4.3 Proving Trigonometric Identities

Do not write down the following puzzles but follow along

Try these trickledown puzzle. There are two simple rules:

1) you may only change one letter at a time
2) Each new line must make a new word.

COAT	PLUG	SLANG
COST	SLUG	SLING
CAST	SLAG	SWING
VAST	STAG	SWINE
VASE	STAY	TWINE

Did you notice that you had to look ahead to what your "goal" word was. When proving identities, you need to look ahead and you can only do one step at a time

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Rules "To prove an identity"

1) The RS and LS should be dealt with seperately
2) Use algebra (simplify, common denominator, factor etc...)
3) Use known identities to transform one side to another
4)Express in terms of sine and cosine (except if both sides are the same trigonometric function.

link

example 1: Prove
a) cotx sinx = cosx

$LS = \cot x \sin x$ $RS = \cos x$ $\therefore LS = RS$ $\therefore$ QED

$= \frac{\cos x}{\cancel{\sin x}} \cancel{\sin x}$

$= \cos x$

b) $(1-\cos^2 x)(\csc x)=\sin x$

$LS = (1-\cos^2 x)(\csc x)$ $RS = \sin x$

$= \sin^2 x \frac{1}{\cancel{\sin x}}$

$\because LS = RS$

$= \sin x$

$\therefore$ QED

Don't write... Read the Hints

Hints for Proving Identities

*Have your formula sheet on the top of your desk and keep looking at it for hints.
*Work on both sides simultaneously to try to get them looking the same.
* Change as many expressions as you can to the primary trigonometric functions.
*Remember that 1 can be expressed in a different way.
*Factor and cancel as much as you can.
*Multiply top and bottom by a common denominator to help you reduce complex fractions.
*When only sines and cosines are present, multiplying numerator and denominator by a conjugate may help

c) $$\frac{\sin(\pi - x)}{\tan(\pi + x)} \bullet \frac{\cot(\frac{\pi}{2} - x)}{\tan(\frac{\pi}{2} + x)} \bullet \frac{\cos(2\pi - x)}{\sin(-x)} = \sin x$$

$$LS = \frac{\overset{Q2}{\sin(\pi - x)}}{\underset{Q3}{\tan(\pi + x)}} \cdot \frac{\overset{Q1}{\cot(\frac{\pi}{2} - x)}}{\underset{Q2}{\tan(\frac{\pi}{2} + x)}} \cdot \frac{\overset{Q4}{\cos(2\pi - x)}}{\underset{Q4}{\sin(-x)}}$$

$$= \frac{\cancel{\sin x}}{\cancel{\tan x}} \cdot \frac{\cancel{\tan x}}{-\cot x} \cdot \frac{\cos x}{-\cancel{\sin x}}$$

$$= \frac{\cos x}{\cot x}$$

$$= \cos x \div \frac{\cos x}{\sin x}$$

$$= \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}$$

$$= \sin x$$

$\because LS = RS$

$\therefore$ QED

d) $$\frac{\sin(\pi + x)}{\cos(2\pi + x)} + \frac{\sec(\pi - x)}{\csc(2\pi - x)} = 0$$

$$LS = \frac{\overset{Q3}{\sin(\pi + x)}}{\underset{Q1}{\cos(2\pi + x)}} + \frac{\overset{Q2}{\sec(\pi - x)}}{\underset{Q4}{\csc(2\pi - x)}}$$

$RS = 0$

$$= \frac{-\sin x}{\cos x} + \frac{-\frac{1}{\cos x}}{-\frac{1}{\sin x}}$$

$\because LS = RS$

$\therefore$ QED

$$= \frac{-\sin x}{\cos x} + \left[\frac{1}{\cos x} \cdot \frac{\sin x}{1}\right]$$

$$= \frac{-\sin x}{\cos x} + \frac{\sin x}{\cos x}$$

$$= 0$$

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☆ Sometimes you may have to factor when proving identities

Example 2: Factor and state the type of factoring

Types of factoring

common factoring

difference of squares

trinomial

grouping

sum or diff of cubes

COMMON FACTORING

a) sinx+2cosxsinx

$= \sin x(1+2\cos x)$

AKA

$b+2ab$

$= b(1+2a)$

COMMON FACTORING

b) $\sin^2 2x+\sin 2x$

$= \sin 2x(\sin 2x+1)$

AKA

b^2+b

$= b(b+1)$

DIFFERENCE OF SQUARES!

c) $\sin^2 x-\cos^2 x$

$= (\sin x-\cos x)(\sin x+\cos x)$

AKA

b^2-a^2

$= (b+a)(b-a)$

d) $\tan^2 x-1$

$= (\tan x-1)(\tan x+1)$

e) $-10\cos^2 x+9-3\sin x$

$= -10(1-\sin^2 x)+9-3\sin x$

$= -10+10\sin^2 x+9-3\sin x$

$= 10\sin^2 x-3\sin x-1$

let $m=\sin x$

$= 10m^2-3m-1$

$= (2m-1)(5m+1)$

$= (2\sin x-1)(5\sin x+1)$

M -10

A -3

N $\frac{10}{-5}$ $\frac{10}{2}$

$\frac{2}{-1}$ $\frac{5}{1}$