MHF 4U Quiz Unit 4 $\frac{\quad}{26} + \frac{\quad}{2}$ Name: Solutions

1. Express each of the following as a function of its <u>co-related acute angle</u>, then evaluate using exact values.

[6]

a) $\sin\frac{2\pi}{3}$ Q2

$= \sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

b) $\tan\frac{\pi}{6}$ Q1

$= \tan\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$

$= \cot\frac{\pi}{3}$

$= \frac{1}{\sqrt{3}}$

c) $\sec\frac{5\pi}{4}$ Q3

$= \sec\left(\frac{3\pi}{2}-\frac{\pi}{4}\right)$

$= -\csc\frac{\pi}{4}$

$= -\sqrt{2}$

2. Express as a single trigonometric function. Simplify only. [4,2]

a) $\cos\left(\frac{3\pi}{2}-x\right)+\sin(\pi+x)-\sin(x-2\pi)+\cos\left(\frac{\pi}{2}+x\right)$

Q3 Q3 Q2

$= -\sin x - \sin x - \sin(-(2\pi-x)) + (-\sin x)$

$= -3\sin x - (-\sin\theta)$

$= -3\sin x + \sin(2\pi - x)$

$= -3\sin x - \sin x$

$= -4\sin x$

b) $\cos\frac{\pi}{6}\cos\frac{\pi}{4}+\sin\frac{\pi}{6}\sin\frac{\pi}{4}$

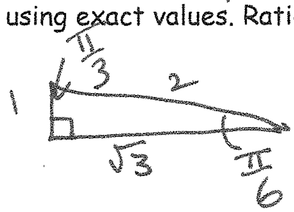
$= \cos\left(\frac{\pi}{6}-\frac{\pi}{4}\right)$

$= \cos\left(\frac{2\pi}{12}-\frac{3\pi}{12}\right)$

$= \cos\left(-\frac{\pi}{12}\right)$

3. Simplify then evaluate using exact values. Rationalize the denominator [4,4]

a) $\tan\frac{11\pi}{12}$



$= \tan\left(\frac{\pi}{6}+\frac{3\pi}{4}\right)$

$= \dfrac{\tan\frac{\pi}{6}+\tan\frac{3\pi}{4}}{1-\tan\frac{\pi}{6}\cdot\tan\frac{3\pi}{4}}$

$= \dfrac{\frac{1}{\sqrt{3}}-\tan\frac{\pi}{4}}{1-\frac{1}{\sqrt{3}}\cdot\left(-\tan\frac{\pi}{4}\right)}$

$= \dfrac{\frac{1}{\sqrt{3}}-1}{1-\frac{1}{\sqrt{3}}(-1)}$

$= \dfrac{\frac{1}{\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$

$= \frac{1-\sqrt{3}}{\cancel{\sqrt{3}}}\cdot\frac{\cancel{\sqrt{3}}}{\sqrt{3}+1}$

$= \frac{1-\sqrt{3}}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{\sqrt{3}+\sqrt{3}-\sqrt{9}+1}{3-1}$

$= \frac{2\sqrt{3}-4}{2}$

$= \sqrt{3}-2$

b) $\sin\frac{\pi}{30}\cos\frac{2\pi}{15}-\cos\frac{\pi}{30}\sin\frac{2\pi}{15}$

$= \sin\left(\frac{\pi}{30}-\frac{2\pi}{15}\right)$

$= \sin\left(\frac{\pi}{30}-\frac{4\pi}{30}\right)$

$= \sin\left(\frac{-3\pi}{30}\right)$

$= \sin\left(-\frac{\pi}{10}\right)$

mistake should have been

$\sin\left(\frac{\pi}{30}\right)\cos\left(\frac{2\pi}{15}\right)+\cos\left(\frac{\pi}{30}\right)\sin\left(\frac{2\pi}{15}\right)$

$= \sin\left(\frac{\pi}{30}+\frac{4\pi}{30}\right)$

$= \sin\left(\frac{5\pi}{30}\right)$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$

4. Prove the following identities [3,3]

a) $\dfrac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y} = \tan(x+y)\tan(x-y)$

b) $\csc^2 y - \csc y \cot y = \dfrac{1}{1+\cos y}$

$$RS = \frac{\tan x + \tan y}{1 - \tan x \tan y} \cdot \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{\tan^2 x - \cancel{\tan x \tan y} + \cancel{\tan x \tan y} - \tan^2 y}{1 - \tan^2 x \tan^2 y}$$

$$= \frac{\tan^2 x - \tan^2 y}{1 - \tan^2 x \tan^2 y}$$

$$= \frac{\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 y}{\cos^2 y}}{1 - \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\sin^2 y}{\cos^2 y}}$$

$$= \frac{\frac{\sin^2 x \cdot \cos^2 y - \sin^2 y \cdot \cos^2 x}{\cos^2 x \cdot \cos^2 y}}{\frac{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}{\cos^2 x \cdot \cos^2 y}}$$

$$= \frac{\sin^2 x \cdot \cos^2 y - \sin^2 y \cdot \cos^2 x}{\cancel{\cos^2 x \cdot \cos^2 y}} \cdot \frac{\cancel{\cos^2 x \cdot \cos^2 y}}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\sin^2 x (1 - \sin^2 y) - \sin^2 y (1 - \sin^2 x)}{\cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y}$$

$$= \frac{\sin^2 x - \cancel{\sin^2 x \sin^2 y} - \sin^2 y + \cancel{\sin^2 y \sin^2 x}}{\cos^2 x - \cancel{\cos^2 x \sin^2 y} - \sin^2 y + \cancel{\cos^2 x \sin^2 y}}$$

$$= \frac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$$

$\because LS = RS$

$\therefore$ QED yuck!

$$LS = \frac{1}{\sin^2 y} - \frac{1}{\sin y} \cdot \frac{\cos y}{\sin y}$$

$$= \frac{1}{\sin^2 y} - \frac{\cos y}{\sin^2 y}$$

$$= \frac{1 - \cos y}{1 - \cos^2 y}$$

$$= \frac{\cancel{1 - \cos y}}{\cancel{(1 - \cos y)}(1 + \cos y)}$$

$$= \frac{1}{1 + \cos y}$$

$\because LS = RS$

$\therefore$ QED