).



b) Determine the equation of a sine function for the given graph.

Find k
 $k = \frac{360}{24}$
 $= 15$

$$T = -3\sin(15t) + 26$$



c) The pool is comfortable for swimming when the temperature is at least 23°C. The pool is open from 6 A.M. to 11 P.M. every day. How many hours of comfortable swimming are available on a sunny day?

- Always at least 23°C

- 11pm \Rightarrow 23

$$\rightarrow 23 - 6 = 17 \text{ hrs}$$

\therefore 17 hours of comfortable swimming

Ex4. The rodent population in a region varies approximately according to the equation $r(t) = 1200 + 300\sin 90t$, where t is the number of years since 1970 and r is the number of rodents.

a) Find the maximum and minimum number of rodents.

$$\begin{aligned} \text{Max} &= 1200 + 300 \\ &= 1500 \end{aligned}$$

$$\begin{aligned} \text{Min} &= 1200 - 300 \\ &= 900 \end{aligned}$$

$$\begin{aligned} c &= 1200 \\ a &= 300 \end{aligned}$$

b) What is the period of the function?

$$\frac{360}{90} = 4$$

$\therefore 4$ years

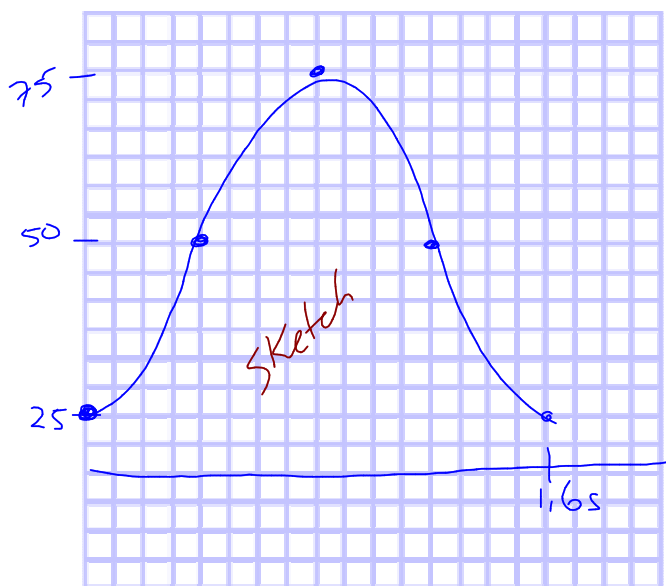
c) How many rodents could be expected in 2010?

$$\begin{aligned} t &= 2010 - 1970 \\ &= 40 \end{aligned}$$

$$\begin{aligned} r(40) &= 1200 + 300\sin(90 \cdot 40) \\ &= 1200 \end{aligned}$$

\therefore Population will be 1200

Ex5. A weight is supported by a spring. The weight rests 50 cm above a tabletop. The weight is pulled down 25 cm and released at time $t=0$. This creates a periodic up-and-down motion. It takes 1.6 s for the weight to return to the low position each time. Determine an equation for the sinusoidal function.



$$\begin{aligned} \text{Max} &= 50 + 25 \\ &= 75 \end{aligned}$$

$$c = 50$$

$$\begin{aligned} \text{Min} &= 50 - 25 \\ &= 25 \end{aligned}$$

$$a = 25$$

$$\text{period} = 1.6\text{s}$$

$$k = \frac{360}{1.6}$$

$$= 225$$

\therefore an equation is

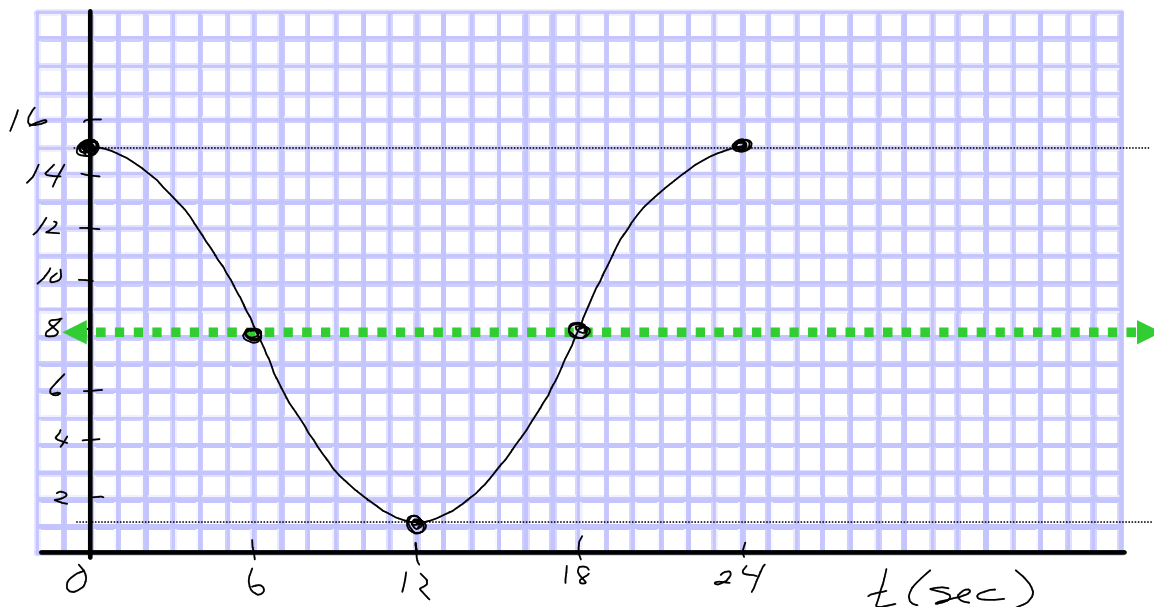
$$h = -25 \cos(225t) + 50$$

Ex6. A ferris wheel has a radius of 7 m. The centre of the wheel is 8m above the ground. The Ferris wheel rotates at a constant speed of $15^\circ/\text{s}$. There is only one red seat on the Ferris wheel.

$$\begin{aligned} \text{period} &= \frac{360}{15} & \text{Scale} &= \frac{24}{4} \\ &= 24\text{s} & &= 6\text{s} \end{aligned}$$

a) Graph one rotation of the wheel if the red seat starts at the maximum height.

$$\begin{aligned} \text{Max} &= 8 + 7 & \text{Min} &= 8 - 7 \\ &= 15 & &= 1 \end{aligned}$$



b) Determine an equation of a cosine function which describes the height of the red seat, where h is the height in metres and t is the time in seconds.

$$h = 7 \cos(15t) + 8$$

c) Determine an equation of a sine function which describes the height of the red seat where h is the height in metres and t is the time in seconds.

$$h = 7 \sin[15(t+6)] + 8$$

Homework

5.5 - p. 376 #13, 15-19

5.6 - p. 389 #12-16, 19, 21