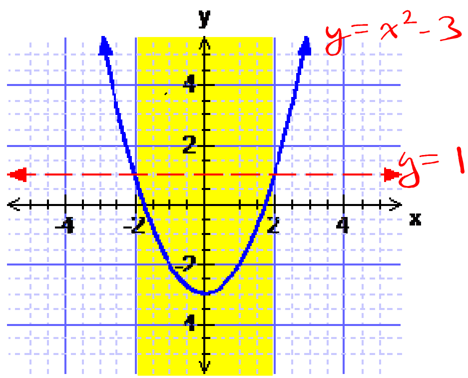
2.8 Solving Inequalities II

Recall:
- To solve a polynomial equation, set it equal to zero and solve. For many equations, you will need to factor.
- Solving inequalities yields an interval as the solution set.

Ex. 1 Solve $x^2 - 3 < 1$

$\rightarrow$ We want to find the interval (x-values) over which the y-value of the parabola defined by $f(x) = x^2 - 3$ is less than 1.



Graphically we get $-2 < x < 2$ as a solution.

$(-2, 2)$

Algebraically:

$x^2 - 3 < 1$ $x^2 - 4 < 0$	$\leftarrow$ this is equivalent to finding the interval where $y = x^2 - 4$ is below the x-axis

The endpoints of the solution set we are looking for are the roots of the new function $y = x^2 - 4$.

$\rightarrow$ Find the roots:

Set $y = 0$

$0 = x^2 - 4$

$= (x-2)(x+2)$

$x = \pm 2$

→ Use a table to look at the sign (positive or negative) of the function on either side of each root:

Roots: -2, 2

	Intervals →	$x < -2$	$-2 < x < 2$	$x > 2$
Factors	$x + 2$	−	+	+
	$x - 2$	−	−	+
Polynomial →	$x^2 - 4$	+	−	+

$y = (x+2)(x-2)$

	-2		2	
+		−		+

$\therefore \ x^2 - 4 < 0$ when $-2 < x < 2$

$\therefore \ x^2 - 3 < 1$ when $-2 < x < 2$

$= (-2, 2)$

Ex. 2 Solve the following inequality algebraically.

$$-x^4 + x^3 + 17x^2 - 21x - 30 > 6$$

Rearrange the inequality so that one side is equal to 0 then solve the corresponding equation.

$P(x) = -x^4 + x^3 + 17x^2 - 21x - 36 > 0$

$P(-1) = 0$

$\vdots$ Synthetic Division

$P(x) = (x+1)(-x^3 + 2x^2 + 15x - 36)$

$Q(x) = -3x^3 + 2x^2 + 15x - 36$

$Q(3) = 0$

$\vdots$ Synthetic Division

$Q(x) = (x-3)(-x^2 - x + 12)$

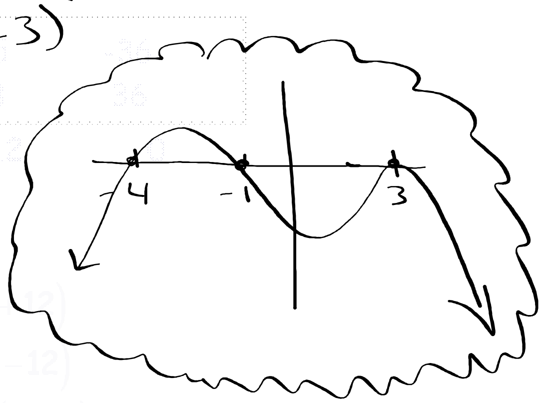
$\therefore P(x) = (x+1)(x-3)(-x^2 - x + 12)$

$= -(x+1)(x-3)(x^2 + x - 12)$

$= -(x+1)(x-3)(x+4)(x-3)$

$= -(x+4)(x+1)(x-3)^2$

Double Root



We need to solve:

$$-(x+1)(x-3)^2(x+4) > 0$$

OR

$$(x+1)(x-3)^2(x+4) < 0$$

	-4	-1	3	
	$x < -4$	$-4 < x < -1$	$-1 < x < 3$	$x > 3$
$x + 1$	−	−	+	+
$(x-3)^2$	+	+	+	+
$x + 4$	−	+	+	+
product	+	−	+	+

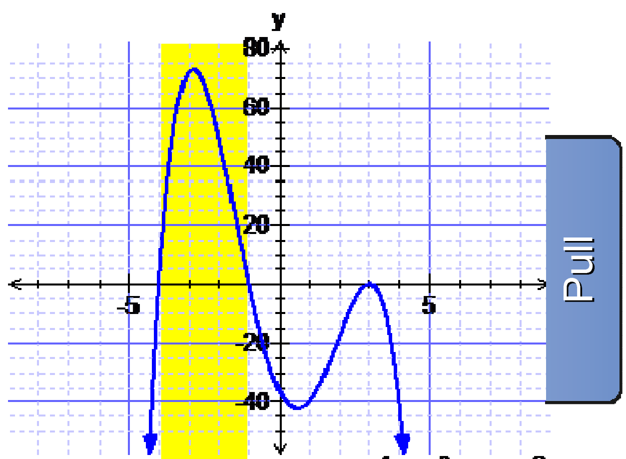
Answer $(-4,-1)$

	-4	-1	3	
$f(x)$	+	−	+	+

Double Root

Sign stays the same

Sketching confirms the result:



$y = -x^4 + x^3 + 17x^2 - 21x - 36$

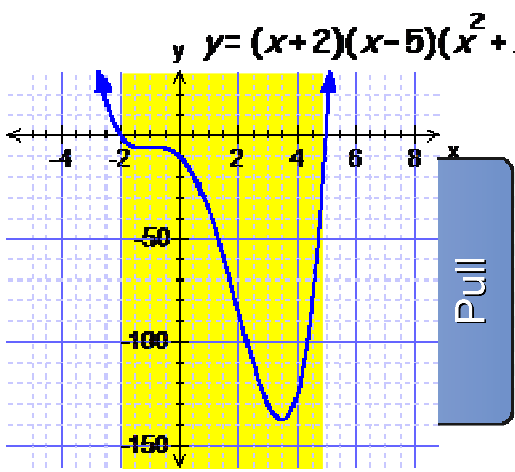
Ex. Solve $(x+2)(x-5)(x^2+x+1)<0$

$\rightarrow$ If we try to solve $x^2+x+1=0$ we get a negative discriminant $\therefore$ the only real roots are -2 and 5.

	$x<-2$	$-2<x<5$	$x>5$
$(x+2)$	$-$	$+$	$+$
$(x-5)$	$-$	$-$	$+$
(x^2+x+1)	$+$	$+$	$+$
$f(x)$	$+$	$-$	$+$

$(-2,5)$

$\therefore$ $(x+2)(x-5)(x^2+x+1)<0$ when $(-2,5)$



Homework
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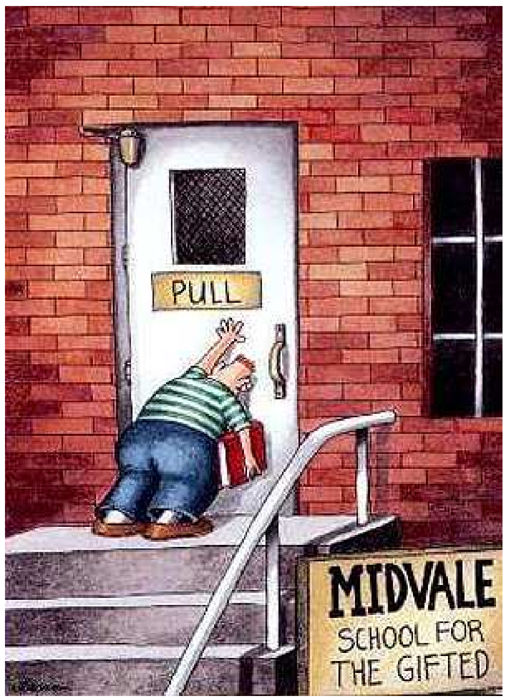

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