A Factoring Trigonometric Expressions/Equations

1. Factor/for the last one, set factors equal to zero and stop.
   a) $\sin x\cos x-\sin^2 x$
   b) $\tan^2 x-\tan x$
   c) $\csc x\sec x-\sec x\tan x$
   d) $\tan^2 x-\sec^2 x$
   e) $\cos^2 x-\sin^2 x$
   f) $\sin^2 x-\sin x$
   g) $2\sin^2 x+\sin x-1$
   h) $2\cos^2 x-7\cos x+3$
   i) $2\sin^2 x-1-\sin x$
   j) $2\cos^2 x+5\cos x-3$
   k) $\tan^2 x-1$
   l) $2\sin^2 x+1+3\sin x$
   m) $\cos x=2\sin x\cos x$

2. Use a Pythagorean identity to convert one of the trigonometric functions to the same functions, then factor. Set factors equal to zero and stop.
   a) $2\cos^2 x+3\sin x-3=0$
   b) $\sin^2 x-1=\cos^2 x$
   c) $1-2\cos^2 x=-\sin x$

Solutions:
1.a) $\sin x(\cos x-\sin x)$ b) $\tan x(\tan x-1)$ c) $\sec x(\csc x-\tan x)$
   d) $(\tan x-\sec x)(\tan x+\sec x)$ e) $(\cos x-\sin x)(\cos x+\sin x)$ f) $\sin x(\sin x-1)$
   g) $(\sin x+1)(2\sin x-1)$ h) $(\cos x-3)(2\cos x-1)$ i) $(\sin x-1)(2\sin x+1)$
   j) $(\cos x+3)(2\cos x-1)$ k) $(\tan x-1)(\tan x+1)$ l) $(\sin x+1)(2\sin x+1)$
   n) $0=\cos x(2\sin x-1)$

2. a) $(\sin x-1)(2\sin x-1)=0$ b) $2(\sin x-1)(\sin x+1)=0$ c) $(\sin x+1)(2\sin x-1)=0$

Identities and Formulas:

1) Reciprocal: $\csc x=$ $\sec x=$ $\cot x=$

$\csc^2 x=$ $\sec^2 x=$ $\cot^2 x=$

2) Quotient: $\tan x=$ $\cot x=$

$\tan^2 x=$ $\cot^2 x=$

3) Pythagorean : $\sin^2 x+\cos^2 x=$ $1+\tan^2 x=$ $1+\cot^2 x=$

4) Addition and Subtraction

$\sin(x+y)=$ $\sin(x-y)=$
$\cos(x+y)=$ $\cos(x-y)=$
$\tan(x+y)=$ $\tan(x-y)=$

5) Double Angle : $\sin 2x=$ $\tan 2x=$

$\cos 2x=$ $\cos 2x=$ $\cos 2x=$

6) Related Angle :

$\sin(\pi-x)=$ $\cos(\pi-x)=$ $\tan(\pi-x)=$
$\sin(\pi+x)=$ $\cos(\pi+x)=$ $\tan(\pi+x)=$
$\sin(2\pi-x)=$ $\cos(2\pi-x)=$ $\tan(2\pi-x)=$
$\sin(-x)=$ $\cos(-x)=$ $\tan(-x)=$

7) Co-Related Angle:

QI $\sin(\frac{\pi}{2}-x)=$ $\cos(\frac{\pi}{2}-x)=$ $\tan(\frac{\pi}{2}-x)=$

QII $\sin(\frac{\pi}{2}+x)=$ $\cos(\frac{\pi}{2}+x)=$ $\tan(\frac{\pi}{2}+x)=$

QIII $\sin(\frac{3\pi}{2}-x)=$ $\cos(\frac{3\pi}{2}-x)=$ $\tan(\frac{3\pi}{2}-x)=$

QIV $\sin(\frac{3\pi}{2}+x)=$ $\cos(\frac{3\pi}{2}+x)=$ $\tan(\frac{3\pi}{2}+x)=$

Co-functions:
$\sin x \leftrightarrow \cos x$ *and* $\csc x \leftrightarrow \sec x$ *and* $\cot x \leftrightarrow \tan x$

4.3 and 4.4 Trig Identities
PROVE IT!!!!

1. $\sin(a+b)+\sin(a-b)=2\sin a\cos b$
2. $\sin x\tan x = \sec - \cos x$
3. $\sin(p+q)\sin(p-q)=\sin^2 p-\sin^2 q$
4. $\csc^2 x+\sec^2 x=\csc^2 x\sec^2 x$
5. $\dfrac{\cos(x-y)}{\cos x\sin y}=\tan x+\cot y$
6. $\cos^2 x\cos^2 y+\sin^2 x\sin^2 y+\sin^2 x\cos^2 y+\sin^2 y\cos^2 x=1$
7. $2\cos x\cos y=\cos(x+y)+\cos(x-y)$
8. $\sec^2 x-\sec^2 y=\tan^2 x-\tan^2 y$
9. $\cot\left(\dfrac{5\pi}{4}+x\right)=-\dfrac{\tan x-1}{\tan x+1}$
10. $\dfrac{\tan x+\tan y}{\cot x+\cot y}=\tan x\tan y$
11. $\sec^6 x-\tan^6 x=1+3\tan^2 x\sec^2 x$
12. $(\sec x-\cos x)(\csc x-\sin x)=\dfrac{\tan x}{t+\tan^2 x}$
13. $\dfrac{\cos(x-y)-\cos(x+y)}{\cos(x-y)+\cos(x+y)}=\tan x\tan y$
14. $\cos^6 x+\sin^6 x=1-3\sin^2 x+3\sin^4 x$

The following identities require an understanding of double identity formulas.

15. $\dfrac{\sin^3 x-\cos^3 x}{\sin x-\cos x}=\dfrac{2+\sin 2x}{2}$
16. $\sin^2 x=\dfrac{1-\cos 2x}{2}$
17. $\cos^4 x-\sin^4 x=1-2\sin^2 x$
18. $\cot a+\tan a=2\csc 2a$
19. $\sin 4x=4\cos x\sin x(2\cos^2 x-1)$
20. $\dfrac{\sin x+\sin 2x+\sin 3x}{\cos x+\cos 2x+\cos 3x}=\tan 2x$

(4.3 & 4.4)

B

The following identities involve the reciprocal, quotient, and Pythagorean relationships. Prove each one.

1. $\sin x\tan x=\sec x-\cos x$
2. $\cos^4 x-\sin^4 x=1-2\sin^2 x$
3. $\csc^2 x+\sec^2 x=\csc^2 x\sec^2 x$
4. $\cos^2 x\cos^2 y+\sin^2 x\sin^2 y+\sin^2 x\cos^2 y+\sin^2 y\cos^2 x=1$
5. $\sec^2 x-\sec^2 y=\tan^2 x-\tan^2 y$
6. $\dfrac{\tan x+\tan y}{\cot x+\cot y}=(\tan x)(\tan y)$
7. $(\sec x-\cos x)(\csc x-\sin x)=\dfrac{\tan x}{1+\tan^2 x}$
8. $\cos^6 x+\sin^6 x=1-3\sin^2 x+3\sin^4 x$
9. $\sec^6 x-\tan^6 x=1+3\tan^2 x\sec^2 x$

The following involve the addition and subtraction formulas.

10. $1+\cot x\tan y=\dfrac{\sin(x+y)}{\sin x\cos y}$
11. $\cos(x+y)\cos y+\sin(x+y)\sin y=\cos x$
12. $\sin x-\tan y\cos x=\dfrac{\sin(x-y)}{\cos y}$
13. $\cos\left(\dfrac{3\pi}{4}+x\right)+\sin\left(\dfrac{3\pi}{4}-x\right)=0$
14. $\dfrac{\tan\left(\frac{\pi}{4}+x\right)-\tan\left(\frac{\pi}{4}-x\right)}{\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)}=2\sin x\cos x$
15. $\sin(x+y)\sin(x-y)=\cos^2 y-\cos^2 x$
16. $\tan(x+y)\tan(x-y)=\dfrac{\sin^2 x-\sin^2 y}{\cos^2 x-\sin^2 y}$
17. $\dfrac{\tan(x-y)+\tan y}{1-\tan(x-y)\tan y}=\tan x$
18. $\sin 5x=\sin x(\cos^2 2x-\sin^2 2x)+2\cos x\cos 2x\sin 2x$

The following involve related and co-related angles.

19. $\sin\left(\dfrac{\pi}{2}-x\right)\cot\left(\dfrac{\pi}{2}+x\right)=-\sin x$
20. $\cos(-x)+\cos(\pi-x)=\cos(\pi+x)+\cos x$
21. $\dfrac{\sin(\pi-x)}{\tan(\pi+x)}\dfrac{\cot\left(\frac{\pi}{2}-x\right)}{\tan\left(\frac{\pi}{2}+x\right)}\dfrac{\cos(2\pi-x)}{\sin(-x)}=\sin x$
22. $\dfrac{\sin(-x)}{\sin(\pi+x)}-\dfrac{\tan\left(\frac{\pi}{2}+x\right)}{\cot x}+\dfrac{\cos x}{\sin\left(\frac{\pi}{2}+x\right)}=3$
23. $\dfrac{\csc(\pi-x)}{\sec(\pi+x)}\dfrac{\cos(-x)}{\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$
24. $\dfrac{\cos\left(\frac{\pi}{2}+x\right)\sec(-x)\tan(\pi-x)}{\sec(2\pi+x)\sin(\pi+x)\cot\left(\frac{\pi}{2}-x\right)}=-1$
25. $\dfrac{\sin(\pi-x)\cos(\pi+x)\tan(2\pi-x)}{\sec\left(\frac{\pi}{2}+x\right)\csc\left(\frac{3\pi}{2}-x\right)\cot\left(\frac{3\pi}{2}+x\right)}=\sin^4 x-\sin^2 x$

The following involve the double angle formulas.

26. $\dfrac{\sin 2x}{1+\cos 2x}=\tan x$
27. $\dfrac{1+\cos x}{\sin x}=\cot\dfrac{x}{2}$
28. $2\csc 2x=\sec x\csc x$
29. $2\cot 2x=\cot x-\tan x$
30. $\dfrac{\cos 2x}{1+\sin 2x}=\tan\left(\dfrac{\pi}{4}-x\right)$
31. $\dfrac{\cos x-\sin x}{\cos x+\sin x}=\sec 2x-\tan 2x$
32. $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=\tan x$
33. $\cos^6 x-\sin^6 x=\cos 2x\left(1-\frac{1}{4}\sin^2 2x\right)$
34. $4(\cos^6 x+\sin^6 x)=1+3\cos^2 2x$
35. $\sec x-\tan x=\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$
36. $\dfrac{\sin 2x}{1+\cos 2x}\dfrac{\cos x}{1+\cos x}=\tan\dfrac{x}{2}$