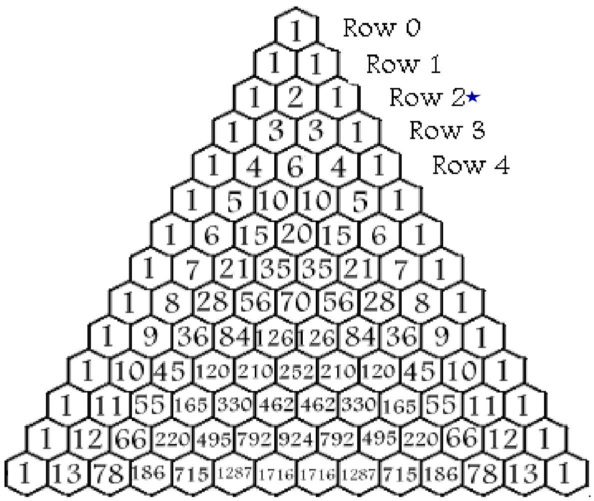
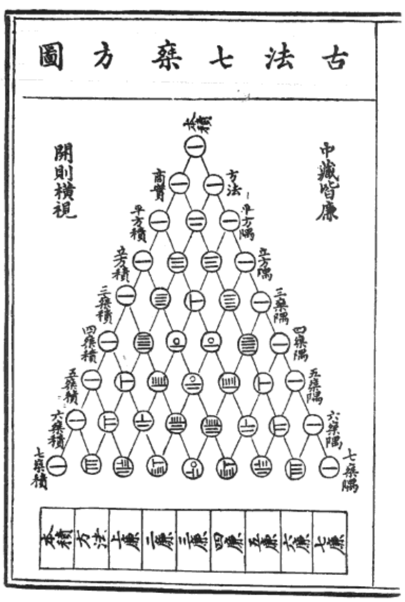
6.7 Pascal's Triangle



This is Pascal's Triangle:

Each term is equal to the sum of the two terms immediately above it.

The term at the end of each row is 1.



Chu Shi-kie's triangle, 700 years old, 300 years older than Pascal!

Pascal's Triangle is named after mathematician, Blaise Pascal (1623-1662).

He discovered the triangle and many of its applications at age 13. Since he published his findings, western history attaches his name to the triangle.

Pascal was not the first to know this triangle though. Chinese mathematician, Chu Shi-kie also wrote of the triangle and many of its applications in the 14th century.

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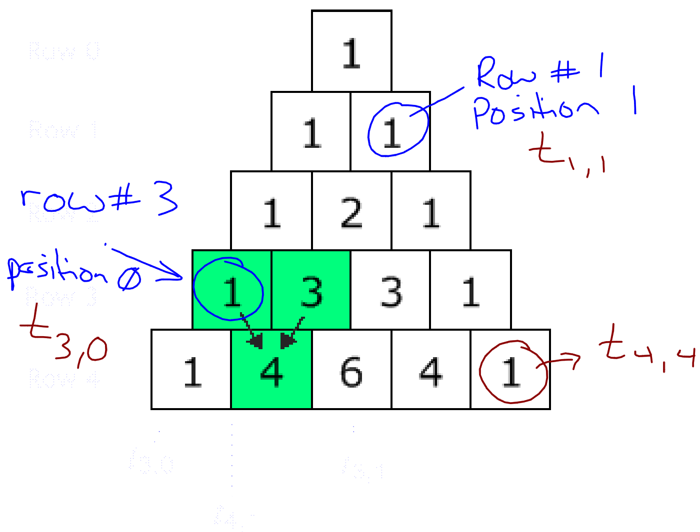

Rows and positions in rows are numbered starting from ZERO.

So, the 3rd row is actually Row 2.

Also, the term equal to 4 is found in row 4, position 1.

$t_{n,r}$ represents the term in row n, position r.

Thus, $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$

As terms we have:

$t_{0,0}$

$t_{1,0}$ $t_{1,1}$

$t_{2,0}$ $t_{2,1}$ $t_{2,2}$

$t_{3,0}$ $t_{3,1}$ $t_{3,2}$ $t_{3,3}$

etc...

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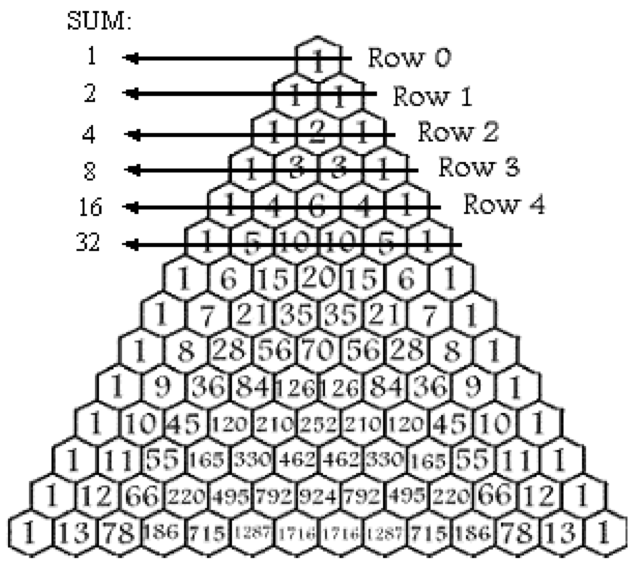
Pascal's Triangle isn't just cool for the sake of being cool...it has the most amazing properties and applications. For instance:

ROW SUMS

The sum of the terms in any row n in Pascal's Triangle is 2^n.



eg. sum of row 9 = 2^9 = 512

eg. row whose sum is 4096 is row 12, $\because 2^{12} = 4096$

What row has the sum of 4096?

$2^n = 4096$

then:

$$n = \frac{\log 4096}{\log 2}$$

$= 12$

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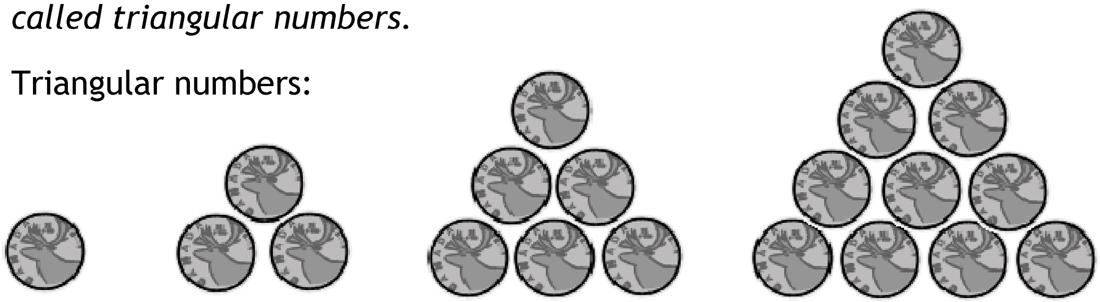
TRIANGULAR NUMBERS | Recursive Formula

Numbers that correspond to the number of items stacked in a triangular array are called triangular numbers.

$t_n = t_{n-1} + n$

Triangular numbers:

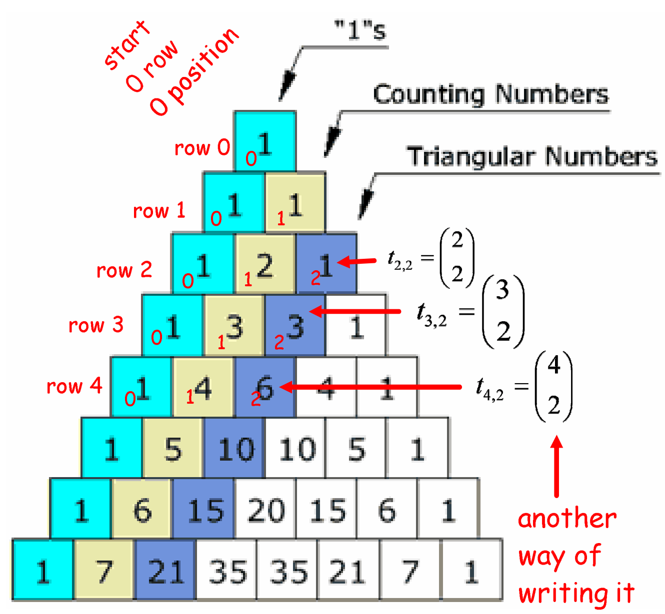


1 3 6 10 . . .

We find the list of triangular numbers in the third diagonal of Pascal's Triangle.

Ex 2. Relate Pascal's Triangle to the number of coins in a triangle with n rows.

$t_{n+1,2}$



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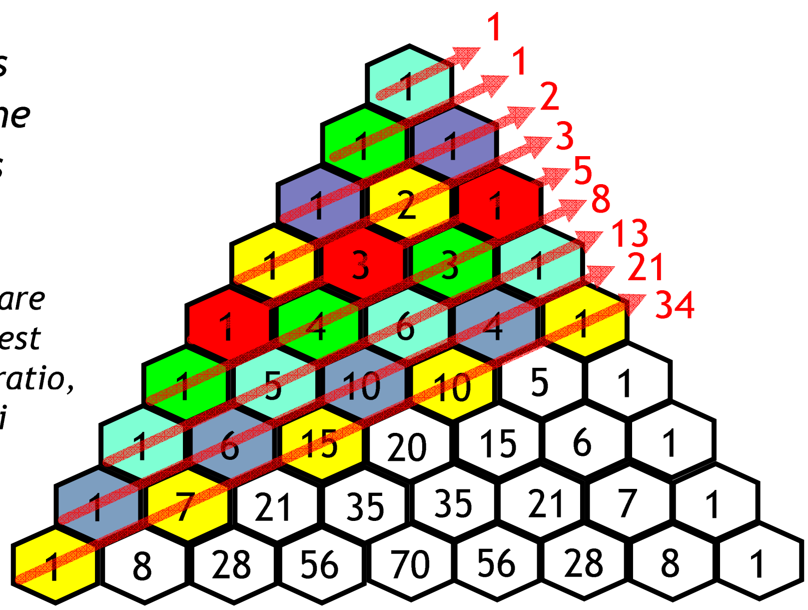
FIBONACCI NUMBERS

The fibonacci sequence begins with 1,1, and then each subsequent term is found by taking the sum of the prevoius two terms.

1, 1, 2, 3, 5 ,8, 13, 21, 34, 55, 89, 144, 233, 377,...

The fibonacci numbers are found by adding the terms of the diagonals of Pascal's Triangle:

Fibonacci numbers are found in the strangest places: the golden ratio, nature, The DaVinci Code,...



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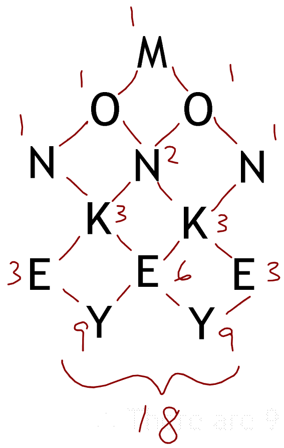
Pascal's Triangle is often used to determine the number of paths between points.

Ex 1: How many paths will spell MONKEY by starting at the top and proceeding downwards, moving diagonally left or right?



∴ There are 9 + 9 = 18 different paths to spell MONKEY.

Ex 2:

In Plinko, a token slides down a board. If the token cannot go through a shaded square, in how many ways could you win $500?

START ↓ ↓ ↓ ↓

1		1		1		1	
	2		2		2		1
2		4		4		3	
	6		8		7		3
6		14		15		10	
	20		29		25		10
20		49		54		35	
	69		$500		$500		35
			103		89		

103 + 89 = 192

∴ 192 ways to win $500

Feb 8-4:28 PM

Ex. 3

a) Write out row 0 to 4 of Pascal's Triangle
b) Express the rows and positions of these rows using recursive sequences with $t_{n,r}$ and create a formula for the Pascal's Triangle sequence for generating new terms
*c) Finally express the rows as combinations *(we discuss this more next slide)**

a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

b)

$t_{n,r}$ represents the term in row n, position r.

Thus, $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$

$t_{0,0}$
$t_{1,0}$ $t_{1,1}$
$t_{2,0}$ $t_{2,1}$ $t_{2,2}$
$t_{3,0}$ $t_{3,1}$ $t_{3,2}$ $t_{3,3}$
$t_{4,0}$ $t_{4,1}$ $t_{4,2}$ $t_{4,3}$ $t_{4,4}$

c)

$\binom{0}{0}$
$\binom{1}{0}$ $\binom{1}{1}$
$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$
$\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$

↑ Combinations

$\binom{n}{r}$ ON CALC

nCr

ex. $\binom{5}{3}$ → [5] [nCr] [3]

"n choose r"

6.7 - More on Pascal's Triangle (lead up to Binomial Theorem)

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There is a more efficient way of finding a specific number within the triangle without drawing it out. We can use Combinatorics (a type of Math used mostly for graph theory and probability theory) to help us.

Definition of a Combination

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

factorial (the product of all natural numbers less than or equal to n)

pronounced "n choose r"

On your calculators, nCr button can be used instead of using the formal algebraic definition

In the case of Pascal's Triangle, *n CHOOSE r* means find the number that is in row n, position r in the triangle.

Ex. 1 Evaluate the following combinations.

$\binom{n}{r} = \frac{n!}{(n-r)!r!}$

a) $\binom{7}{3} = \frac{7!}{4!\,3!}$

$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1}$

$= 35$

b) $\binom{6}{0}$

6 nCr 0

$= 1$

c) $\binom{4}{4} = 1$

d) ${}_9C_7 = 36$

Ex. 2

a) find the term in row 2, position 2 of Pascal's Triangle

look at △ $= 1$

$\binom{2}{2} = 1$

☆ b) find the term in the 3rd row, 3rd position in Pascal's Triangle

Remember! 3rd Row = Row #2

3rd Pos = Pos #2

$\binom{2}{2} = 1$

What about Row 20, position 18?

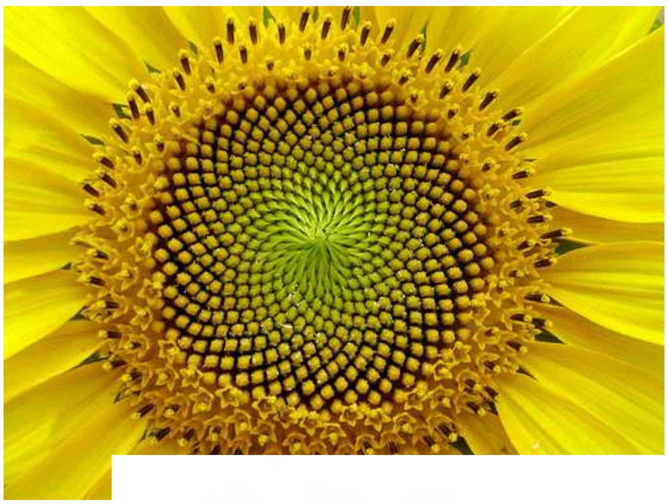
$\binom{20}{18} = 190$

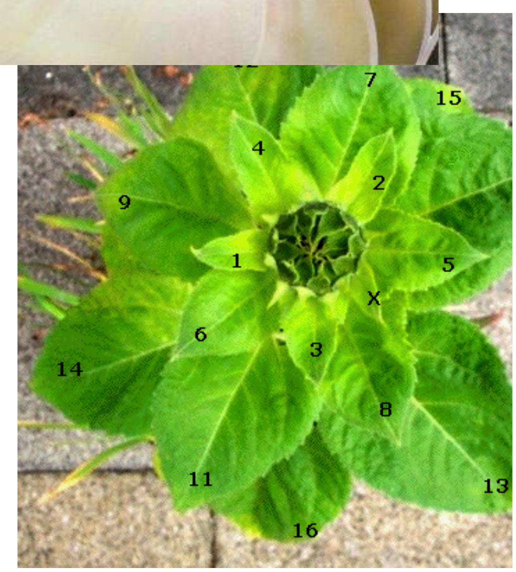
20 nCr 18

Sep 11-7:33 AM

Homework: Handout

Golden Ratio of the human body





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