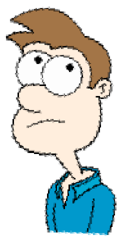
4.6 Solving Quadratic Trigonometric Equations



In order to solve trig equations, we need to find a way of writing the quadratic expression as a product of linear expressions.

ideas:

- change all terms to the same trig function
- remove a common factor
- use identities to make the angle the same in all terms
- remember the domain
- check for restrictions

Ex. 1 Solve for x , $0 \leq x \leq 2\pi$.

$$\cos^2\theta - 1 = 0$$

The LS is a difference of squares:

$$(\cos\theta - 1)(\cos\theta + 1) = 0$$

$\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

$\cos\theta = 1$ $\cos\theta = -1$

$\theta = 0, 2\pi$ $\theta = \pi$

$$\therefore \theta = \{0, \pi, 2\pi\}$$

Ex. 2 Solve for x , $0 \le x \le 2\pi$.

a) $2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ OR $2\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = -\frac{1}{2}$

$x_r = \frac{\pi}{3}$

Q2 $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q3 $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\therefore x = \left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$

b) $1 - 2\cos^2 x = -\sin x$

Need same function

$1 - 2(1 - \sin^2 x) = -\sin x$

$1 - 2 + 2\sin^2 x + \sin x = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

M -2
A 1
N $\frac{2}{2}$ $\frac{2}{-1}$

$2\sin x - 1 = 0$ OR $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$

$\sin x = -1$

$x_r = \frac{\pi}{6}$

Q1 $x = \frac{\pi}{6}$

Q2 $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$x = \frac{3\pi}{2}$

$x = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

c) $3\cos^2 x - 4\cos x + 1 = 0$

M 3
A -4
N $\frac{3}{-3}$ $\frac{3}{-1}$ $\frac{1}{-1}$

$(\cos x - 1)(3\cos x - 1) = 0$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0, 2\pi$

$3\cos x - 1 = 0$

$\cos x = \frac{1}{3}$

$x_r = \cos^{-1}\left(\frac{1}{3}\right)$

$x_r \doteq 1.2310$

Q1 $x = 1.2310$

Q4 $x = 2\pi - 1.2310$

$x \doteq 5.0522$

$\therefore x = \{0, 1.23, 5.05, 2\pi\}$

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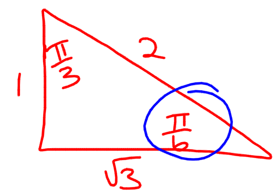
Eg. 3 Solve for x in the indicated domain.

a) $3\tan^2(2x) = 1$ $[0, 2\pi]$



$$\tan^2(2x) = \frac{1}{3}$$

$$\tan(2x) = \pm\sqrt{\frac{1}{3}}$$

$$= \pm\frac{1}{\sqrt{3}}$$

(RA) $2x = \frac{\pi}{6}$

✓	✓
✓	✓

Q1

$2x = \frac{\pi}{6}$

$x = \frac{\pi}{12}$

Q2

$2x = \pi - \frac{\pi}{6}$

$2x = \frac{5\pi}{6}$

$x = \frac{5\pi}{12}$

Q3

$2x = \pi + \frac{\pi}{6}$

$2x = \frac{7\pi}{6}$

$x = \frac{7\pi}{12}$

Q4

$2x = 2\pi - \frac{\pi}{6}$

$2x = \frac{11\pi}{6}$

$x = \frac{11\pi}{12}$

Need to keep adding a period to each angle until we are outside the required domain!

$per = \frac{\pi}{k}$

$= \frac{\pi}{2}$

$x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ $x = \frac{5\pi}{12} + \frac{\pi}{2} = \frac{11\pi}{12}$ $x = \frac{7\pi}{12} + \frac{\pi}{2} = \frac{13\pi}{12}$ $x = \frac{11\pi}{12} + \frac{\pi}{2} = \frac{17\pi}{12}$

$x = \frac{7\pi}{12} + \frac{\pi}{2} = \frac{13\pi}{12}$ $x = \frac{11\pi}{12} + \frac{\pi}{2} = \frac{17\pi}{12}$ $x = \frac{13\pi}{12} + \frac{\pi}{2} = \frac{19\pi}{12}$ $x = \frac{17\pi}{12} + \frac{\pi}{2} = \frac{23\pi}{12}$

$$\therefore x = \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$$

b) $2\sin^2 x + 5\sin x - 5 = 0$ $[0, 2\pi]$

Can't factor!
Use quadratic formula!

let $m = \sin x$

$$2m^2 + 5m - 5 = 0$$

$a = 2$
$b = 5$
$c = -5$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{65}}{4}$$

$$\sin x = \frac{-5 \pm \sqrt{65}}{4}$$

+'ve

$$\sin x = \frac{-5 - \sqrt{65}}{4} \quad \text{OR} \quad \sin x = \frac{-5 + \sqrt{65}}{4}$$

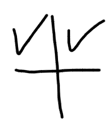
$$x = \sin^{-1}\left(\frac{-5 - \sqrt{65}}{4}\right)$$

$= \text{ERROR}$

NO SOL$^{\underline{N}}$

$$x = \sin^{-1}\left(\frac{-5 + \sqrt{65}}{4}\right)$$

$$x_r \doteq 0.8719$$



Q1: $x = 0.8719$

Q2: $x = \pi - 0.8719$
$\doteq 2.2697$

$$\therefore x = \{0.87, 2.27\}$$

Eg. 4 Determine the solutions to the following for $-2\pi \leq x \leq 2\pi$.

$$\frac{(\cos x)}{(\cos x)}\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}\frac{(1+\sin x)}{(1+\sin x)}=2$$

Add the two terms on the L.S.

FOIL

$$\frac{\cos^2 x+(1+\sin x)^2}{(1+\sin x)(\cos x)}=2$$

=1

$$\frac{\cos^2 x+1+2\sin x+\sin^2 x}{(1+\sin x)(\cos x)}=2$$

$$\frac{2+2\sin x}{(1+\sin x)(\cos x)}=2$$ Factor out 2

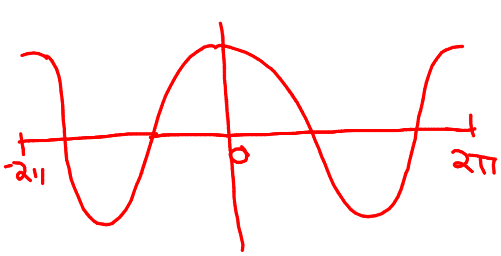
$$\frac{2\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(\cos x)}=2$$

$$\frac{2}{(\cos x)}=2$$

$$\cos x=\frac{2}{2}$$

$$\cos x=1$$

$\therefore x=\{-2\pi, 0, 2\pi\}$



Be sure to check Domain!

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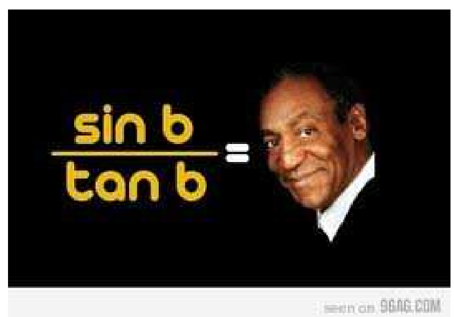

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