5.4: Laws of Logarithms

Recall: $\log_a a^x = x$

1. Product Law

Given $m = a^x$ and $n = a^y$, determine

i) the logarithmic forms $\log_a m = x$ $\log_a n = y$

ii) $mn = a^x \cdot a^y$
$= a^{x+y}$

iii) $\log_a (mn)$
$= \log_a (a^{x+y})$
$= x+y$
$= \log_a m + \log_a n$

∴ The logarithm of a product is the sum of the logarithms.

$$\log_a (mn) = \log_a m + \log_a n$$

Nov 14-1:50 PM

2. Quotient Law

Given $m = a^x$ and $n = a^y$, determine

i) the logarithmic form $\log_a m = x$ $\log_a n = y$

ii) $\left(\frac{m}{n}\right) = \frac{a^x}{a^y}$

$= a^{x-y}$

iii) $\log_a\left(\frac{m}{n}\right) = \log_a(a^{x-y})$

$= x - y$

$= \log_a m - \log_a n$

∴ The logarithm of a quotient is the difference of the logarithms.

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Nov 14-2:07 PM

3. Power Law

Given $m = a^x$, determine

$\downarrow$

i) the logarithmic form $\log_a m = x$

ii) $m^n = (a^x)^n$

$= a^{xn}$

iii) $\log_a(m^n)$

$= \log_a(a^{xn})$

$= xn$

$= \log_a m \cdot n$ OR $= n \cdot \log_a m$

$\therefore$ The logarithm of a power is equal to the exponent multiplied by the logarithm of the base.

$$\log_a m^p = p \cdot \log_a m$$

Nov 15-6:23 PM

Ex. 1 Rewrite as a single logarithm.

a) $\log_5 12 - \log_5 2$

$= \log_5\left(\frac{12}{2}\right)$

$= \log_5 6$

b) $\log_6 5 + \log_5 4$

CANNOT WRITE as a single log because the bases are DIFFERENT!

c) $\frac{1}{2}\log 7 + \log 1$

$= \log_{10} 7^{\frac{1}{2}} + \log_{10} 1$

$= \log_{10}(7^{\frac{1}{2}} \cdot 1)$

$= \log_{10} 7^{\frac{1}{2}}$

ex: $\log_{10} 3^2 = 2\log_{10} 3$

Nov 15-6:27 PM

Ex. 2 Evaluate each of the following.
Show more than one way, if possible.

a) $\log_6 4 + \log_6 9$

$= \log_6 (4 \cdot 9)$
$= \log_6 36 \quad (36 \sim 6^2)$
$= 2$

b) $\log_2 144 - \log_2 9$

$= \log_2 \frac{144}{9}$
$= \log_2 16 \quad (16 \sim 2^4)$
$= 4$

c) $\log_7 \left(\sqrt[5]{49}\right)$

$= \log_7 49^{\frac{1}{5}}$
$= \log_7 (7^2)^{\frac{1}{5}}$
$= \log_7 7^{\frac{2}{5}}$
$= \frac{2}{5}$

$= \log_7 49^{\frac{1}{5}}$
$= \frac{1}{5} \log_7 49$
$= \frac{1}{5} \cdot 2$
$= \frac{2}{5}$

d) $\log_8 2 + 3\log_8 2 + \frac{1}{2}\log_8 16$

$= \log_8 8^{\frac{1}{3}} + 3\log_8 8^{\frac{1}{3}} + \frac{1}{2}\log_8 2^4$

$= \log_8 8^{\frac{1}{3}} + 3\log_8 8^{\frac{1}{3}} + \frac{1}{2}\log_8 (8^{\frac{1}{3}})^4$

$= \frac{1}{3} + 3 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{4}{3}$

$= \frac{2+6+4}{6}$

$= 2$

$= \log_8 2 + \log_8 2^3 + \log_8 16^{\frac{1}{2}}$
$= \log_8 (2 \cdot 2^3 \cdot 16^{\frac{1}{2}})$
$= \log_8 (2 \cdot 8 \cdot 4)$
$= \log_8 (64)$
$= 2$

e) $\log_5 \left[(25)\left(\sqrt[3]{5}\right)\right]$

$= \log_5 (5^2 \cdot 5^{\frac{1}{3}})$
$= \log_5 (5^{2+\frac{1}{3}})$
$= 2 + \frac{1}{3}$
$= \frac{7}{3}$

$= \log_5 25 + \log_5 \sqrt[3]{5}$
$= \log_5 5^2 + \log_5 5^{\frac{1}{3}}$
$= 2 + \frac{1}{3}$
$= \frac{7}{3}$

Nov 15-6:30 PM

Ex. 3 Express as a single logarithm.

a) $3\log_7 3 - \log_7 8$

$= \log_7 3^3 - \log_7 8$

$= \log_7\left(\frac{27}{8}\right)$

b) $2\log_3 5 + 3\log_3 4$

$= \log_3 5^2 + \log_3 4^3$

$= \log_3(5^2 \cdot 4^3)$

$= \log_3(1600)$

*Remember to do the power first!

c) $2\log 7 + \log 8 - 3\log 4$

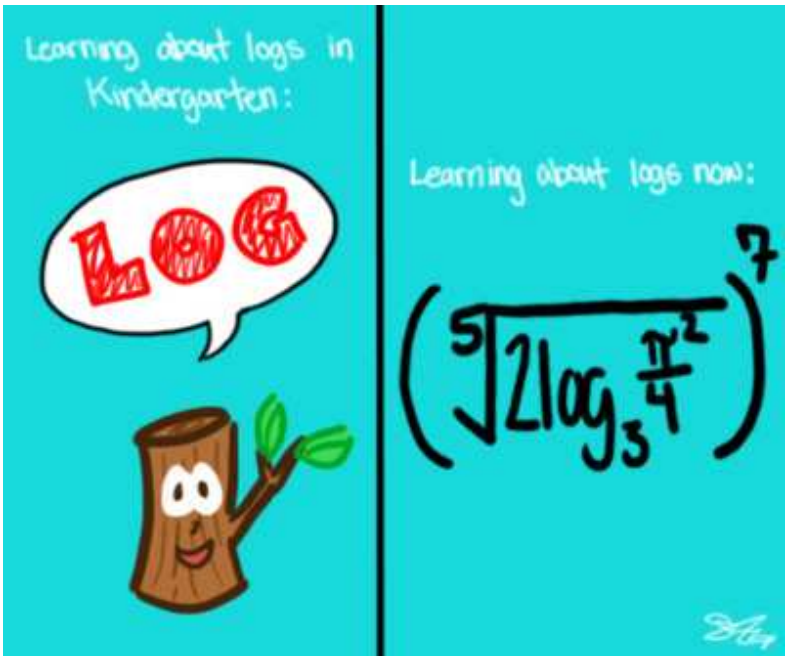
$= \log_{10} 7^2 + \log_{10} 8 - \log_{10} 4^3$

$= \log_{10}\left(\frac{49 \cdot 8}{64}\right)$

$= \log_{10}\left(\frac{49}{8}\right)$

Nov 15-6:48 PM

Homework
Page 475 #2abf, 3ad, 4aef, 6f, 9ef, 11cf, 17



Nov 15-6:31 PM