

4.4 Proving Trigonometric Identities Day 2

example: Prove

a) $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$

$$\begin{aligned} \text{RS} &= \cancel{\sin x \cos y} + \cancel{\sin y \cos x} + \sin x \cos y - \cancel{\sin y \cos x} \\ &= 2 \sin x \cos y \end{aligned}$$

$$\therefore \text{LS} = \text{RS}$$

$$\therefore \text{QED}$$

b) $\frac{\cos(x - y)}{\cos x \cdot \sin y} = \cot y + \tan x$

$$\text{LS} = \frac{\cos x \cos y + \sin x \sin y}{\cos x \sin y}$$

$$\text{RS} = \frac{\cos y}{\sin y} + \frac{\sin x}{\cos x}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\sin y \cos x}$$

$$\therefore \text{LS} = \text{RS}$$

$$\therefore \text{QED}$$

c) $\sin(a+b) \cdot \sin(a-b) = \cos^2 b - \cos^2 a$

$$\begin{aligned}
 LS &= (\sin a \cos b + \sin b \cos a)(\sin a \cos b - \sin b \cos a) \\
 &= (ad+bc)(ad-bc) \quad \begin{array}{l} a = \sin a \\ b = \sin b \\ c = \cos a \\ d = \cos b \end{array} \\
 &\quad \text{DIFFERENCE OF SQUARES} \\
 &= \sin^2 a \cos^2 b - \sin^2 b \cos^2 a \\
 &= (1 - \cos^2 a) \cos^2 b - (1 - \cos^2 b) \cos^2 a \\
 &= \cos^2 b - \cancel{\cos^2 a \cos^2 b} - \cos^2 a + \cancel{\cos^2 a \cos^2 b} \\
 &= \cos^2 b - \cos^2 a \quad \therefore LS = RS \\
 &\quad \therefore QED
 \end{aligned}$$

d) $\frac{\tan(a-b) + \tan b}{1 - \tan(a-b)\tan b} = \tan a$

$$\begin{aligned}
 &\frac{\tan x + \tan y}{1 - \tan x \tan y} \\
 &= \tan(x+y) \\
 &= \tan(a-b+b) \\
 &= \tan(a)
 \end{aligned}$$

e) $\cos 2x = 1 - 2\sin^2 x$

$$\begin{aligned}
 LS &= \cos 2x \\
 &= \cos(x+x) \\
 &= \cos x \cos x - \sin x \sin x \\
 &= \cos^2 x - \sin^2 x \\
 &= 1 - \sin^2 x - \sin^2 x \\
 &= 1 - 2\sin^2 x
 \end{aligned}$$

DO NOT USE DOUBLE ANGLE FORMULAS

$\therefore LS = RS$

$\therefore QED$

Are you having an IDENTITY CRISIS?

Homework 4.4

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Handout #10-12

