

7.2 Problems with Compound Interest

Present Value:

- principal that needs to be invested/borrowed now to achieve a future goal
- PV can be calculated when the interest rate, compounding period and length of term are known

Formula: use compound interest formula, but rearrange for "P" or "PV"

$$A = P(1 + i)^n$$

$$\frac{A}{(1 + i)^n} = P$$

$$A(1 + i)^{-n} = P$$

where

A = amount at end of investment

P = present value/principal

i = interest rate per compound pd.

n = # of compound periods

Ex. 1 Julia wants to have \$5000 in 2 years to use as a down-payment for a car. How much does she need to invest now at 6.3%/a compounded monthly?

$$P = ?$$

$$A = 5000$$

$$i = \frac{0.063}{12}$$

$$n = 2 \times 12 = 24$$

$$\begin{aligned} P &= A(1+i)^{-n} \\ &= 5000 \left(1 + \frac{0.063}{12}\right)^{-24} \\ &= 4409.53 \end{aligned}$$

∴ She should invest \$4409.53

Ex.2 David plans to put money into an RESP so that he has \$9000 in 5 years. Which option is the best deal for David?

a) 7.2% compounded semi-annually

$$P = ?$$

$$A = 9000$$

$$i = \frac{0.072}{2}$$

$$n = 5 \times 2 \\ = 10$$

$$P = 9000 \left(1 + \frac{0.072}{2}\right)^{-10} \\ = 6318.95$$

b) 6.5% compounded bi-weekly

$$P = ?$$

$$A = 9000$$

$$i = \frac{0.065}{26}$$

$$n = 5 \times 26 \\ = 130$$

$$P = 9000 \left(1 + \frac{0.065}{26}\right)^{-130} \\ = 6505.38$$

Option A is best.

Ex. 3 Sam borrowed \$8000 at 7.5%/a compounded monthly for ~~5~~ years. He lands a good paying job after 2 years and wants to make a lump sum payment to pay off his debt. How much should he have to pay?

We don't care

$$P = 8000$$

$$A = ?$$

$$i = \frac{0.075}{12}$$

$$n = 2 \times 12 \\ = 24$$

$$A = 8000 \left(1 + \frac{0.075}{12}\right)^{24} \\ = 9290.34$$

∴ He would pay back \$9290.34 after 2 years

The TVM Solver

- a program on the graphing calculator used for financial calculations

To find the TVM Solver program:

APPS , 1: Finance..., **ENTER** , 1: TVM Solver... **ENTER**

| | |
|------|--|
| N | = # of years |
| I% | = interest rate/a as a percent |
| PV | = present value (P) |
| PMT | = the payment amount (put as "0" if there are no payments) |
| FV | = future value (A) |
| P/Y | = number of payments per year (put "1" if no payments) |
| C/Y | = number of compound periods per year |
| PMT: | = choose END |

To determine a value:

- move the cursor to the appropriate line and press **ALPHA** **ENTER**

Ex. 1 Harriet invested \$4 000 in an investment fund that pays 6.5%/a compounded monthly. How much money is in the account after 8 years?

| |
|--|
| N= 8 |
| I%= 6.5 |
| PV= -4000 |
| PMT= 0 |
| → FV= <input type="text" value="6718.68"/> ? ALPHA Enter |
| P/Y= 1 |
| C/Y= 12 |
| PMT: <u>END</u> BEGIN |

∴ Harriet has \$6718.68

Ex. 2 How much do you need to invest now at 8.2%/a compounded quarterly to have \$20 000 in 5 years?

| |
|--|
| N= 5 |
| I%= 8.2 |
| → PV= <input type="text" value="-13328.15"/> |
| PMT= 0 |
| FV= 20000 |
| P/Y= 1 |
| C/Y= 4 |
| PMT: <u>END</u> BEGIN |

Ex. 3 Graham deposits \$4500 into an account paying 3.5%/a compounded monthly. How long will it take him to have \$8000 to buy a motorcycle?

$$\begin{aligned} \Rightarrow N &= \boxed{} \doteq 16.5 \\ I\% &= 3.5 \\ PV &= -4500 \\ PMT &= 0 \\ FV &= 8000 \\ P/Y &= 1 \\ C/Y &= 12 \\ PMT: & \text{END} \text{ BEGIN} \end{aligned}$$

\therefore It will take him approx. $16\frac{1}{2}$ yrs.

Ex. 4 Jason invested \$1200 in a fund that compounded interest semi-annually. At what rate did he invest at if he earned \$500 interest in 6 years?

$$\begin{aligned} N &= 6 \\ \Rightarrow I\% &= \boxed{} \doteq 5.9 \\ PV &= -1200 \\ PMT &= 0 \\ FV &= 1700 \\ P/Y &= 1 \\ C/Y &= 2 \\ PMT: & \text{END} \text{ BEGIN} \end{aligned}$$

\therefore Interest was 5.9%

Ex. 5 Which investment will reach \$10 000 faster?

A: \$7500 invested at
4.5%/a compounded monthly

OR B: \$8200 invested at
4.2% compounded semi-annually

→ N = ~~12~~ 6.4
 I% = 4.5
 PV = -7500
 PMT = 0
 FV = 10000
 P/Y = 1
 C/Y = 12
 PMT: END BEGIN

→ N = ~~12~~ = 4.8
 I% = 4.2
 PV = -8200
 PMT = 0
 FV = 10000
 P/Y = 1
 C/Y = 2
 PMT: END BEGIN

∴ Definitely B

Ex.6 Brian is investing \$6800 at an interest rate of 7% per annum, compounded quarterly, for 2 years. Then, he will invest the amount plus additional money at 6.5%/a, compounded semi-annually, for 3 years. At the end of the second investment, he wants to have \$15000. How much extra must he invest for the second investment?

| |
|--|
| $N = 2$ |
| $I\% = 7$ |
| $PV = -6800$ |
| $PMT = 0$ |
| $\rightarrow FV = \text{[scribble]} = 7812.40$ |
| $P/Y = 1$ |
| $C/Y = 4$ |
| $PMT: \text{END} \text{ BEGIN}$ |

| |
|---|
| $N = 3$ |
| $I\% = 6.5$ |
| $\rightarrow PV = \text{[scribble]} - 12380.86$ |
| $PMT = 0$ |
| $FV = 15000$ |
| $P/Y = 1$ |
| $C/Y = 2$ |
| $PMT: \text{END} \text{ BEGIN}$ |

Need 12380.86 to reach \$15000

So we need $12380.86 - 7812.40$ additionally.

\therefore We need an additional 4568.46

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#4 - 7 (use calculator)

#8-11,15,16

