1. Determine the exact value of each of the following.

a) $\sin\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$ b) $\cos\left(-\frac{\pi}{6}-\frac{\pi}{4}\right)$ c) $\tan\left(-\frac{3\pi}{4}+\frac{2\pi}{3}\right)$

2. Evaluate and state the exact value of each of the following.

a) $\sin\left(\frac{11\pi}{12}\right)$ b) $\cos\left(\frac{13\pi}{12}\right)$ c) $\tan\left(-\frac{7\pi}{12}\right)$

d) $\tan\left(-\frac{5}{12}\pi\right)$ e) $\sin\left(\frac{5}{12}\pi\right)$ f) $\cos\left(-\frac{\pi}{12}\right)$

3. Determine the exact value of each of the following.

a) $\sin\left(\frac{5\pi}{18}\right)\cos\left(\frac{\pi}{9}\right)-\cos\left(\frac{5\pi}{18}\right)\sin\left(\frac{\pi}{9}\right)$ b) $\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{4\pi}{21}\right)-\sin\left(\frac{\pi}{7}\right)\sin\left(\frac{4\pi}{21}\right)$

c) $\dfrac{\tan\left(\frac{7\pi}{180}\right)+\tan\left(\frac{2\pi}{45}\right)}{1-\tan\left(\frac{7\pi}{180}\right)\tan\left(\frac{2\pi}{45}\right)}$ d) $\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{18}\right)+\cos\left(\frac{5\pi}{36}\right)\sin\left(\frac{5\pi}{18}\right)$

4. Simplify

a) $\dfrac{\sin\left(x-\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{3}-x\right)}{\sin x}$ b) $\dfrac{\tan\left(\frac{\pi}{4}-x\right)-\tan\left(\frac{\pi}{4}+x\right)}{\tan x}$

Answers:

1a) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ c) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ 2a) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ b) $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ c) $-\frac{1+\sqrt{3}}{1-\sqrt{3}}$ d) $\frac{\sqrt{3}+1}{1-\sqrt{3}}$ e) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ f) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

3a) $\frac{1}{2}$ b) $\frac{1}{2}$ c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ d) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ 4a) $\sqrt{3}$ b) $\frac{-4}{1-\tan^2 x}$ c) $\frac{\cot x}{2}$

7.3 Double Angle Formulas

1. $cosx = -\frac{4}{5}, \frac{\pi}{2} \leq x \leq \pi$ find the value of $sin2x$ and $cos2x$. Determine the quadrant of angle 2x.
2. $sinx = \frac{12}{13,0} \leq x \leq \frac{\pi}{2}$ find the value of $sin2x$ and $cos2x$. Determine the quadrant of angle 2x.
3. $sinx = \frac{2}{3,0} \leq x \leq \frac{\pi}{2}$ find the value of $sin4x$.
4. $cosx = \frac{\frac{2}{5,3\pi}}{2} \leq x \leq 2\pi$ find the value of $csc2x$ and $sec2x$.
5. $tanx = \frac{1}{2,0} \leq x \leq \frac{\pi}{2}$ find the value of $tan2x$.
6. $tanx = 2, -2\pi \leq x \leq -\frac{3\pi}{2}$ evaluate $tan4x$.
7. Develop formulas for:
   a) $sin3x$ in terms of $sinx$
   b) $\cos 3x$ in terms of $\cos x$
   c) $tan3x$ in terms of $tanx$
   d) $cos4x$ in terms of $cosx$
8. Express
   a) $\sin\left(\frac{x}{2}\right)$ in terms of $cosx$
   b) $\cos\left(\frac{x}{2}\right)$ in terms of $cosx$
   c) $\tan\left(\frac{x}{2}\right)$ in terms of $cosx$
9. If $cosx = -\frac{12}{13}$ and x is in the interval $\left(\pi, \frac{3\pi}{2}\right)$ find the following:
   a) $\sin\left(\frac{x}{2}\right)$ b) $\cos\left(\frac{x}{2}\right)$ c) $\tan\left(\frac{x}{2}\right)$

Answers:

1a) $sin2\theta = \frac{-24}{25}, cos2\theta = \frac{7}{25}, 4th$ 2) $sin2\theta = \frac{120}{169}, cos2\theta = \frac{119}{169}, 2nd$ 3) $\frac{8\sqrt{5}}{81}$

4) $csc2\theta = \frac{-25}{4\sqrt{21}}, sec2\theta = \frac{-25}{17}$

5) $\frac{4}{3}$ 6) $\frac{24}{7}$ 7a) $3sin\theta - 4sin^3\theta$ b) $4cos^3\theta - 3cos\theta$ c) $\frac{3tan\theta - tan^3\theta}{1 - 3tan^2\theta}$ d) $8cos^4\theta - 8cos^2\theta + 1$

8a) $\sqrt{\frac{1 - cos\theta}{2}}$ b) $\sqrt{\frac{1 + cos\theta}{2}}$ c) $\sqrt{((1 - cos\theta)/(1 + cos\theta))}$ 9a) $\frac{5}{\sqrt{26}}$ b) $-\frac{1}{26}$ c) -5