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Unit 6 pm Test

Name: Solutions

Date: May 27th, 2014

S1. Demonstrate an understanding of recursive sequences, represent recursive sequences in a variety of ways, and make connections to Pascal's triangle
S2. Demonstrate an understanding of the relationships involved in arithmetic and geometric sequences and series, and solve related problems

Part A: Write your simplified answer in the space provided. [1 each] [13]

1. Determine the third term of the sequence given $t_n = \frac{2n^3 - 3}{n^2}$ $\frac{17}{3}$

2. Find the simplified general term if
   a) the first term is -21 and the common ratio is 8 $t_n = -21 \cdot 8^{n-1}$
   b) the first term 12 and the common difference is -3 $t_n = -3n + 15$

3. Find the third term of the sequence defined recursively for $n \in N$ and $n>1$
   $t_1 = -2, \quad t_n = 5(t_{n-1}) - 4$ -74

4. Find the next two terms of each sequence:
   a) -3, 9, -27, ... $81, -243$
   b) 8, - 1, -10 $-19, -28$

5. A term of each sequence is represented by a variable. If the sequence is
   a) Geometric, what is the value of m: 16, m, 4... 8
   b) Arithmetic, what is the value of m: 16, m, 4, ... 10

6. What row number of Pascal's triangle has a row sum of 8192? 2^{13} row 13

7. Determine a recursion formula for 23,-46, 92,... $t_1 = 23, t_n = -2t_{n-1}$

8. Use patterns in the terms of the expansion to determine the following of $(x-y)^{17}$.
   a) The number of terms in the expansion 18
   b) The value of k in the term $-6188x^k y^5$ $K = 12$
   c) The coefficient of the term kx^3y^{14} $\binom{17}{14} = 680$

Part B: For full marks you must <u>show all of your work and state the formula</u> used for sequence and series questions.

9. Calculate the sum of the first 24 terms for the following series: $-6, \frac{-7}{2}, -1 \ldots$ [3]

$a = -6$

$d = t_2 - t_1$
$= -\frac{7}{2} - (-6)$
$= -\frac{7}{2} + \frac{12}{2}$
$= \frac{5}{2}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{24} = \frac{24}{2}[2(-6) + 23(\frac{5}{2})]$
$= 12[-12 + \frac{115}{2}]$
$= 12[\frac{-24 + 115}{2}]$
$= 6[91]$
$= 546$

$\therefore S_{24} = 546$

10. Describe how Pascal's Triangle and expanding binomials are related. [2]

n=0 1
n=1 1 1
n=2 1 2 1
n=3 1 3 3 1
n=4 1 4 6 4 1
r=0 r=1 r=2

→ binomial coefficients can be obtained by the use of Pascal's triangle.

→ Each row of triangle gives the combinatorial numbers

ex. $4C_2 = 6$, same coefficient as middle term of $(a+b)^4$.

11. Determine the general simplified term for the following sequence: 4096, 2048, 1024,.... [3]

$a = 4096$

$r = \frac{t_2}{t_1}$

$= \frac{2048}{4096}$

$= \frac{1}{2}$

$t_n = a \cdot r^{n-1}$

$= 4096 \cdot \frac{1}{2}^{n-1}$

$= 2^{12} \cdot (2^{-1})^{n-1}$

$= 2^{12} \cdot 2^{-n+1}$

$= 2^{-n+13}$

$\therefore t_n = 2^{-n+13}$

12. Determine the number of terms for the following sequence: 5, 20, 80, ... 81920 [3]

$n = ?$

$a = 5$

$r = \frac{20}{5}$

$= 4$

$t_n = 81920$

$t_n = a \cdot r^{n-1}$

$81920 = 5 \cdot 4^{n-1}$

$16384 = 4^{n-1}$

$4^7 = 4^{n-1}$

$\therefore 7 = n - 1$

$n = 8$

$\therefore$ there are 8 terms

13. The 10th term of an arithmetic series is 34, and the sum of the first 20 terms is 710. Determine the 25th term. [4]

$t_{10} = 34$

$34 = a + (10-1)d$

$34 = a + 9d$ (1)

$S_{20} = 710$

$710 = \frac{20}{2}[2a + (20-1)d]$

$710 = 10[2a + 19d]$

$71 = 2a + 19d$ (2)

$a = 34 - 9d$ (1a)

subst (1a) into (2)

$71 = 2(34 - 9d) + 19d$

$71 = 68 - 18d + 19d$

$d = 3$

$a = 34 - 9(3)$

$= 7$

$t_n = a + (n-1)d$

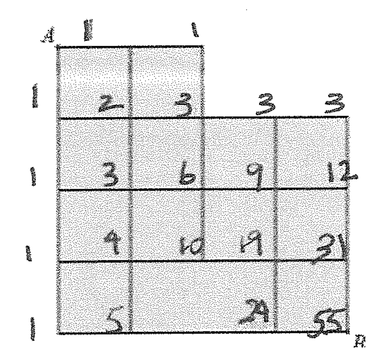
$t_{25} = 7 + (25-1)3$

$= 79$

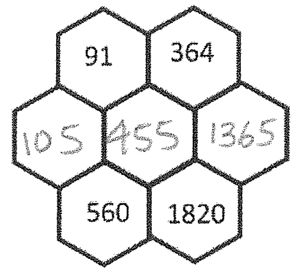
$\therefore t_{25} = 79$

14. How many paths are there from A to B? Assume there is no backtracking. [2]

there are 55 paths from A to B



15. Use your knowledge of Pascal's Triangle to fill in the missing numbers [3]



16. In a lecture hall there are 16 seats in the first row. The number of seats in each successive row increases by 3. How many seats in the 15^{th} row? [3]

$a = 16$

$d = 3$

$t_{15} = ?$

$t_n = a + (n-1)d$

$t_{15} = 16 + (15-1)3$

$= 16 + 42$

$= 58$

$\therefore$ there will be 58 seats in the 15^{th} row.

17. You agree to do the household chores every day for a month (30 days). You have a choice of being paid in one of two ways: 1 cent on day one, 2 cents on day two, 4 cents on day three, etc., doubling each day; or $10 for each day. Which option would you choose and why? [4]

$a = 1$

$r = 2$

$n = 30$

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_{30} = \dfrac{1(2^{30} - 1)}{2 - 1}$

$= 1\,073\,741\,823$ ¢

$= \$10\,737\,418.23$

or option 2 $= 30(10)$

$= 300$

you would be a multi-millionaire with option 1.

18. Find the 6th simplified term in the expansion $\left(2x - \dfrac{3}{\sqrt[4]{x}}\right)^{10}$ [3]

$a = 2x$

$b = -3x^{-\frac{1}{4}}$

$n = 10$

$\binom{10}{5}(2x)^5\left(-3x^{-\frac{1}{4}}\right)^5$

$= (252)(32x^5)\left(-243x^{-\frac{5}{4}}\right)$

$= 1\,959\,552\, x^{\frac{20-5}{4}}$

$= 1\,959\,552\sqrt[4]{x^{15}}$

19. The terms given by x-2, x+7, 48 form a <u>geometric</u> sequence. Find the value(s) of x. [3]

$t_1 = x - 2$

$t_2 = x + 7$

$t_3 = 48$

$r = r$

$\dfrac{t_2}{t_1} = \dfrac{t_3}{t_2}$

$\dfrac{x+7}{x-2} = \dfrac{48}{x+7}$

$(x+7)(x+7) = 48(x-2)$

$x^2 + 14x + 49 = 48x - 96$

$x^2 + 14x - 48x + 49 + 96 = 0$

$0 = x^2 - 34x + 145$

$x = \dfrac{34 \pm \sqrt{(-34)^2 - 4(1)(145)}}{2(1)}$

$= \dfrac{34 \pm 24}{2}$

$x = \dfrac{34+24}{2}$ $\quad x = \dfrac{34-24}{2}$

$= 29$ $\quad = 5$

$\therefore x = 29, 5$