3.1 Higher-order Derivatives

The derivative of a derivative is called the second derivative and can be represented as f''(x),

Other notations y'', or $\frac{d^2y}{dx^2}$

The derivative of the second derivative is called the third derivative and can be represented as f'''(x),

Other notations y''', or $\frac{d^3y}{dx^3}$

Feb 22-7:30 PM

Ex.1 Find the second derivative of

a) $y = (2x-1)^4$

$y' = 4(2x-1)^3(2)$

$= 8(2x-1)^3$

$y'' = 24(2x-1)^2(2)$

$= 48(2x-1)^2$

b) $y = \dfrac{x}{x^2-1}$

$y' = \dfrac{1(x^2-1) - x(2x)}{(x^2-1)^2}$

$= \dfrac{-x^2-1}{(x^2-1)^2}$

$y'' = \dfrac{(-2x)(x^2-1)^2 - (-x^2-1)2(x^2-1)(2x)}{(x^2-1)^4}$

$= \dfrac{-2x(x^2-1)\left[(x^2-1) + (-x^2-1)(2)\right]}{(x^2-1)^4}$

$= \dfrac{-2x(x^2-1)(x^2-1-2x^2-2)}{(x^2-1)^4}$

$= \dfrac{-2x(x^2-1)(-x^2-3)}{(x^2-1)^{4\,3}}$

$= \dfrac{-2x(-x^2-3)}{(x^2-1)^3}$

Ex.2 Find f''(1), if $f(x) = (2-x^2)^{10}$

$f'(x) = 10(2-x^2)^9(-2x)$

$= -20x(2-x^2)^9$

PRODUCT RULE!

$f''(x) = (-20)(2-x^2)^9 + (-20x)(9)(2-x^2)^8(-2x)$

$f''(1) = -20(1) + (-20)(9)(1)(-2)$

$= 340$

Mar 7-9:57 AM

Ex.3 a)Find the first five derivatives of $y=x^4+2x^3-e^x+2\cos x$
b) Can you predict the 8th derivative? 21st?

a) $y' = 4x^3 + 6x^2 - e^x - 2\sin x$

$y'' = 12x^2 + 12x - e^x - 2\cos x$

$y''' = 24x + 12 - e^x + 2\sin x$

$y^{(4)} = 24 - e^x + 2\cos x$

$y^{(5)} = -e^x - 2\sin x$

b) $y^{(8)} = -e^x + 2\cos x$

$y^{(21)} = -e^x - 2\sin x$

Mar 7-9:58 AM

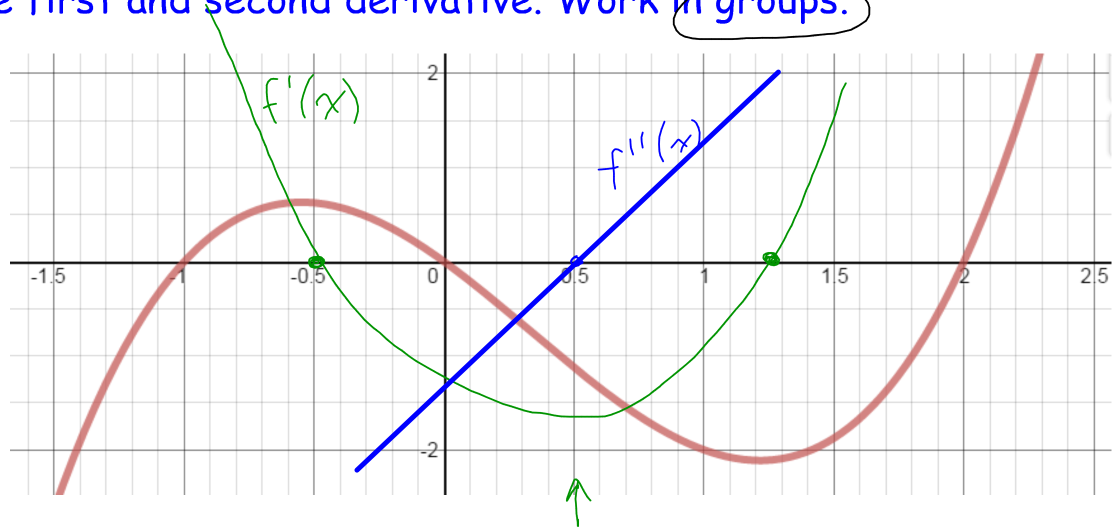
Since a derivative can be described as a rate of change or the slope of the tangent line.

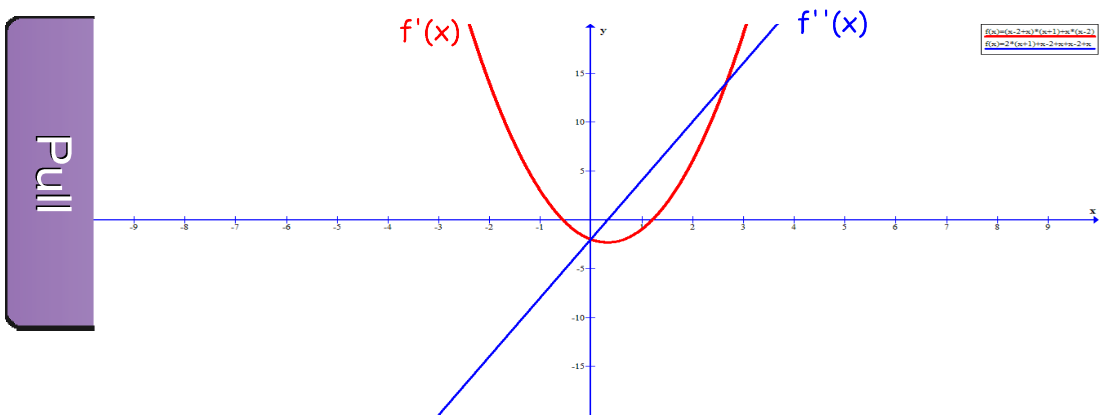
What would the derivative of the derivative represent?

A derivative of the derivative (second derivative) is the rate of change of the slope of the tangent line.

Ex.4 Given the following functions, sketch the graphs of the first and second derivative. Work in groups.

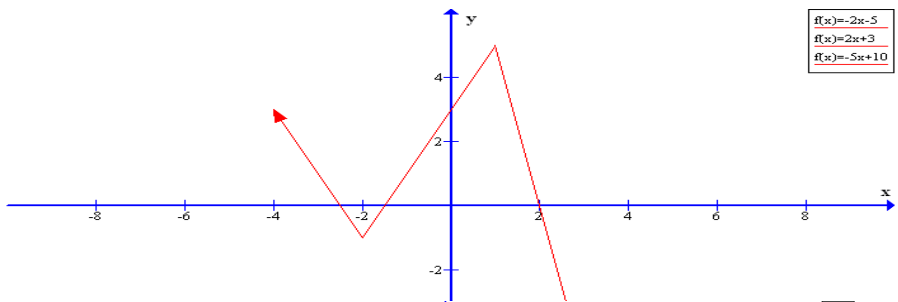
a)

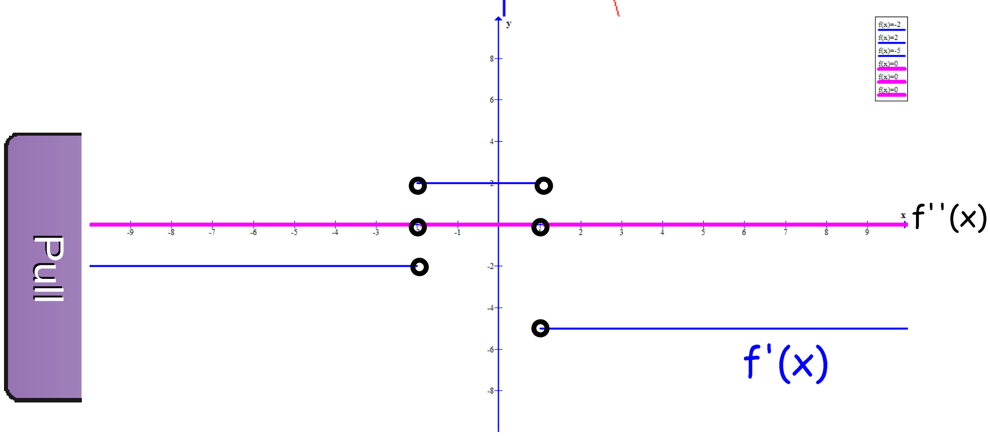




Mar 6-7:43 PM

b)





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Feb 22-8:52 PM