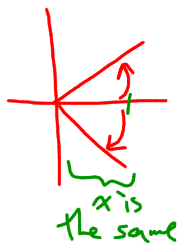


4.1 B Equivalent Trigonometric Functions

Recall

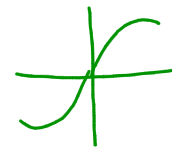
- cosine: even

$$\cos(-\theta) = \cos \theta$$



- sine & tangent: odd

$$\begin{aligned} \sin(-\theta) &= -\sin \theta \\ \tan(-\theta) &= -\tan \theta \end{aligned}$$



Example 1: Simplify

Related! Q2

$$\text{a) } \cos(\pi - x) = -\cos x$$

CoRelated! Q2

$$\text{b) } \sec\left(\frac{\pi}{2} + x\right) = -\csc x$$

CoRelated! Q3

$$\text{c) } \cot\left(\frac{3\pi}{2} - x\right) = \tan x$$

Related Q2

$$\begin{aligned} \text{d) } \cos(x - \pi) &= \cos(-\pi + x) \\ &= \cos[-1(\pi - x)] \\ &= \cos(\pi - x) \\ &= -\cos x \end{aligned}$$

Even!
 $\cos(-\theta) = \cos \theta$
 $\cos \theta = \cos(-\theta)$

CoRelated Q4

$$\begin{aligned} \text{e) } \tan\left(-x - \frac{3\pi}{2}\right) &= \tan\left[-\left(\frac{3\pi}{2} + x\right)\right] \\ &= -\tan\left(\frac{3\pi}{2} + x\right) \\ &= -(-\cot x) \\ &= \cot x \end{aligned}$$

ODD!
 $\tan(-\theta) = -\tan \theta$

CoRelated Q2

$$\begin{aligned} \text{f) } \sin\left(-x - \frac{\pi}{2}\right) &= \sin\left[-\left(\frac{\pi}{2} + x\right)\right] \\ &= -\sin\left(\frac{\pi}{2} + x\right) \\ &= -\cos x \end{aligned}$$

g) $\cos\left(\frac{\pi}{2} + x\right) \cdot \tan\left(\frac{3\pi}{2} - x\right)$

$$\begin{aligned} &= -\sin x \cdot \cot x \\ &= -\sin x \cdot \frac{\cos x}{\sin x} \\ &= -\cos x \end{aligned}$$

h) $\frac{\sin(\pi + x) \sin\left(\frac{3\pi}{2} - x\right)}{\cos\left(\frac{3\pi}{2} + x\right)}$

$$\begin{aligned} &= \frac{-\sin x \cdot (-\cos x)}{\sin x} \\ &= \cos x \end{aligned}$$

Ex: 2. Write the following angles with a denominator of 12

Q1 $\frac{\pi}{6} = \frac{2\pi}{12}$ $\frac{\pi}{4} = \frac{3\pi}{12}$ $\frac{\pi}{3} = \frac{4\pi}{12}$ $\frac{\pi}{2} = \frac{6\pi}{12}$

Q2 $\frac{5\pi}{6} = \frac{10\pi}{12}$ $\frac{3\pi}{4} = \frac{9\pi}{12}$ $\frac{2\pi}{3} = \frac{8\pi}{12}$

Q3 $\frac{7\pi}{6} = \frac{14\pi}{12}$ $\frac{5\pi}{4} = \frac{15\pi}{12}$ $\frac{4\pi}{3} = \frac{16\pi}{12}$ $\frac{3\pi}{2} = \frac{18\pi}{12}$

Q4 $\frac{11\pi}{6} = \frac{22\pi}{12}$ $\frac{7\pi}{4} = \frac{21\pi}{12}$ $\frac{5\pi}{3} = \frac{20\pi}{12}$

Ex. 3 Write each as the sum or difference of two values.
(use special angles, related angle identities or co-related angle identities)

$\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}$ $\pi, 2\pi$ $\frac{\pi}{2}, \frac{3\pi}{2}$ OR MULTIPLES OF

a) $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ b) $\frac{7\pi}{4} = \frac{3\pi}{2} + \frac{\pi}{4}$ c) $\frac{4\pi}{3} = \pi + \frac{\pi}{3}$

Check!
 $\frac{6\pi}{4} + \frac{\pi}{4} = \frac{7\pi}{4}$
✓

OR $2\pi - \frac{\pi}{4}$

d) $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$ e) $\frac{-7\pi}{12} = -\frac{\pi}{3} - \frac{\pi}{4}$ f) $\frac{-5\pi}{12} = \frac{\pi}{4} - \frac{2\pi}{3}$

Check!
 $\frac{9\pi}{12} + \frac{2\pi}{12} = \frac{11\pi}{12}$
✓

OR $\frac{2\pi}{3} - \frac{5\pi}{4}$
OR $\frac{\pi}{6} - \frac{3\pi}{4}$
.....

OR $-\frac{\pi}{6} - \frac{\pi}{4}$
.....

Homework: 4.1 Handout

