2.5 Solving Polynomial Equations

- When solving an equation, you are finding the x-intercepts. Any polynomial equation of the form f(x)=0 can be solved.

- ***** Factor!!!!!!*****

example 1: Solve

a) $x^3-x^2-2x=0$

$x(x^2-x-2)=0$ ← always take out common factor

$x(x-2)(x+1)=0$ ← if quadratic is left, try Product and Sum "MAN"

← Set each factor equal to zero and solve

$x=0$ $x=2$ $x=-1$

b) $3x^3+x^2-12x-4=0$ ← CF? if more than 3 terms, try grouping

$=x^2(3x+1)-4(3x+1)$

$=(3x+1)(x^2-4)$

$=(3x+1)(x+2)(x-2)$

$x=-2, -\frac{1}{3}, 2$

c) $2x^3+3x^2=11x+6$

$0 = 2x^3+3x^2-11x-6$ ← Bring every term to one side

NOPE ← CF? Grouping?

$\cancel{f(1) = -12}$ ← Use factor theorem then long or synthetic division

$f(2) = 0$ ✓

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -11 & -6 \\ & & 4 & 14 & 6 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

M 6
A 7
N $\frac{2}{6}$ $\frac{2}{1}$

$= (x-2)(2x^2+7x+3)$

$= (x-2)(x+3)(2x+1)$

$\therefore x = -3, -\frac{1}{2}, 2$

d) $x^3-8=0$ ← Don't forget sum & diff of cubes

$= (x-2)(x^2+2x+4)$ ← use quad. formula for quadratics that are not factorable

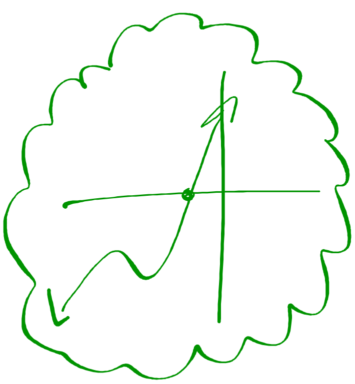
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\therefore$

$x = 2$

b^2-4ac
$= 4-4(1)(4)$
$= -12$

NO REAL ROOTS!



example 2: The volume, V, in cm³, of a block of ice that a sculptor uses can be modelled by $V(x)=9x^3+60x^2+249x$ where x represents the thickness of the block, in cm. What is the maximum thickness that can be carved from a block of ice with volume 2532 cm³?

What thickness(es) give us this volume?

$2532=9x^3+60x^2+249x$ ← SOLVE

$0=9x^3+60x^2+249x-2532$

$=3(3x^3+20x^2+83x-844)$

$f(4)=0$

$=(x-4)(\underbrace{3x^2+32x+211}_{b^2-4ac<0})$

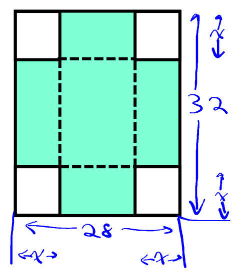
4	3	20	83	−844
		12	128	844
	3	32	211	0

∴ No real roots.

∴ 4 cm is the max. thickness

example 3: Open top boxes are constructed by cutting equal squares from the corners of cardboard sheets that measure 32 by 28. Determine possible dimensions of the boxes if each has a volume of 1344 cm³.

let x be the width of the cutout squares.



$V=L\cdot W\cdot h$

$1344=(32-2x)(28-2x)x$

$0=(896-64x-56x+4x^2)x-1344$

$=4x^3-120x^2+896x-1344$

$=4(x^3-30x^2+224x-336)$

$f(2)=0$

2	1	−30	224	−336
		2	−56	336
	1	−28	168	0

$=4(x-2)(\underbrace{x^2-28x+168}_{\text{QUAD Formula}})$

$x=2$

$x \doteq 8.7$

$x \doteq 19.3$

	$x=2$	$x\doteq 8.7$
L : $32-2x$	28	14.6
W : $28-2x$	24	10.6
H : x	2	8.7

~~$x \doteq 19.3$~~ Inadmissable

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example 4: Solve

$x^3-3x=-1$

$0=x^3-3x+1$

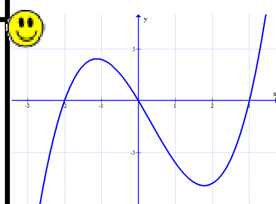
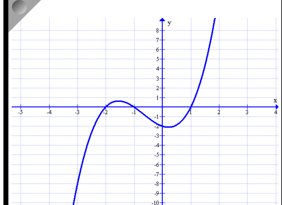
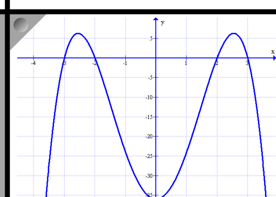
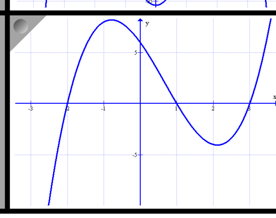
$f(1)\neq 0$?

$f(-1)\neq 0$

use graphing calculators when not factorable

(ANY TECH)

Ex: 5. Factor the following polynomial equations. Using graphing technology, sketch the polynomial equation and determine the x-intercepts .

Polynomial Equation	Factor	Sketch	x-intercepts
$y=x^3-x^2-6x$	$=x(x^2-x-6)$ $=x(x-3)(x+2)$		-2,0,3
$y=x^3+2x^2-x-2$	$=x^2(x+2)-(x+2)$ $=(x+2)(x^2-1)$ $=(x+2)(x-1)(x+1)$		-2,-1,1
$y=-x^4+13x^2-36$			-3,-2,2,3
$y=x^3-2x^2-5x+6$			-3,1,2

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