B **1.** Find the first and second derivatives of the given functions.

(a) $f(x) = x^5 - 4x^2 + 1$

(b) $g(x) = 7x^4 + 12x^3 - 4x + 8$

(c) $f(t) = 2t - \dfrac{1}{t+1}$ (d) $g(t) = \dfrac{4}{\sqrt{t}}$

(e) $y = (2x + 1)^8$ (f) $y = t^3 + \dfrac{1}{t^3}$

(g) $y = \sqrt{x^2 + 1}$ (h) $y = \dfrac{t}{t-1}$

2. Find the third derivative.

(a) $f(x) = 1 - 12x + 4x^2 - x^3$ (b) $f(x) = \dfrac{1}{x^5}$

(c) $y = \dfrac{3}{(4-x)^2}$ (d) $y = \sqrt{1 + 2x}$

3. Find the first six derivatives of the function $y = x^5 + x^4 + x^3 + x^2 + x + 1$.

4. If $f(x) = \sqrt{1 + x^3}$, find $f''(2)$.

5. If $g(x) = \dfrac{1}{\sqrt{3x + 4}}$, find $g'''(4)$.

6. If $f(x) = x^n$, find $f^{(n)}(x)$.

7. Find y'' by implicit differentiation.

(a) $x^4 + y^4 = 1$ (b) $x^2 - y^2 = 1$ (c) $x^3 + y^3 = 6xy$

8. Find a quadratic function f such that $f(3) = 33$, $f'(3) = 22$, and $f''(3) = 8$.

C **9.** Suppose that $f(x) = g(x)h(x)$.

(a) Express f'' in terms of g, g', g'', h, h', h''.

(b) Find a similar expression for f'''.

10. (a) If $f(x) = |x^2 - 1|$, find f' and f'' and state their domains.

(b) Sketch the graphs of f, f', and f''.

EXERCISE 2.8

1. (a) $f'(x) = 5x^4 - 8x$, $f''(x) = 20x^3 - 8$

(b) $f'(x) = 28x^3 + 36x^2 - 4$.
$f''(x) = 84x^2 + 72x$

(c) $f'(t) = 2 + \dfrac{1}{(t+1)^2}$; $f''(t) = -\dfrac{2}{(1+t)^3}$

(d) $g'(t) = -2t^{-\frac{3}{2}}$; $g''(t) = 3t^{-\frac{5}{2}}$

(e) $y' = 16(2x + 1)^7$, $y'' = 224(2x + 1)^6$

(f) $y' = 3t^2 - 3t^{-4}$, $y'' = 6t + 12t^{-5}$

(g) $y' = x(x^2 + 1)^{\frac{1}{2}}$, $y'' = (x^2 + 1)^{\frac{3}{2}}$

(h) $y' = -\dfrac{1}{(t-1)^2}$, $y'' = \dfrac{2}{(t-1)^3}$

2. (a) $f''''(x) = -6$ (b) $f''''(x) = -\dfrac{210}{x^8}$

(c) $y'''' = \dfrac{72}{(4-x)^5}$ (d) $y'''' = 3(1 + 2x)^{\frac{5}{2}}$

3. $y' = 5x^4 + 4x^3 + 3x^2 + 2x + 1$, $y'' = 20x^3 + 12x^2 + 6x + 2$, $y''' = 60x^2 + 24x + 6$. $y^{(4)} = 120x + 24$, $y^{(5)} = 120$, $y^{(6)} = 0$

4. $\frac{2}{3}$ **5.** $-\frac{405}{131\,072}$ **6.** $n!$

7. (a) $-\dfrac{3x^2}{y^7}$ (b) $-\dfrac{1}{y^3}$ (c) $\dfrac{16xy}{(2x - y^2)^3}$

8. $f(x) = 4x^2 - 2x + 3$

9. (a) $f''' = g'''h + 2g''h' + gh''$

(b) $f'''' = g''''h + 3g'''h' + 3g''h'' + gh'''$

10. (a)
$$f'(x) = \begin{cases} 2x & \text{if } |x| > 1 \\ -2x & \text{if } |x| < 1 \end{cases}$$
$\text{dom}(f') = \{x \mid x \neq \pm 1\}$

$$f''(x) = \begin{cases} 2 & \text{if } |x| > 1 \\ -2 & \text{if } |x| < 1 \end{cases}$$
$\text{dom}(f'') = \{x \mid x \neq \pm 1\}$

(b)