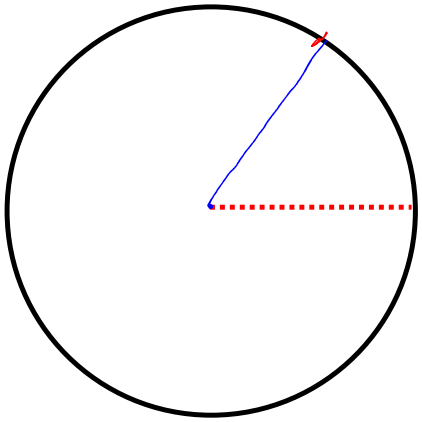
Unit 3 - Trigonometric Functions

What are Radians?

- Radians are another way to measure an angle!

Oct 3-12:01 PM

Draw a circle with a radius!



Take a piece of string the same length as the radius and bend it around the circle.

Tada! A radian!

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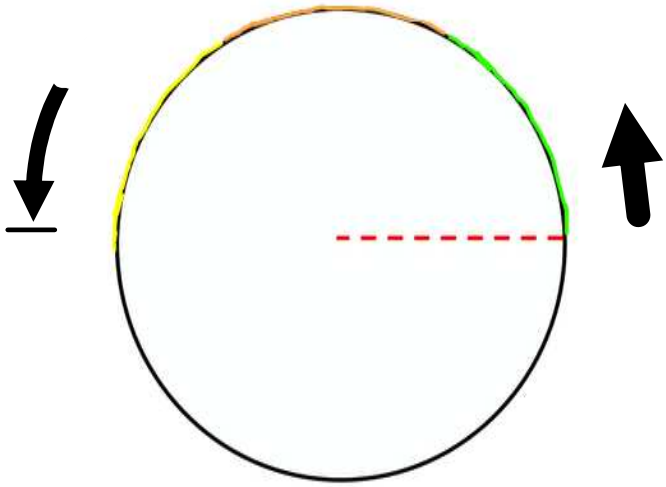
About how many Radians does it take to get half way around the circle?



~3 ?

~3 radians same as half the circumference?

Would you be ok with

$3.14r = \frac{C}{2}$

$\pi r = \frac{C}{2}$

$2\pi r = C$ ← cool!

Ok... so $\frac{1}{2}$ Circumference

is π radians (~3.14)

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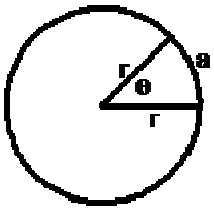
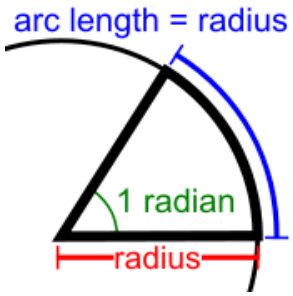
A convenient way to measure some angles, such as central angles, is radian measure, the ratio of the arc length to the radius of a circle.

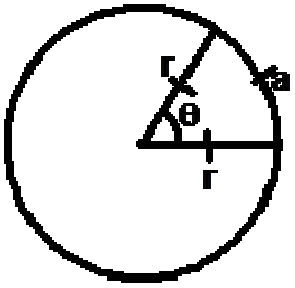


On a circle of radius r, the radian measure of a central angle θ that intercepts an arc of length a is given by

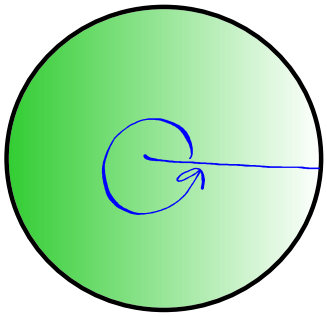
$$\text{radian measure} = \theta = \frac{\text{arc length}}{\text{radius}} = \frac{a}{r}$$

1 radian is defined as the angle subtended by an arc length, a, equal to the radius, r since $\theta = \frac{r}{r} = 1$.





What is the arc length of a 360° angle?
→ Circumference of the circle ∴ $2\pi r$
The corresponding angle, in radians is:



Theta →

$$\theta = \frac{a}{r}$$
$$= \frac{2\pi r}{r}$$
$$= 2\pi$$

theta is always measured in radians

360° = 2π radians
or
180° = π radians

∴ 1 radian = $\frac{180°}{\pi}$ and $1° = \frac{\pi}{180}$ radians

Ex. 1 Convert 60° to radians.

$$60° = 60° \times \frac{\pi}{180} \text{ radians}$$
$$= \frac{60\pi}{180_3}$$
$$= \frac{\pi}{3} \text{ radians}$$

Convert 90° to radians

$$90° \frac{\pi}{180°}$$
$$= \frac{\pi}{2}$$

Ex. 2 Convert $\frac{5\pi}{4}$ to degrees.

$\frac{5\pi}{4}$ radians $= \frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right)$ ($180^\circ \to 45^\circ$)

$= 5(45^\circ)$

$= 225^\circ$

radians → degrees

$\times \frac{180^\circ}{\pi}$

degrees → radians

$\times \frac{\pi}{180^\circ}$

Try:

a) Convert 30° to radians

$30^\circ\left(\frac{\pi}{180^\circ}\right)$ ($30 \to 1$, $180 \to 6$)

$= \frac{\pi}{6}$

b) Convert $\frac{7\pi}{6}$ to degrees.

$\frac{7\pi}{6}\left(\frac{180^\circ}{\pi}\right)$ ($6 \to 1$, $180 \to 30$)

$= 210^\circ$

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Ex. 3 Two cities are separated by 120° central angle (from the centre of the earth). Determine the arc distance between the cities.

? Radians for this

r of the Earth?

$r \doteq 6378$ km

$\theta = 120^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{2\pi}{3}$

$\theta = \frac{a}{r}$

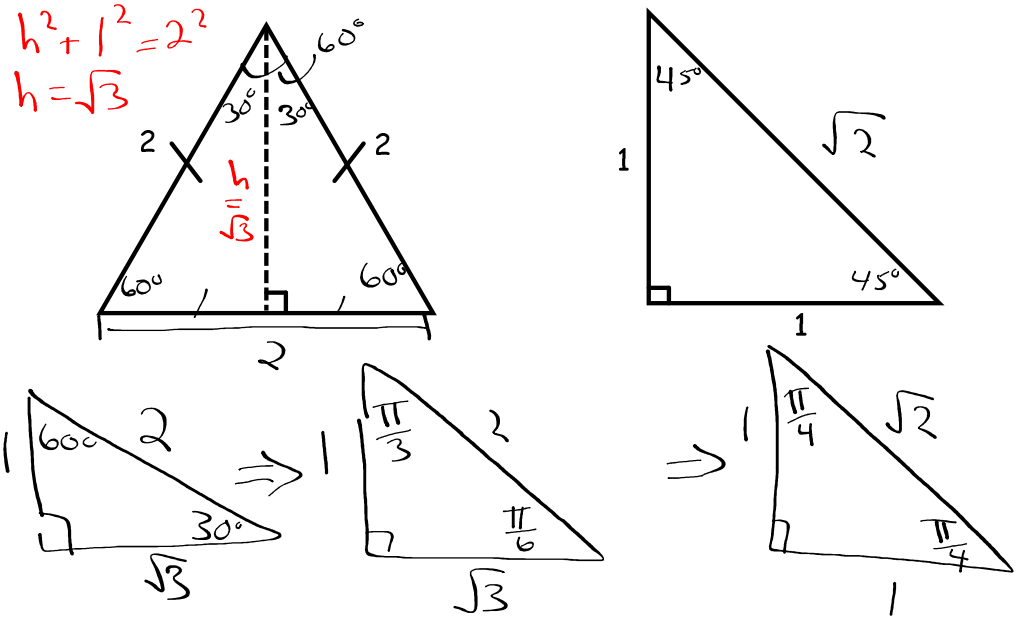
$a = \theta r$

$= \frac{2\pi}{3}(6378)$

$= 4252\pi$ (exact)

$\doteq 13358.1$ km (approx. Because of π)

Recall - special triangles:



Ex. 4 Determine the exact value of each trigonometric ratio using special triangles then check your answers using a calculator.

SOH CAH TOA

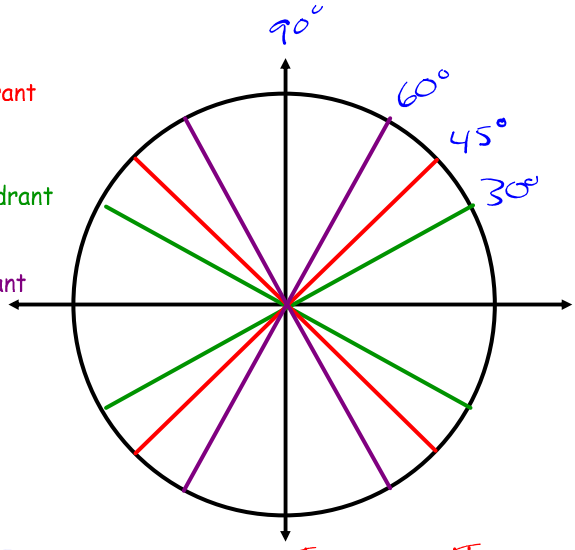
a) $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

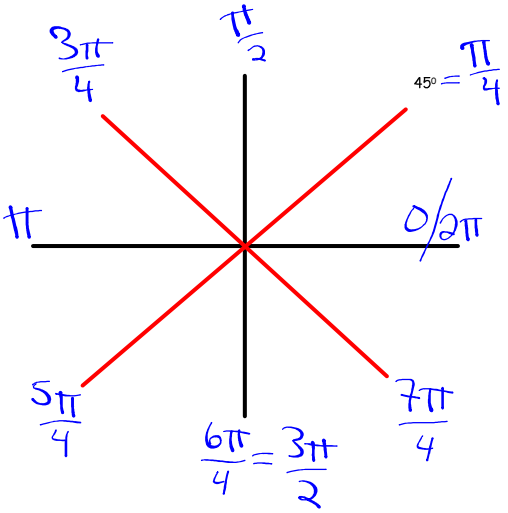
b) $\csc\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{1}$

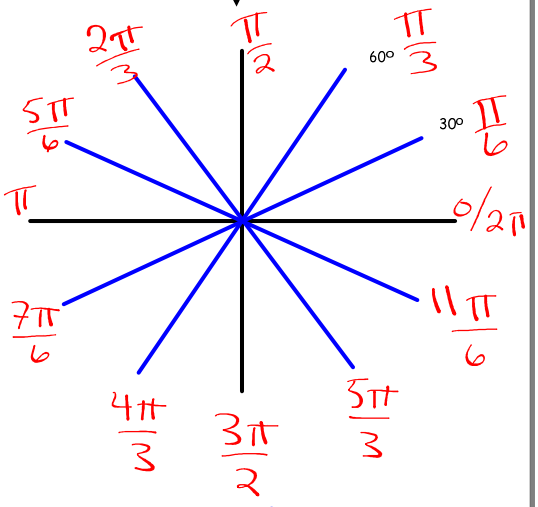
$= \sqrt{2}$

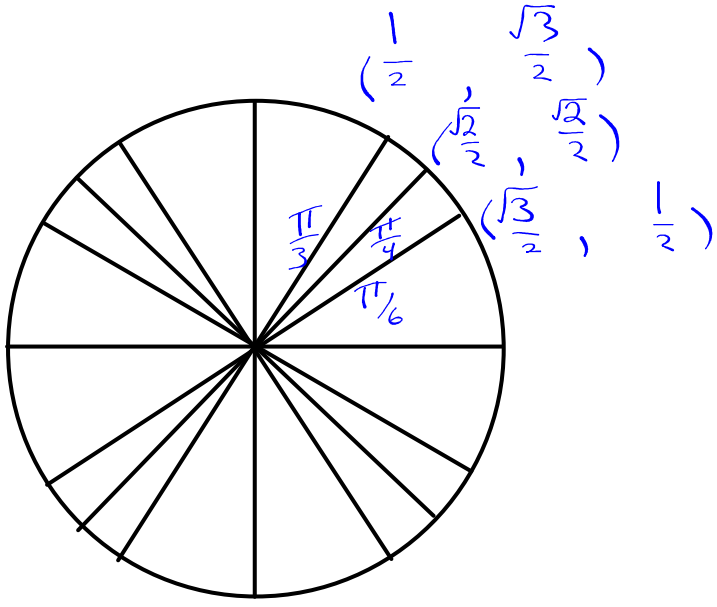
Creating the Unit Circle:

- start with a circle where r=1
- consider the terminal arm in each quadrant where the related angle is 45°
- consider the terminal arm in each quadrant where the related angle is 30°
- consider the terminal arm in each quadrant where the related angle is 60°









Oct 5-1:09 PM

Homework:
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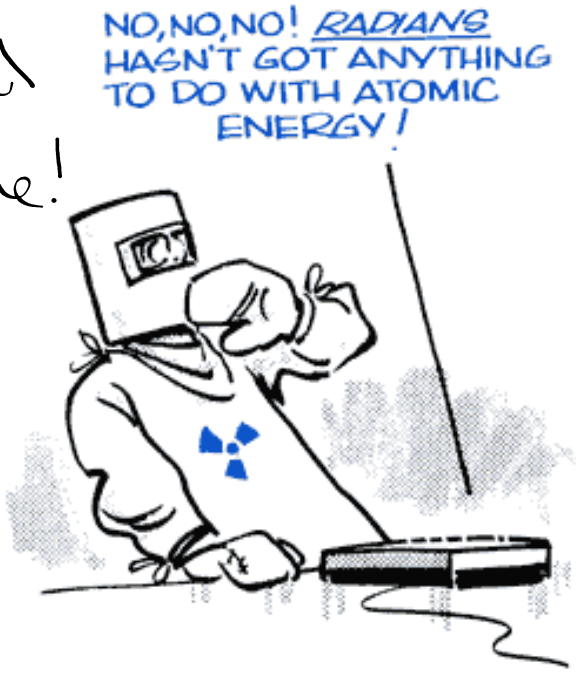
AND a beautiful unit circle!



Nov 2-9:11 AM