# 3.4 Vertical, Horizontal, and Oblique Asymptotes

## A. Vertical Asymptotes

The graph of f(x) has a vertical asymptote at $x=a$, if $f(x)\Rightarrow\infty$ or $-\infty$ as $x\Rightarrow a^+$ or $x\Rightarrow a^-$.

In the case of rational functions, vertical asymptotes occur where the function is undefined and no "hole" exists.

Eg.1 Determine the equations of all vertical asymptotes.

a) $f(x)=\dfrac{3x-2}{(2x+1)(x-4)}$

$x=-\frac{1}{2}, 4$

b) $f(x)=\dfrac{x-3}{x^2+x-12}$

$f(x)=\dfrac{\cancel{x-3}}{(\cancel{x-3})(x+4)}$

$\therefore$ Asymptote @ $x=-4$

Hole @ $x=3$

## B. Horizontal Asymptotes

- describes the behaviour of f(x) as $x\Rightarrow\pm\infty$
- exists when the degree of the numerator is equal to or less than the degree of the denominator
- note: it is possible for the function to cross the H.A. many times!!!!

RECALL method for determining the limit of f(x) as $x\Rightarrow\pm\infty$:

method:
- write f(x) as a limit
- divide all terms by the highest power of x in the denominator
- simplify
- evaluate the limit as $x\Rightarrow\pm\infty$
- the resulting value is the equation of the H.A.

Eg. 2 Determine the equation of the H.A. if one exists.

a) $f(x)= \dfrac{5x - 7}{x + 3}$

$\lim_{x\to\infty} \dfrac{5x-7}{x+3}$

$= \lim_{x\to\infty} \dfrac{5 - \frac{7}{x}\to 0}{1 + \frac{3}{x}\to 0}$ $\Rightarrow = \dfrac{5}{1}$

$\therefore$ HA @ $y = 5$

b) $f(x)= \dfrac{x^2 - 5x +3}{x^3 - 7x^2 + 2}$

$\lim_{x\to\infty} \dfrac{x^2-5x+3}{x^3-7x^2+2}$

$= \lim_{x\to\infty} \dfrac{\frac{1}{x}\to 0 - \frac{5}{x^2}\to 0 + \frac{3}{x^3}\to 0}{1 - \frac{7}{x}\to 0 + \frac{2}{x^3}\to 0}$

$= 0$

$\therefore$ HA @ $y = 0$

c) $f(x)= \dfrac{3x^3-7x^2 + x - 4}{2x^2 - 5x + 2}$

Degree? $\dfrac{\text{Deg } 3}{\text{Deg } 2}$

$\therefore$ No H.A.

Deg N > Deg D

d) $g(x)= \dfrac{5x^2 + 7x -11}{7 - 15x^2}$

QUICK METHOD

(Look at degree)

$y = \dfrac{5}{-15}$

$y = -\dfrac{1}{3}$

$\therefore$ HA @ $y = -\dfrac{1}{3}$

## C. Oblique Asymptotes

- occurs when the degree of the numerator is is greater than the degree of the denominator

- the equation of the O.A. is determined by using long division to divide the numerator by the denominator then taking the limit as $x\Rightarrow\pm\infty$ of the resulting division statement.

Eg. 3 Determine the equation of any Oblique Asymptotes

a) $f(x)= \dfrac{2x^3 - x^2 +3}{x^2 - 4x +2}$

$$\begin{array}{r|l} & 2x + 7 \\ x^2-4x+2 & 2x^3 - x^2 + 0 + 3 \\ - & 2x^3 - 8x^2 + 4x \\ & 7x^2 - 4x + 3 \\ - & 7x^2 - 28x + 14 \\ & 24x - 11 \end{array}$$

$\therefore$ OA @ $y = 2x + 7$

b) $g(x)= \dfrac{5x^3 + 3x^2 - 5x + 1}{x^2}$

$= 5x + 3 - \dfrac{5}{x} + \dfrac{1}{x^2}$

$\lim_{x\to\infty} g(x)$

$= 5x + 3$

$\therefore$ OA @ $y = 5x + 3$

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