

4.1 Related and Co-Related Angle Formulas

1. Use co-related angle identities to simplify, then evaluate. Give exact values.

a) $\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$ b) $\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$ c) $\cot\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$
 d) $\sec\left(\frac{3\pi}{2} - \frac{\pi}{6}\right)$ e) $\sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right)$ f) $\tan\left(\frac{3\pi}{2} - \frac{\pi}{4}\right)$
 g) $\cos\left(\frac{3\pi}{2} + \frac{\pi}{4}\right)$ h) $\tan\left(\frac{3\pi}{2} + \frac{\pi}{3}\right)$ i) $\csc\left(\frac{3\pi}{2} + \frac{\pi}{6}\right)$

2. Express each of the following as a function of it's co-related acute angle, then evaluate. Give exact values.

a) $\cos\frac{5\pi}{6}$ b) $\tan\frac{5\pi}{4}$ c) $\sin\frac{7\pi}{6}$
 d) $\csc\frac{5\pi}{3}$ e) $\sec\frac{4\pi}{3}$ f) $\tan\frac{11\pi}{6}$
 g) $\cos\frac{7\pi}{4}$ h) $\sin\frac{2\pi}{3}$ i) $\cot\frac{7\pi}{6}$

3. Simplify.

a) $\cos x + \cos(\pi - x) - \cos(\pi + x) - \cos(-x)$ b) $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} - x\right) - \tan(2\pi - x)$
 c) $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right)$ d) $\sin\left(\frac{\pi}{2} + x\right) - \cos\left(\frac{3\pi}{2} - x\right) + \sin\left(\frac{3\pi}{2} - x\right)$
 e) $\sin\left(\frac{\pi}{2} - x\right) + \sin(\pi - x) + \sin\left(\frac{3\pi}{2} - x\right) + \sin(2\pi - x)$

4. Find \csc , \sec , and \cot for each angle. Express your answers in terms of $\csc/\sec/\cot$.

a) $\pi - x$ b) $\frac{\pi}{2} + x$ c) $\pi + x$ d) $\frac{3\pi}{2} + x$

5. Simplify.

a) $\sin(x - \pi)$ b) $\cos\left(x - \frac{\pi}{2}\right)$ c) $\tan(-x - \pi)$

6. Simplify.

a) $\frac{\cos(\pi + x)\cos\left(\frac{\pi}{2} + x\right)}{\cos(\pi - x)} - \frac{\sin\left(\frac{3\pi}{2} - x\right)}{\sec(\pi + x)}$ b) $\frac{\sin\left(x - \frac{\pi}{2}\right)}{\cos(\pi - x)} + \frac{\tan\left(x - \frac{3\pi}{2}\right)}{-\tan(\pi + x)}$

answers:

1a) $\frac{1}{\sqrt{2}}$ b) $-\frac{\sqrt{3}}{2}$ c) $-\frac{1}{\sqrt{3}}$ d) -2 e) $-\frac{1}{2}$ f) 1 g) $\frac{1}{\sqrt{2}}$ h) $-\frac{1}{\sqrt{3}}$ i) $-\frac{2}{\sqrt{3}}$ 2a) $-\frac{\sqrt{3}}{2}$ b) 1 c) $-\frac{1}{2}$ d) $-\frac{2}{\sqrt{3}}$ e) -2 f) $\frac{1}{\sqrt{3}}$ g) $\frac{1}{\sqrt{2}}$ h) $\frac{\sqrt{3}}{2}$ i) $\sqrt{3}$
 3a) 0 b) $2\tan x$ c) 0 d) $\sin x$ e) 0 4a) $\csc x, -\sec x, -\cot x$ b) $\sec x, -\csc x, \frac{-1}{\cot x}$ c) $-\csc x, -\sec x, \cot x$ d) $-\sec x, \csc x, \frac{-1}{\cot x}$
 5a) $-\sin x$ b) $\sin x$ c) $-\tan x$ 6a) $-\sin x - \cos^2 x$ b) $\csc^2 x$

1.1 Related and Co-related Angle Formulas

1 a) $\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$
 $= +\cos\left(\frac{\pi}{4}\right)$
 $= +\frac{1}{\sqrt{2}}$

b) $\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$
 $= -\sin\left(\frac{\pi}{3}\right)$
 $= -\frac{\sqrt{3}}{2}$

c) $\cot\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$
 $= -\tan\left(\frac{\pi}{6}\right)$
 $= -\frac{1}{\sqrt{3}}$

d) $\sec\left(\frac{3\pi}{2} - \frac{\pi}{6}\right)$
 $= -\csc\left(\frac{\pi}{6}\right)$
 $= -2$

e) $\sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right)$
 $= -\cos\left(\frac{\pi}{3}\right)$
 $= -\frac{1}{2}$

f) $\tan\left(\frac{3\pi}{2} - \frac{\pi}{4}\right)$
 $= \cot\left(\frac{\pi}{4}\right)$
 $= 1$

g) $\cos\left(\frac{3\pi}{2} + \frac{\pi}{4}\right)$
 $= \sin\left(\frac{\pi}{4}\right)$
 $= \frac{1}{\sqrt{2}}$

h) $\tan\left(\frac{3\pi}{2} + \frac{\pi}{3}\right)$
 $= -\cot\left(\frac{\pi}{3}\right)$
 $= -\frac{1}{\sqrt{3}}$

i) $\csc\left(\frac{3\pi}{2} + \frac{\pi}{6}\right)$
 $= -\sec\left(\frac{\pi}{6}\right)$
 $= -\frac{2}{\sqrt{3}}$

2 a) $\cos\frac{5\pi}{6}$
 $= \cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$
 $= -\sin\left(\frac{\pi}{3}\right)$
 $= -\frac{\sqrt{3}}{2}$

b) $\tan\frac{5\pi}{4}$
 $= \tan\left(\frac{3\pi}{2} - \frac{\pi}{4}\right)$
 $= +\cot\left(\frac{\pi}{4}\right)$
 $= 1$

c) $\sin\frac{7\pi}{6}$
 $= \sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right)$
 $= -\cos\left(\frac{\pi}{3}\right)$
 $= -\frac{1}{2}$

d) $\csc\frac{5\pi}{3}$
 $= \csc\left(\frac{3\pi}{2} + \frac{\pi}{6}\right)$
 $= -\sec\left(\frac{\pi}{6}\right)$
 $= -\frac{2}{\sqrt{3}}$

e) $\sec\frac{4\pi}{3}$
 $= \sec\left(\frac{3\pi}{2} + \frac{\pi}{6}\right)$
 $= -\csc\left(\frac{\pi}{6}\right)$
 $= -2$

f) $\tan\frac{11\pi}{6}$
 $= \tan\left(\frac{3\pi}{2} + \frac{\pi}{3}\right)$
 $= -\cot\left(\frac{\pi}{3}\right)$
 $= -\frac{1}{\sqrt{3}}$

g) $\cos\frac{7\pi}{4}$
 $= \cos\left(\frac{3\pi}{2} + \frac{\pi}{4}\right)$
 $= +\sin\left(\frac{\pi}{4}\right)$
 $= \frac{1}{\sqrt{2}}$

h) $\sin\frac{2\pi}{3}$
 $= \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$
 $= \cos\left(\frac{\pi}{6}\right)$
 $= \frac{\sqrt{3}}{2}$

i) $\cot\frac{7\pi}{6}$
 $= \cot\left(\frac{3\pi}{2} - \frac{\pi}{6}\right)$
 $= \tan\left(\frac{\pi}{6}\right)$
 $= \frac{1}{\sqrt{3}}$

3 a) $\cos x - \cos x + \cos x - (\cos x)$
 $= 0$

b) $\tan x - \tan x + \tan x + \tan x$
 $= 2 \tan x$

c) $-\sin x + \sin x - \cot x + \cot x$
 $= 0$

d) $-\cos x + \sin x + \cos x$
 $= \sin x$

e) $\cos x + \cos x - \cos x - \cos x$
 $= 0$

4 a) $\csc(\pi - x)$
 $= \csc(x)$
 $= \sec(\pi - x)$
 $= -\sec(x)$
 $= \cot(\pi - x)$
 $= -\cot(x)$

b) $\csc\left(\frac{\pi}{2} + x\right)$
 $= \sec(x)$
 $= \sec\left(\frac{\pi}{2} + x\right)$
 $= -\csc(x)$
 $= \cot\left(\frac{\pi}{2} + x\right)$
 $= -\tan(x)$

c) $\csc(\pi + x)$
 $= -\csc(x)$
 $\sec(\pi + x)$
 $= -\sec(x)$
 $\cot(\pi + x)$
 $= \cot(x)$

d) $\csc\left(\frac{3\pi}{2} + x\right)$
 $= -\sec(x)$
 $\sec\left(\frac{3\pi}{2} + x\right)$
 $= +\csc(x)$
 $\cot\left(\frac{3\pi}{2} + x\right)$
 $= -\tan(x)$

$$5) a) \sin(x - \pi) \\ = \sin(-(\pi - x)) \\ = -\sin(x)$$

$$b) \cos(x - \pi/2) \\ = \cos(-(\pi/2 - x)) \\ = +\sin(x)$$

$$c) \tan(-(\pi + x)) \\ = -\tan(x)$$

$$6) a) \frac{\cos(\pi + x) \cos(\pi/2 + x)}{\cos(\pi - x)} - \frac{\sin(3\pi/2 - x)}{\sec(\pi + x)}$$

$$= \frac{[-\cos(x)][\sin(x)]}{[-\cos(x)]} - \frac{[-\cos(x)]}{[-\sec(x)]}$$

$$= \sin(x) - \cos x \cdot \frac{\cos x}{1}$$

$$= \sin x - \cos^2 x$$

$$b) \frac{\sin(x - \pi/2)}{\cos(\pi - x)} + \frac{\tan(x - 3\pi/2)}{-\tan(\pi + x)}$$

$$= \frac{\sin(-(\pi/2 - x))}{-\cos(x)} + \frac{\tan(-(\pi/2 - x))}{-\tan(x)}$$

$$= \frac{-\cos(x)}{-\cos(x)} + \frac{-\cot(x)}{-\tan(x)}$$

$$= 1 + \left(-\frac{1}{\tan x}\right) \cdot \left(\frac{1}{\tan x}\right)$$

$$= 1 + \frac{1}{\tan^2 x}$$

$$= \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x}$$

$$= \csc^2 x$$