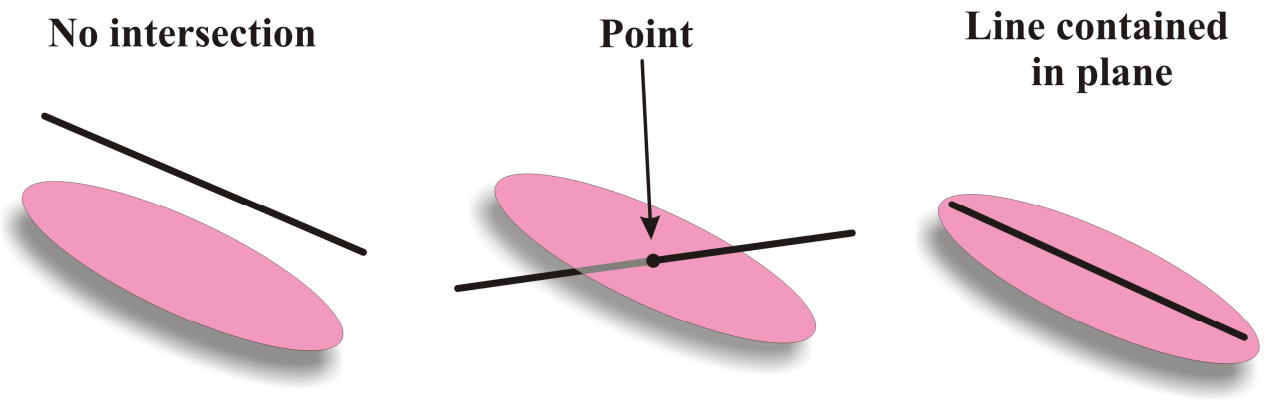
7.1 Intersection of a Line with a Plane
Intersection of Two Lines

Intersection of a line and a plane:



May 18-8:47 AM

Ex. 1 Determine the point of intersection of the line and plane.

a) l: $x=3+4t$, $y=2+3t$, $z=-1+2t$ $\quad \pi: x - y - 2z = 0$

Sub in

$(3+4t) - (2+3t) - 2(-1+2t) = 0$

$3+4t-2-3t+2-4t = 0$

$3 - 3t = 0$

$t = 1$

∴ Yes, they intersect.

where?

$x = 3+4(1) = 7$

$y = 2+3(1) = 5$

$z = -1+2(1) = 1$

at $(7, 5, 1)$

b) l: $\vec{r} = (2,-5,6) + t(3,1,8)$ $\quad \pi: 5x + y - 2z + 2 = 0$

$x = 2+3t$

$y = -5+t$

$z = 6+8t$

$5(2+3t) + (-5+t) - 2(6+8t) + 2 = 0$

$10+15t-5+t-12-16t+2 = 0$

$0t = 5$

$LS \neq RS$

∴ No solution

∴ They do not intersect

c) l: $x = 2 + t$, $y = -1 - 2t$, $z = -3 - 5t$ $\quad \pi: 3x + 19y - 7z - 8 = 0$

$3(2+t) + 19(-1-2t) - 7(-3-5t) - 8 = 0$

$6+3t-19-38t+21+35t-8 = 0$

$0t = 0$

$LS = RS$

but cannot solve for t

$t \in \mathbb{R}$

∴ the line is in the plane

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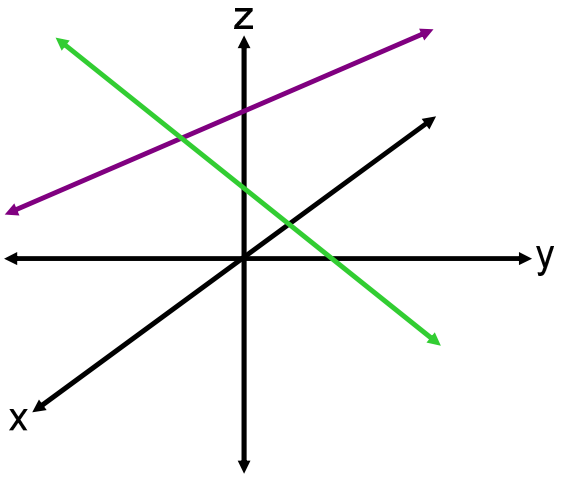
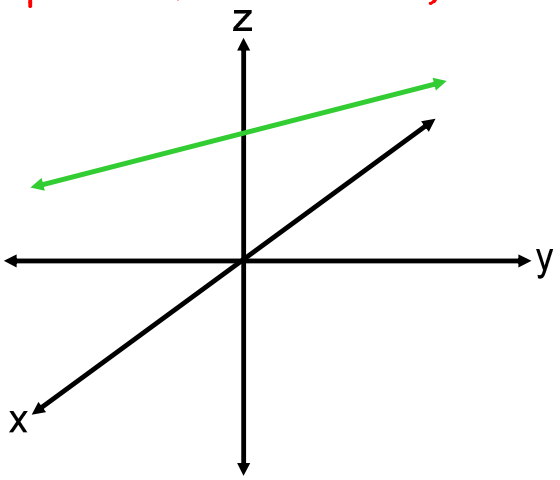
Intersection of Two Lines:

A: Intersecting Lines

Intersect at One Point

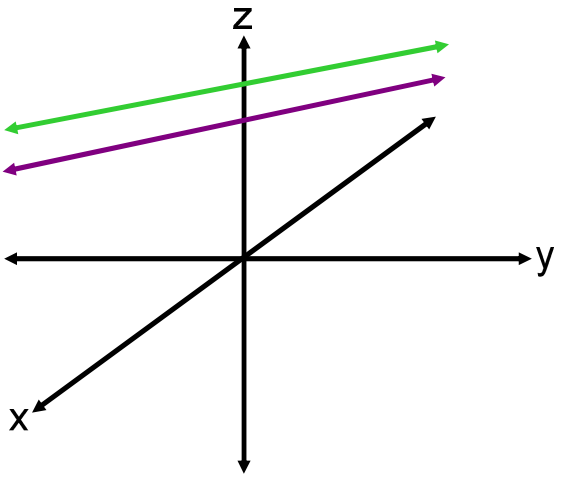


Coincident Lines (all points on line are points of intersection)

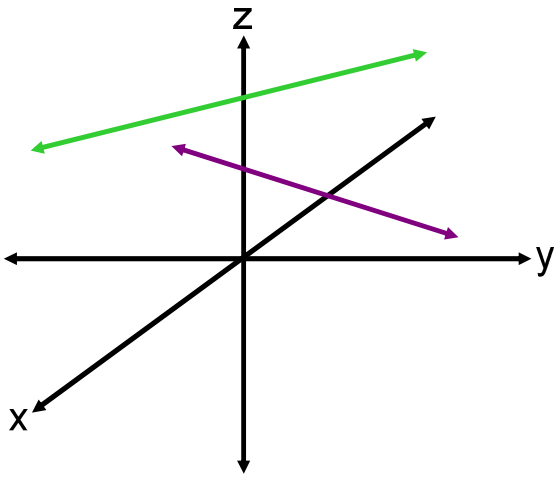


B: Non-Intersecting Lines

Parallel Lines



Skew Lines



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Ex. 2 Determine the point of intersection of the two lines.

l_1: $x=-5+3s$, $y=2+2s$, $z=-7+6s$ $\quad$ l_2: $x=t$, $y=-6-5t$, $z=-3-t$

$\vec{m}_1=(3,2,6)$ $\quad$ $\vec{m}_2=(1,-5,-1)$

NOT parallel.

Must be point or skew.

Set equal, solve, check

① $-5+3s=t$

② $2+2s=-6-5t$

③ $-7+6s=-3-t$

Sub ① into ②

$2+2s=-6-5(-5+3s)$

$2+2s=-6+25-15s$

$17s=17$

$s=1$

Sub back into ①

$-5+3(1)=t$

$-2=t$

Check $s=1$ & $t=-2$ in ③

LS $=-7+6(1)=-1$ $\quad$ RS $=-3-(-2)=-1$

LS = RS

∴ Intersect!

Sub into either set of parametric eqn's

Find point using either $s=1$ or $t=-2$

$(-2,4,-1)$

b) l_1: $\vec{OP}=(-2,1,0)+s(1,3,7)$ $\quad$ l_2: $\vec{r}=(3,-3,4)+t(5,-4,-2)$

$\vec{m}_1=(1,3,7)$ $\quad$ $\vec{m}_2=(5,-4,-2)$

∴ Must be a point or skew

Set equal, solve & check

① $-2+s=3+5t$

② $1+3s=-3-4t$

③ $7s=4-2t$

① $s=5+5t$

Sub into ②

$1+3(5+5t)=-3-4t$

$19t=-19$

$t=-1$

Sub back into ①

$-2+s=3+5(-1)$

$s=0$

Check $t=-1$ & $s=0$ in ③

LS $=7s=0$ $\quad$ RS $=4-2t=6$

LS ≠ RS

∴ The lines are SKEW!

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c) l_1: $x = 3 + 2t$ l_2: $r = (1, -1, 3) + s(-4, 6, -2)$

$y = -1 - 3t$

$z = 1 + t$

$\vec{m}_1 = (2, -3, 1)$ $\vec{m}_2 = (-4, 6, -2)$

$-2\vec{m}_1 = \vec{m}_2$

$\therefore$ Lines are collinear

Check for coincidence using any point on either line.

Sub $(1, -1, 3)$ into l_1

$1 = 3 + 2t$ $-1 = -1 - 3t$ $3 = 1 + t$

$t = -1$ $t = 0$

$\therefore$ Inconsistent!

They are parallel, but are not coincident.

Homework
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#4a,5ab,6-10(not7b),1