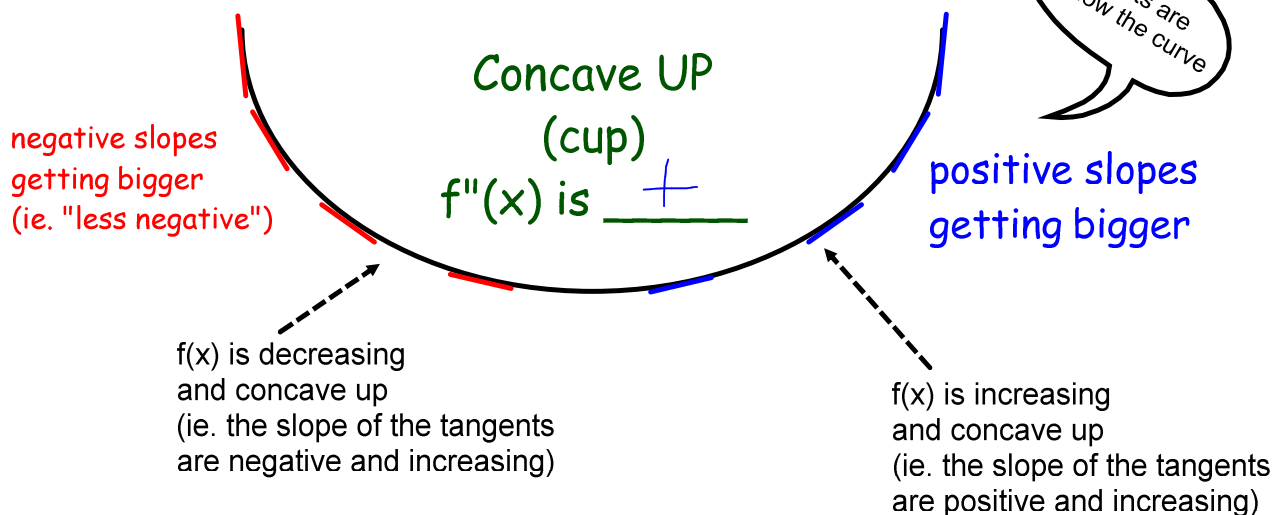


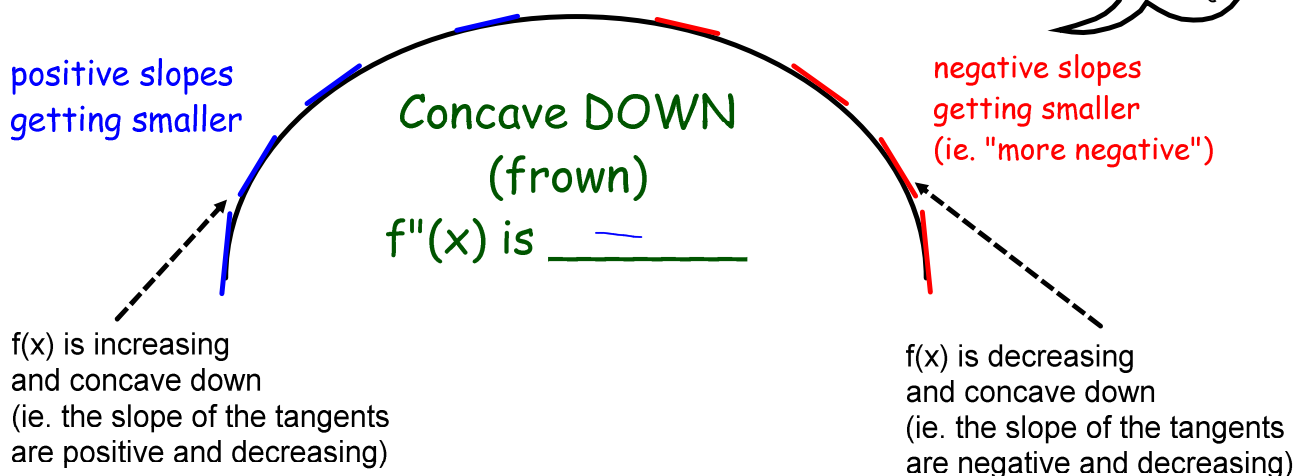
3.5 Concavity and Points of Inflection

Concavity:

Case 1: Consider the shape of $f(x)$ when the slopes of the tangents are increasing (ie. getting larger)

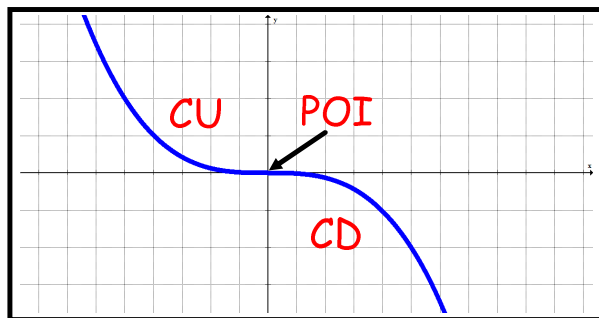


Case 2: Consider the shape of $f(x)$ when the slopes of the tangents are decreasing (ie. getting smaller)

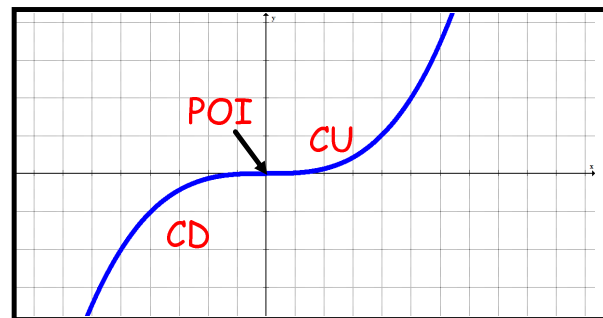


Point of Inflection:

occurs when $f(x)$ changes from CU to CD or from CD to CU.



decreasing



increasing

Summary

concave up: $f'(x)$ is increasing so $f''(x) > 0$

concave down: $f'(x)$ is decreasing so $f''(x) < 0$

point of inflection (POI): where the function changes from CU to CD or vice versa.
 $f''(x) = 0$ or is undefined

critical #'s occur when
 $f''(x) = 0$ or dne

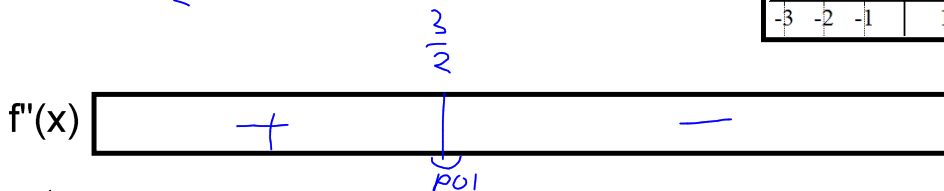
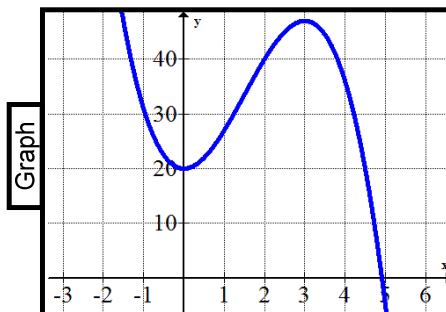
Ex. 1 Determine the intervals of concavity.

a) $f(x) = -2x^3 + 9x^2 + 20$

$f'(x) = -6x^2 + 18x$

$f''(x) = -12x + 18$

$f''(x) = 0$
 $x = \frac{3}{2}$



\therefore
 CU $(-\infty, \frac{3}{2})$
 CD $(\frac{3}{2}, \infty)$

b) $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$

$f'(x) = 4x^3 + 6x^2 - 6x - 4$

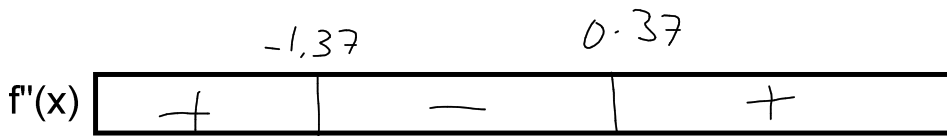
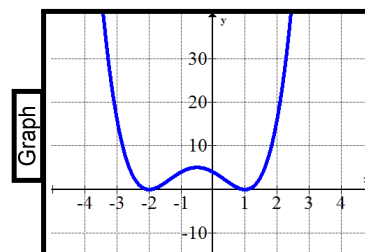
$f''(x) = 12x^2 + 12x - 6$

$0 = 6(2x^2 + 2x - 1)$

Crit #s:

$x = \frac{-1 \pm \sqrt{3}}{2}$

M -2
 A 2
 N ???
 QUAD FORMULA



\therefore CU $(-\infty, -1.37) \cup (0.37, \infty)$

CD $(-1.37, 0.37)$

Ex. 2 Determine the intervals of concavity and points of inflection.

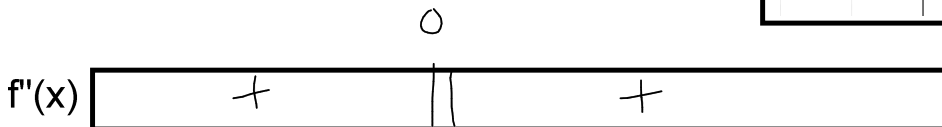
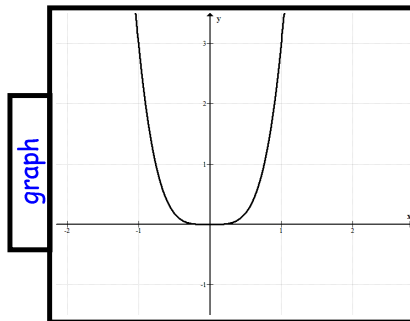
a) $f(x) = 3x^4$

$f'(x) = 12x^3$

$f''(x) = 36x^2$

Crit #s:

$x = 0, 0$



\therefore CU $(-\infty, 0) \cup (0, \infty)$

NO POI

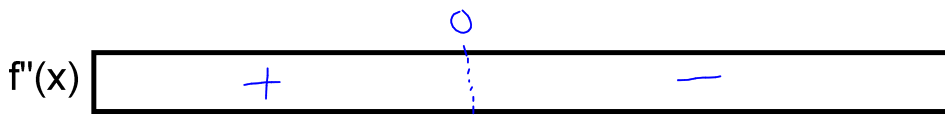
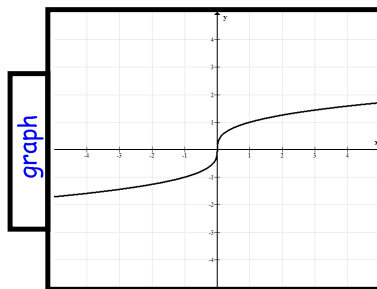
b) $f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f''(x) = -\frac{2}{9}x^{-\frac{5}{3}}$

$0 = -\frac{2}{9x^{\frac{5}{3}}}$

Crit #s: $x \neq 0$



\therefore CU $(-\infty, 0)$

CD $(0, \infty)$

POI @ $x = 0$
 $(0, 0)$

$$c) f(x) = \frac{1}{x^2+3}, f'(x) = \frac{-2x}{(x^2+3)^2}, f''(x) = \frac{6(x^2-1)}{(x^2+3)^3}$$

$$f'(x) = 0$$

$$0 = 6(x^2-1)$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\text{Crit \#s: } x = \pm 1$$

arab



$$\therefore \text{CU } (-\infty, -1) \cup (1, \infty)$$

$$\text{CD } (-1, 1)$$

$$\text{POI } (-1, f(-1)) \Rightarrow \left(-1, \frac{1}{4}\right)$$

$$\left(1, \frac{1}{4}\right)$$

Homework
page 205
1,3,4,5,9,12

Members of Germany's "Point of Inflection"

