

3.6 Curve Sketching (Day 1)

Analysis of $f(x)$

1. Domain
2. Intercepts(x and y)
3. Asymptotes (vertical, horizontal, oblique)

Analysis of $f'(x)$

4. Intervals of increase/decrease

5. Local extrema (max/min) →

Can use the 1st or 2nd derivative test to check if max/min

Analysis of $f''(x)$

6. Intervals of concavity

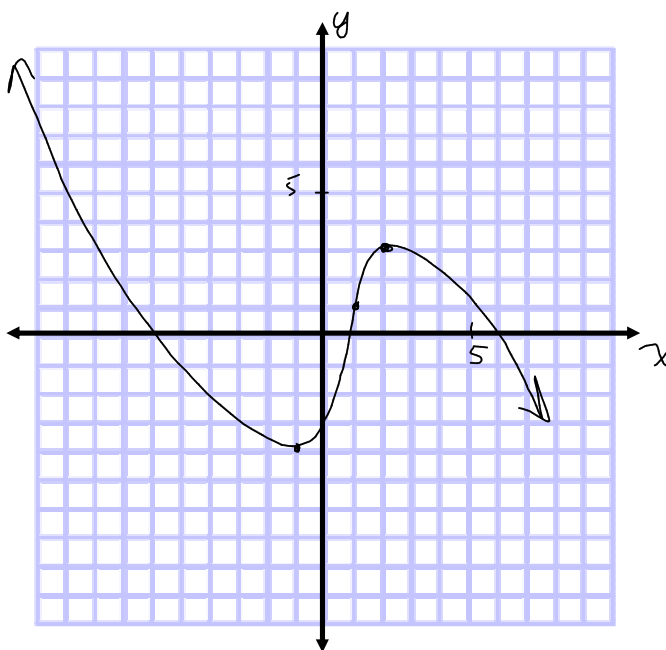
7. Points of inflection

Last step:

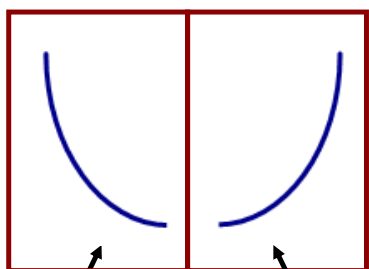
8. SKETCH!

Ex. 1 Sketch, given the following:

- ✓ ● Local max at (2,3) and local min at (-1,-4)
- ✓ ● Decreasing on $(-\infty, -1)$ and $(2, \infty)$; increasing on $(-1, 2)$
- ✓ ● CU on $(-\infty, 1)$; CD on $(1, \infty)$
- ✓ ● POI at (1,1)



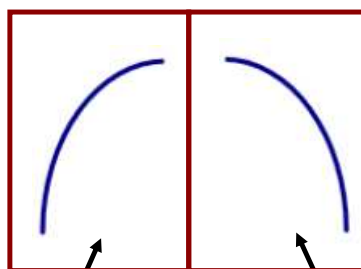
Concave UP



CU and decreasing

CU and increasing

Concave Down

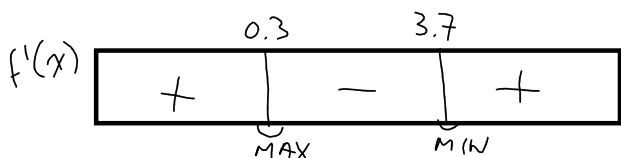
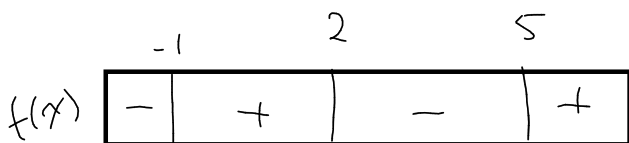


CD and increasing

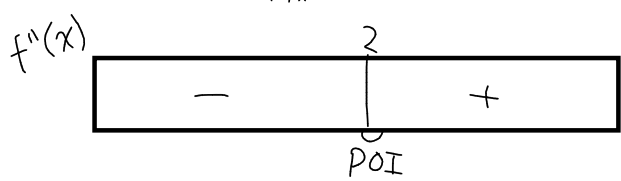
CD and decreasing

Ex. 2 Sketch $y = x^3 - 6x^2 + 3x + 10$

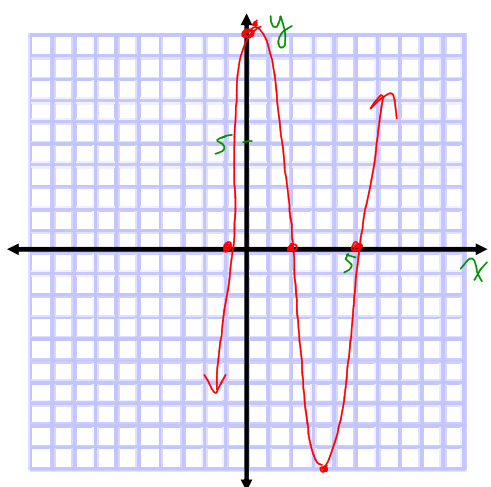
y	y'	y''
$-1 \left \begin{array}{cccc} 1 & -6 & 3 & 10 \\ 1 & -1 & 7 & -10 \\ \hline 1 & -7 & 10 & 0 \end{array} \right.$ $y = (x+1)(x^2 - 7x + 10)$ $y = (x+1)(x-2)(x-5)$ <p>Zeros: $x = -1, 2, 5$ y-int: 10</p>	$y' = 3x^2 - 12x + 3$ $= 3(x^2 - 4x + 1)$ $x = 2 \pm \sqrt{3}$ <p>Crit #s: $x = 0.3, 3.7$</p>	$y'' = 6x - 12$ $= 6(x - 2)$ <p>Crit #s: $x = 2$</p>



MAX $f(0.3) = 10.4$
 MIN $f(3.7) = -10.4$



POI $f(2) = 0$



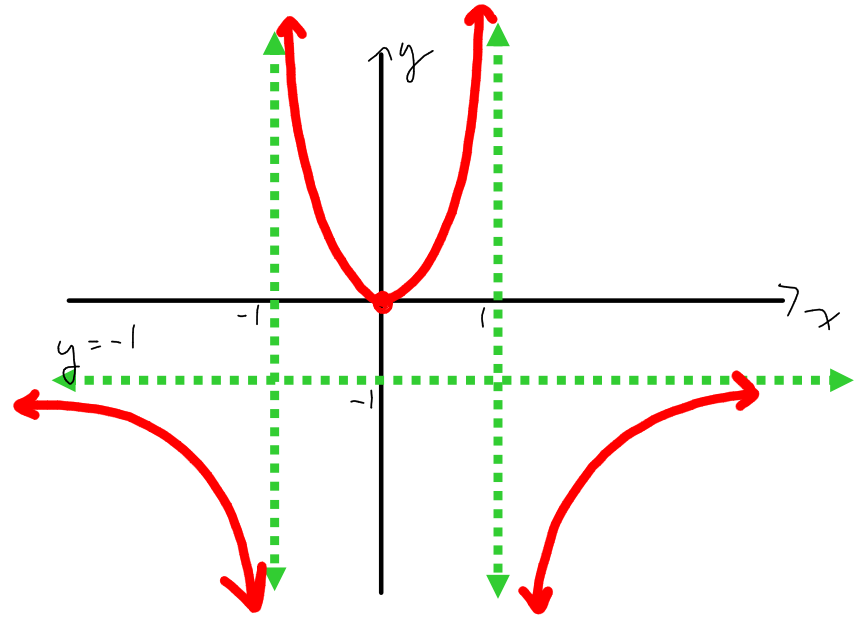
Ex. 3 Sketch $y = \frac{x^2}{1-x^2}$

y Zeros: $x=0,0$ VA: $x=\pm 1$ HA: $y=-1$	y' $y' = \frac{2x(1-x^2) - x^2(-2x)}{(1-x^2)^2}$ $y' = \frac{2x-2x^3+2x^3}{(1-x^2)^2}$ $y' = \frac{2x}{(1-x^2)^2}$ Zeros: $x=0$ VA: $x=\pm 1, \pm 1$	y'' $y'' = \frac{2(1-x^2)^2 - 2x(2)(1-x^2)(-2x)}{(1-x^2)^4}$ $= \frac{2-2x^2+8x^2}{(1-x^2)^3}$ $= \frac{6x^2+2}{(1-x^2)^3}$ $= \frac{2(3x^2+1)}{(1-x^2)^3}$
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Crit #s: $x \neq \pm 1, \pm 1, \pm 1$

y	<table border="0" style="width: 100%; text-align: center;"> <tr> <td style="border-right: 1px dashed black; width: 20%;">-</td> <td style="border-right: 1px dashed black; width: 20%;">+</td> <td style="border-right: 1px solid black; width: 20%;">+</td> <td style="border-right: 1px dashed black; width: 20%;">-</td> <td style="width: 20%;">-</td> </tr> </table>	-	+	+	-	-
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MIN @ (0,0)



Ex. 4 Sketch $y = \frac{x^2 + 2x - 8}{x - 3}$

y
VA @ $x=3$

OA?

$$\begin{array}{r} x+5 \\ x-3 \overline{)x^2+2x-8} \\ \underline{x^2-3x} \\ 5x-8 \\ \underline{5x-15} \\ 7 \end{array}$$

\therefore OA @ $y = x + 5$

Zeros:

$$\begin{aligned} 0 &= x^2 + 2x - 8 \\ &= (x+4)(x-2) \\ x &= -4, 2 \end{aligned}$$

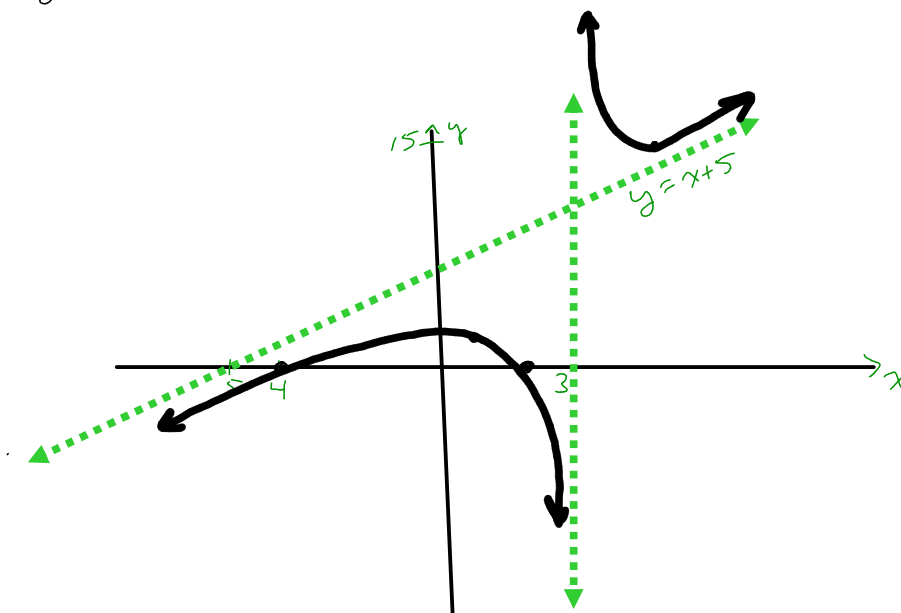
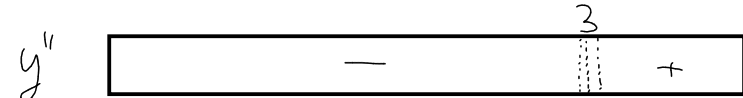
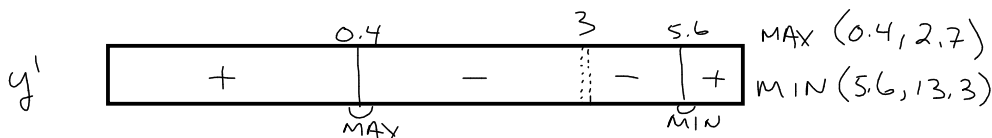
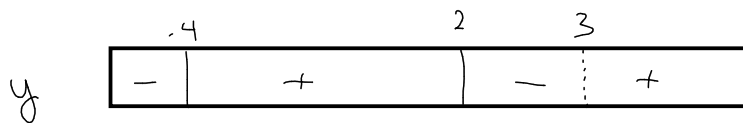
$$\begin{aligned} y' &= \frac{(2x+2)(x-3) - (x^2+2x-8)(1)}{(x-3)^2} \\ &= \frac{2x^2 - 4x - 6 - x^2 - 2x + 8}{(x-3)^2} \\ &= \frac{x^2 - 6x + 2}{(x-3)^2} \end{aligned}$$

Crit #s:

$$\begin{aligned} x &= 3 \pm \sqrt{7} \\ &\approx 0.4, 5.6 \\ x &\neq 3, 3 \end{aligned}$$

$$\begin{aligned} y'' &= \frac{(2x-6)(x-3) - (x^2-6x+2)(2)}{(x-3)^4} \\ &= \frac{2x^2 - 12x + 18 - 2x^2 + 12x - 4}{(x-3)^3} \\ &= \frac{14}{(x-3)^3} \end{aligned}$$

Crit #s: $x \neq 3, 3, 3$



Homework
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