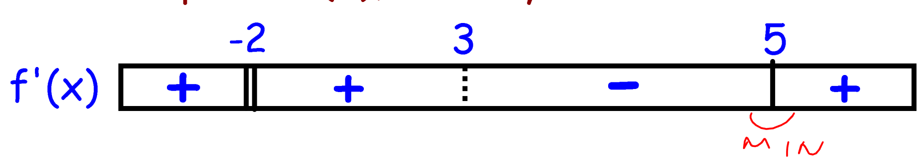
3.3 Critical Points, Local Maxima/Minima

Critical Values for $f'(x)$:

- include all values of x where $f'(x) = 0$ OR $f'(x)$ is undefined
- all local max/mins occur at a critical value of $f'(x)$
- not all critical values yield a max/min (do 1st derivative test)
- critical values where $f'(x)$ is undefined give the locations of cusps, corners, vertical tangents or a discontinuity of $f(x)$

> **1st Derivative Test:**
> If $x=c$ is a critical value, then there is a:
> - local maximum if $f'(x)$ changes from positive to negative at c
> - local minimum if $f'(x)$ changes from negative to positive at c

Eg. 1
Given this strip for $f'(x)$, identify:



critical values: $x: -2, 3, 5$
intervals of increase: $(-\infty, -2)(-2, 3)(5, \infty)$
intervals of decrease: $(3, 5)$
local max: none, asymptote @ $x = 3$
local min: $x = 5$

Ex. 2 Determine the critical values and local extrema.

a) $y = \frac{1}{2}x^4 - \frac{16}{3}x^3 + 16x^2$

$y' = 2x^3 - 16x^2 + 32x$

$= 2x(x^2 - 8x + 16)$

$= 2x(x-4)^2$

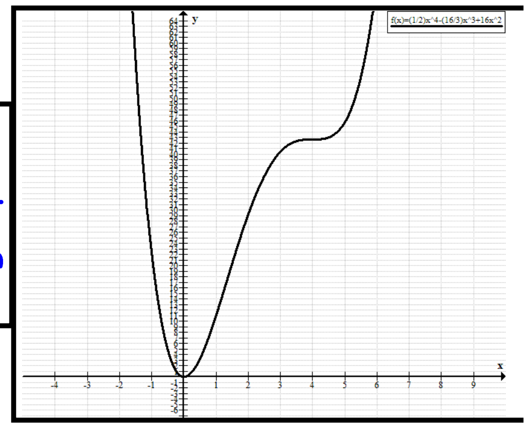
Crit #s: $x = 0, 4$

	0		4	
$-$	|	$+$	||	$+$

MIN

$\therefore$ Min @ $x = 0$

graph



b) $y = x^3$

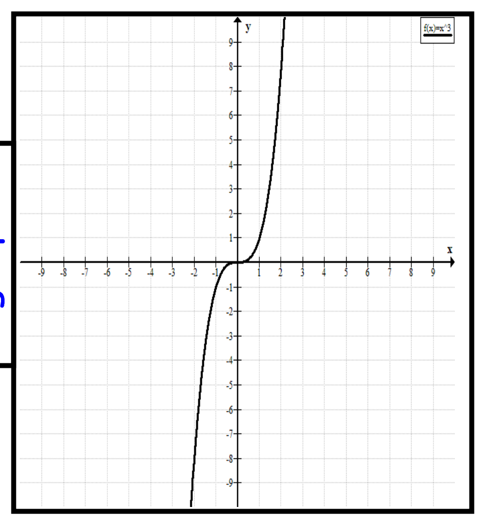
$y' = 3x^2$

Crit #s: $x = 0$

	0	
$+$	||	$+$

$\therefore$ No max/min

graph



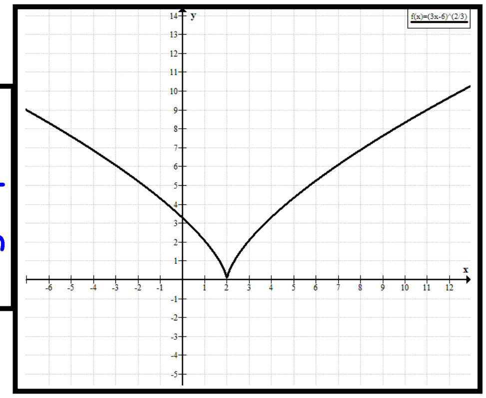
c) $f(x) = (3x-6)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(3x-6)^{-\frac{1}{3}}(3)$

$= \dfrac{2}{(3x-6)^{\frac{1}{3}}}$

Crit #s: $x \neq 2$

graph



2

−	+

MIN

$\therefore$ Min @ $f(2) = 0$, even though $f'(2)$ dne

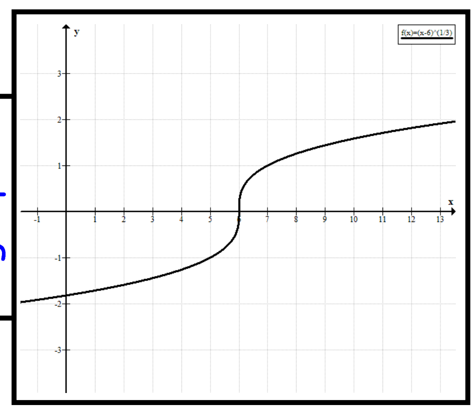
d) $f(x) = \sqrt[3]{x-6}$

$f'(x) = \frac{1}{3}(x-6)^{-\frac{2}{3}}$

$= \dfrac{1}{3(x-6)^{\frac{2}{3}}}$

Crit #s: $x \neq 6$

graph



6

+	+

$\therefore$ No max/min

e) $f(x) = \dfrac{x^2 - 9}{x - 3}$

$= \dfrac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$

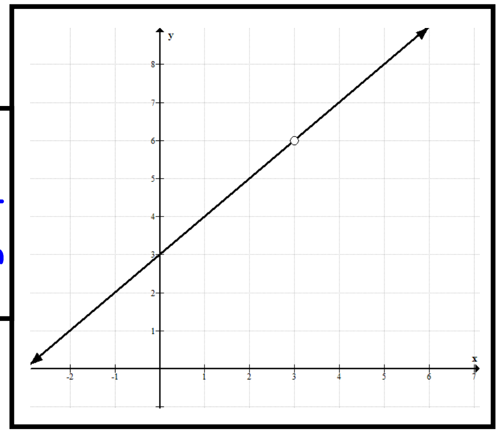
$= x+3, \; x \neq 3$

$f'(x) = 1$

3

+	•	+

graph



f) $f(x) = |x - 3|$

ALWAYS BREAK INTO TWO PARTS

$f(x) = \begin{cases} x-3, & x > 3 \\ -(x-3), & x < 3 \end{cases}$

$= \begin{cases} x-3, & x > 3 \\ -x+3, & x < 3 \end{cases}$

$f'(x) = \begin{cases} 1, & x > 3 \\ -1, & x < 3 \end{cases}$

f'(3) dne

3

−	+

MIN

$\therefore$ Min @ $f(3) = 0$

graph

Homework
page 179
5ac,7ace,9,10,12,

Mar 3-8:53 PM