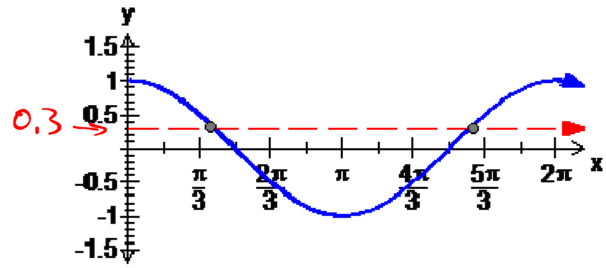
4.5 Solving Linear Trigonometric Equations

Ex. 1 Solve $\cos x = 0.3,\ 0 \le x \le 2\pi$

The solutions to this equation are angles whose cosine equals 0.3.

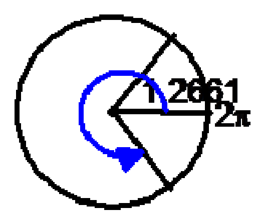
If you think of it in terms of the cosine curve, the solutions are where the curve intersects the line $y = 0.3$.



Using your calculator, you can find one solution to this equation:

$$\cos x = 0.3$$
$$x = \cos^{-1}(0.3)$$
$$x = 1.2661$$

Since cosine is positive in quadrants I and IV, the other solution can be found by subtracting the related acute angle from 2π:



$$x = 2\pi - 1.2661$$
$$= 5.0171$$

$\therefore x = \{1.2661,\ 5.0171\}$

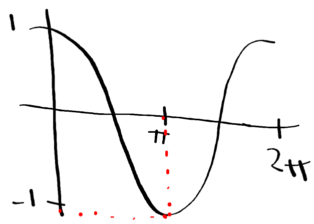
Some Strategies:

- Isolate the trig. function (like you would isolate "x")
- If the equations equals 0, 1 or -1 use the graph or unit circle to get initial solutions
- ALWAYS use exact values when possible (equation equals $\frac{1}{2}$, $\frac{1}{\sqrt{2}}$, $\frac{\sqrt{3}}{2}$, etc.)
  Special Δs, Unit Circle
- Check restrictions for
  a) domain...give ALL solutions in domain
  b) if the domain is unrestricted give a general solution

Ex. 2 Solve for x, $0 \le x \le 2\pi$.

a) $\cos x = -1$

$x = \{\pi\}$

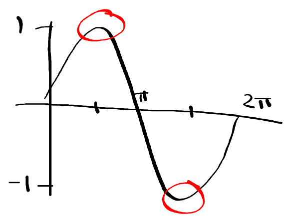


b) $\sin^2 x = 1$

$(\sin x)^2 = 1$ DON'T FORGET!

$\sin x = \pm\sqrt{1}$

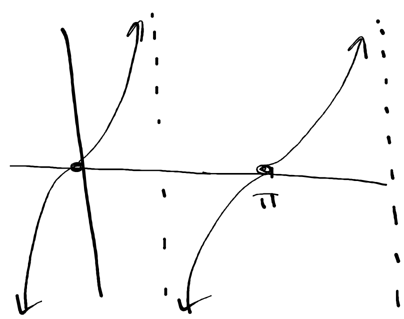
$= \pm 1$



$\therefore x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Ex. 3 Solve $\tan x = 0$ for $x \in R$.

$x = \{0, \pi, 2\pi, \ldots\}$



GENERAL SOLUTION

$x = \{0 + \pi n\}, n \in Z$

= Initial angle + period in $\cdot$ n, $n \in Z$ which it re-occurs

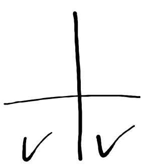
Ex. 4 Use exact values to solve $2\sin x + 1 = 0$ if $0 \le x \le 2\pi$.

$$2\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

Unit Circle/Special Δs

(RA) $x = \frac{\pi}{6}$



Q3

$$x = \pi + \frac{\pi}{6}$$

$$= \frac{7\pi}{6}$$

Q4

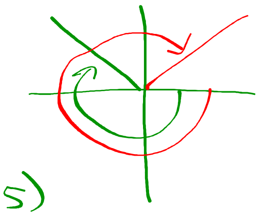
$$x = 2\pi - \frac{\pi}{6}$$

$$= \frac{11\pi}{6}$$

$$\therefore x = \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

Ex. 5 Solve for θ. Use the restrictions given. Round to 2 decimal places.

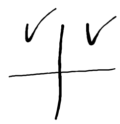
a) $5\sin\theta = 2$, $-2\pi \leq \theta \leq 0$



$\sin\theta = \frac{2}{5}$

(RA) $\theta = \sin^{-1}\left(\frac{2}{5}\right)$

$\doteq 0.4115$



Q2

$\theta = -(\pi + 0.4115)$

$= -\pi - 0.4115$

$\doteq -3.5531$

Q1

$\theta = -(2\pi - 0.4115)$

$= -2\pi + 0.4115$

$\doteq -5.8717$

$\therefore x = \{-3.55, -5.87\}$

b) $3\cos\theta + 1 = 0$, $x \varepsilon R$.

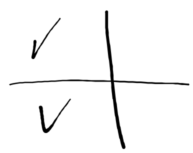
↳ "general solⁿ"

$3\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{3}$

(RA) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

$\doteq 1.2310$



Q2

$\theta = \pi - 1.2310$

$\doteq 1.9106$

Q3

$\theta = \pi + 1.2310$

$\doteq 4.3726$

How often does it re-occur? Every period!

2π

$\therefore \theta = \{1.91 + 2\pi n, 4.37 + 2\pi n\}, n \in Z$

Ex. 6 Solve $\sqrt{2}\sin 2\theta = 1$

Rearranging we get: $\sqrt{2}\sin 2\theta = 1$

$$\sin 2\theta = \frac{1}{\sqrt{2}}$$

What's happening here?

Graph will be horizontally compressed

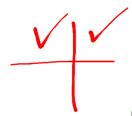
Period $= \pi$

Using the graph of $y = \sin x$ or special triangles, we know that:

Pull

$\sin 2\theta = \frac{1}{\sqrt{2}}$

(RA) $2\theta = \frac{\pi}{4}$



Q1

$2\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{8}$

Q2

Since sine is also positive in quadrant II, the second set of values of θ can be found:

Pull

Q2

$2\theta = \pi - \frac{\pi}{4}$

$2\theta = \frac{3\pi}{4}$

$\theta = \frac{3\pi}{8}$

Period in which it will re-occur

$$\therefore \theta = \left\{ \frac{\pi}{8} + \pi n, \frac{3\pi}{8} + \pi n \right\}, n \in \mathbb{Z}$$

b) If your domain was $0 \le \theta \le 2\pi$

$$\theta = \left\{ \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8} \right\}$$

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Example 7:

The height of a looped-a-looped car is given as $H(t) = 15\cos\left(\frac{\pi}{3}t - \frac{\pi}{6}\right) + 25$

Find when the car is 20 cm in the first 12 seconds. Round to two places.

H is cm

t is seconds

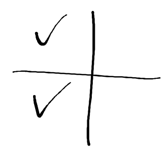
$0 \le t \le 12$

$$20 = 15\cos\left(\frac{\pi}{3}t - \frac{\pi}{6}\right) + 25$$

$$-\frac{5}{15} = \cos\left(\frac{\pi}{3}t - \frac{\pi}{6}\right)$$

(RA) $\frac{\pi}{3}t - \frac{\pi}{6} = \cos^{-1}\left(\frac{5}{15}\right)$

$\doteq 1.2310$



Q2

$\frac{\pi}{3}t - \frac{\pi}{6} = \pi - 1.2310$

$\frac{\pi}{3}t - \frac{\pi}{6} \doteq 1.9110$

$\frac{\pi}{3}t \doteq 2.4342$

$t \doteq 2.3245$

Q3

$\frac{\pi}{3}t - \frac{\pi}{6} = \pi + 1.2310$

$\frac{\pi}{3}t \doteq 4.8962$

$t \doteq 4.6755$

How often re-occurs?

$$per = \frac{2\pi}{\frac{\pi}{3}} = 6$$

$$t = \{2.32, 4.68, 2.32+6, 4.68+6\}$$

$$= \{2.32, 4.68, 8.32, 10.68\}$$