5.5 Solving Exponential Equations

Some exponential equations can be solved by writing both sides as powers with the same base.

Ex. 1 Solve the following

a) $2^{7x-9} = 8^4$

$2^{7x-9} = (2^3)^4$

$2^{7x-9} = 2^{12}$

$\therefore 7x-9 = 12$

$7x = 21$

$x = 3$

b) $5^{4x-1} = \frac{1}{125}$

$5^{4x-1} = 5^{-3}$

$\therefore 4x-1 = -3$

$4x = -2$

$x = -\frac{1}{2}$

c) $36^{2x+4} = \sqrt{1296^x}$

$36^{2x+4} = (1296^x)^{\frac{1}{2}}$

$36^{2x+4} = (36^2)^{\frac{x}{2}}$

$36^{2x+4} = 36^x$

$\therefore 2x+4 = x$

$x = -4$

★ Isolate the power BEFORE getting a common base!

d) $4^{3x-1} - 8 = 1016$

$4^{3x-1} = 1024$

$4^{3x-1} = 4^5$

$\therefore 3x-1 = 5$

$3x = 6$

$x = 2$

e) $3^{x+1} + 3^x = 324$

$3^x(3^1+1) = 324$

$3^x = \frac{324}{4}$

$3^x = 81$

$3^x = 3^4$

$\therefore x = 4$

$3^{x+1} = 3^x \cdot 3^1$

f) $3 = 768\left(\frac{1}{2}\right)^{\frac{x}{1916}}$

$\frac{3}{768} = \left(\frac{1}{2}\right)^{\frac{x}{1916}}$

$\frac{1}{256} = \left(\frac{1}{2}\right)^{\frac{x}{1916}}$

$\left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{\frac{x}{1916}}$

$\therefore 8 = \frac{x}{1916}$

$15328 = x$

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Ex. 2 Solve $2^{2x} - 2^x - 12 = 0$

$= 2^x 2^x - 2^x - 12$

$= m \cdot m - m - 12$

$= m^2 - m - 12$

? let $2^x = m$

$0 = m^2 - m - 12$

$0 = (m-4)(m+3)$

$0 = (2^x - 4)(2^x + 3)$

$2^x - 4 = 0$

$2^x = 4$

$2^x = 2^2$

$\therefore x = 2$

$2^x + 3 = 0$

$2^x = -3$

NO SOL'N

$y = 2^x$ will never have negative "y"

Ex. 3 Solve

HOW??? The bases are not the same!?!

Take log (base 10) of each side then use the laws of logarithms to simplify:

a) $2^x = 17$

$\log_{10} 2^x = \log_{10} 17$

$x \cdot \log_{10} 2 = \log_{10} 17$

$x = \dfrac{\log_{10} 17}{\log_{10} 2}$

$\doteq 4.1$

b) $6^{3x} = 4^{2x-3}$

$\log_{10} 6^{3x} = \log_{10} 4^{2x-3}$

$3x \cdot \log_{10} 6 = (2x-3)\log_{10} 4$

$3x \cdot \log_{10} 6 = 2x \log_{10} 4 - 3\log_{10} 4$

$3x \cdot \log_{10} 6 - 2x \log_{10} 4 = -3\log_{10} 4$

$x(3\log_{10} 6 - 2\log_{10} 4) = -3\log_{10} 4$

c.f.

$x = \dfrac{-3\log_{10} 4}{3\log_{10} 6 - 2\log_{10} 4}$

$\doteq -1.6$

Keep exact until the very last step!

...>round to 1 decimal place

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How do you change the base of a logarithm?

Consider the following:

let $\log_a x = b$

write in exponential form

$a^b = x$

take the log of both sides

$b \cdot \log_{10} a = \log_{10} x$

solve for b

$b = \frac{\log_{10} x}{\log_{10} a}$

$$\log_a x = \frac{\log x}{\log a}$$

Change of base FORMULA!

ex $\log_3 81 = \frac{\log_{10} 81}{\log_{10} 3}$

$= 4$

ex $\log_2 \sqrt{2} = \frac{\log_{10} \sqrt{2}}{\log_{10} 2}$

$= 0.5$

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Ex. 4 After 10 days, a bacteria colony has grown from 12 to 130. How long does the colony take to double?

$y \rightarrow$ final, $C \rightarrow$ initial (same units)

$y = 130$

$C = 12$

$a \rightarrow$ rate or factor $\rightarrow 2$

$x \rightarrow$ time

$x = \frac{10}{D}$

$y = C\,a^{x}$

$130 = 12(2)^{\frac{10}{D}}$

$D \approx 2.9$

$\frac{t}{D}$ (doubling time)

$\frac{t}{H}$ (half-life) (same units!)

Ex. 5 An initial investment of \$800 will earn 8% compounded quarterly. How long will it take for the investment to grow to \$3000?

≈ 16.7 years